

Stochastic Inventory Model using Exponential Lead Time Distribution with Price Breaks

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Abstract: This paper deals with the probabilistic nature of demand with exponential lead time distribution function and price breaks. The Inventory policy is to order the quantity y , whenever the inventory drops to reorder level r . The reorder level r is a function of the lead time between placing and receiving an order. The aim of this paper is to find the optimal values of y and r by minimizing the expected total cost for a given price break. A numerical illustration and sensitivity analysis using R-language is given at the end of this paper.

Keywords:

Exponential distribution, Lead time demand, Optimum order quantity, Probabilistic model, Price breaks, Reorder level, Sensitivity analysis

I. INTRODUCTION

Inventory decisions such as when to order and how much to order for different items of consumption or sales are indeed very important. These affect financial performance of a firm through capital being tied up due to lost sales or production. Delivery delays or stock outs caused by such decisions may affect customer satisfaction. There are many inventory models available in the literature to determine economic order quantity. In those classical EOQ models it has been considered that demand is deterministic but in actual practice it is not possible to have a fixed demand. Therefore it is necessary to consider a stochastic demand. In probabilistic Inventory models demand is described by a probability distribution. The developed models categorized broadly under continuous and periodic review situations.

The periodic review model includes both single – period and Multi – period cases. probabilistic inventory models with probabilistic demand and supply are more appropriate in many real situations.

In some cases, EOQ is a bound for the optimal order quantity.

Many probabilistic models have been proposed by several authors by using various demand patterns. Most of the literature dealing with probabilistic inventory models assumes that the demand rate as the probability distribution of the demand rate rather than the exact value of the demand rate itself is known. Nita Shah (1993) has given a model on probabilistic time scheduling for an exponentially decaying inventory when delays in payments are permissible. An EOQ Model for items with Weibull distribution, shortages are trended demand was given by Chakrabarthy. T and Stephen C H, Leung, Zhong Yoa,(2011) have considered Stochastic Inventory model with uniformly distributed demand for ordering or returning policies in single periods and H. Hausman and Thomas L.J (1972) have considered Inventory Control with Probabilistic demand and periodic withdrawals.

In this study we assume that the demand rate is probabilistic in nature with lead time distribution as exponential. The price breaks for the quantity to be ordered is also considered in this model. The objective is to find the optimal order quantity with price breaks and optimum reorder level.

II NOTATIONS OF THE MODEL

d =Expected demand per unit time

k =Setup cost per order

$f(x)$ = pdf of demand x , during lead time is exponential distribution= $\lambda e^{-\lambda x}$

$1/\lambda$ = Expected rate of demand

r =Reorder level

q =Price break quantity

c_1 =Purchasing cost per unit time if $y \leq q$

c_2 =Purchasing cost per unit time if $y > q$
 h =Holding cost per Inventory unit per unit time
 p =Shortage cost per Inventory unit
 y_m =Order quantity

Based on the notations, the elements of the total cost function are determined as

Expected holding cost:

The Average Inventory is the expected holding cost per unit time hI where $I = y_m/2 + r - E\{x\}$

Expected Shortage Cost:

Shortage occurs when $x > r$. Thus the Expected

Shortage quantity per cycle is $S = \int_r^\infty (x - r) f(x) dx$

The Expected Shortage cost per unit time is pS/y_m

Expected Purchasing Cost:

It may be offered at q discount that depends on the size of the order.

Purchasing cost per unit time = $\begin{cases} Dc_1, & y \leq q \\ Dc_2, & y > q \end{cases}$

where $c_1 > c_2$

$$Dc_1 = \frac{c_1 y}{t} = \frac{c_1 y}{(\frac{y}{p})} \quad Dc_2 = \frac{c_2 y}{t} = \frac{c_2 y}{(\frac{y}{p})}$$

Setup cost: The approximate number of orders per unit time is d/y_m ; so that the Setup cost per unit time is kd/y_m

III DESCRIPTION OF THE MODEL

In this model probabilistic nature of demand with the exponential distribution function during lead time is considered. The shortage cost per inventory unit is also considered. In purchasing cost, the price breaks for the quantity to be ordered is also incorporated into the model. The Inventory policy calls for ordering the quantity y whenever the inventory drops to level r . The Reorder level r is a function of the lead time between placing and receiving an order. The total cost function with price breaks involving setup, holding, purchasing, and shortage costs are given by

$TCU_{(y_m, r)}$ is followed by

$$TCU_{1(y_m, r)} = dk/y_m + h[y_m/2 + r - 1/\lambda] + pd/y_m \int_r^\infty (x - r) \lambda e^{-\lambda x} dx + Dc_1 \quad y \leq q$$

$$TCU_{2(y_m, r)} = dk/y_m + h[y_m/2 + r - 1/\lambda] + pd/y_m \int_r^\infty (x - r) \lambda e^{-\lambda x} dx + Dc_2 \quad y > q$$

Solving the integration in the above equations we get

$TCU_{(y_m, r)}$ is given by

$$TCU_{1(y_m, r)} = dk/y_m + h[y_m/2 + r - 1/\lambda] + \frac{pd}{y_m} \frac{e^{-r\lambda}}{\lambda} + Dc_1 \quad y \leq q$$

$$TCU_{2(y_m, r)} = dk/y_m + h[y_m/2 + r - 1/\lambda] + \frac{pd}{y_m} \frac{e^{-r\lambda}}{\lambda} + Dc_2 \quad y > q$$

By differentiating the above equations

$$\frac{\partial TCU_{1(y_m, r)}}{\partial y_m} = \frac{-dk}{y_m^2} + \frac{h}{2} - \frac{pde^{-r\lambda}}{\lambda y_m^2} = 0$$

$$\frac{\partial TCU_{2(y_m, r)}}{\partial r} = h + \left(\frac{pd}{y_m} \right) (e^{-r\lambda}) = 0$$

The optimal values of Total cost unit are,

$$y_m^* = \sqrt{dk - h/2 + pd \frac{e^{-r\lambda}}{\lambda}}$$

$$r^* = 1/\lambda \log \frac{pd}{h y_m^*}$$

y_m^* and r^* cannot be determined directly from the above equations. The following algorithm is used to find solutions. The algorithm is to converge in a finite number of iterations, provided a feasible solution exists.

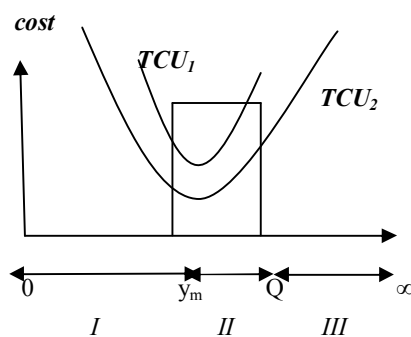
The solution procedure recognizes that the smallest value of y_m^* is $\sqrt{dk - \frac{h}{2} + pd/\lambda}$ which is achieved when $r=0$

The steps of the algorithm are,

Step(i) Use the initial solution of $y_1 = y_m^* = \sqrt{dk - \frac{h}{2} + \frac{pd}{\lambda}}$ by letting $R_0=0$ go to step(ii)

Step(ii) Use y_m^* to determine r from equation (ii). If $r_i = r_{i-1}$ stop; the optimal solution is $y_m^* = y_i$ and $r^* = r_i$ otherwise use r_i in Equation(i) To compute y_m^* , set $i=i+1$ and repeat step (i)

Thus the values of y_m^* and r^* are obtained by using the above algorithm by minimizing the total cost functions. Now using these values and the price break point q , the optimal value of y_m is obtained. The following diagram is considered for the total cost functions with price breaks.



The cost function $TCU(y_m)$ starts on left with $TCU_1(y_m)$ and drops to $TCU_2(y_m)$ at the price break point q , the above figure reveals that the determination of the optimum order quantity y_m^* depends on where the price break point q lies with respect to zones I, II and III delineated by $(0, y_m)$, (y_m, Q) and (Q, ∞) respectively. The value of Q ($> y_m$) is determined from the equation.

$$TCU_2(Q) = TCU_1(y_m, r)$$

The optimal order quantity with price breaks for different zones is determined from the following steps

Step:1

$$\text{Determine } y_m = \sqrt{dk - h/2 + pd \frac{e^{-r\lambda}}{\lambda}}$$

If q is in zone I, then $y^* = y_m$ otherwise go to step 2

Step2: Determine Q from the equation $TCU_2(Q) = TCU_1(y_m, r)$ and define zones II and III. If q is in zone II, $y_m^* = q$. Otherwise q is in zone III and $y^* = y_m$

$$y^* = \begin{cases} y_m, & \text{if } q \text{ is in zones I or III} \\ q, & \text{if } q \text{ is in zone II} \end{cases}$$

IV NUMERICAL ILLUSTRATION

For example let us consider the stochastic inventory system with lead time distribution as exponential with mean $1/\lambda$ in which the values of different parameters are $d=1000$, $h=3$, $p=50$, $k=10$, $q=1000$, $\lambda=1/10$, $c_1=3$, $c_2=2.50$

Using R- Language, output results is as follows

$y_m^* = 116.1466$, $r^* = 49.6631$, $Q^* = 165351.8638$ the optimal reorder level is order if the number of units reaches 50 units and q is in zone II (116.1466, 165351.8638). Therefore the optimal order quantity is $y_m^* = q = 1000$ units

V Sensitivity Analysis:

The table below gives the optimal order quantities for different values of λ as 1/20, 1/30, 1/40, 1/50, 1/60 and for different values of $q = 50, 200, 400, 500, 600$. It is found that the effect the optimal order quantity differs with price break. To check sensitivity of the model we have performed a sensitivity analysis by changing value for the parameter λ and for different values of q . and it is seen that for different parameter values there is a slight variations are given below in table

λ	y_m	R	q	Zones	Optimal order quantity
1/20	348.13	45.182	50	I or III	348.13
1/20	348.13	45.182	200	I or III	348.13
1/20	348.13	45.182	400	II	400
1/20	348.13	45.182	500	II	500
1/20	348.13	45.182	600	II	600
1/30	364.58	66.389	50	I or III	364.5805
1/30	364.58	66.389	200	I or III	364.5805
1/30	364.58	66.389	400	II	400
1/30	364.58	66.389	500	II	500
1/30	364.58	66.389	600	II	600
1/40	381.93	86.659	50	I or III	381.9336
1/40	381.93	86.659	200	I or III	381.9336
1/40	381.93	86.659	400	II	400
1/40	381.93	86.659	500	II	500
1/40	381.93	86.659	600	II	600
1/50	400.15	105.99	50	I or III	400.1515
1/50	400.15	105.99	200	I or III	400.1515
1/50	400.15	105.99	400	II	400
1/50	400.15	105.99	500	II	500
1/50	400.15	105.99	600	II	600
1/60	418.84	124.45	50	I or III	418.8461
1/60	418.84	124.45	200	I or III	418.8461
1/60	418.84	124.45	400	II	400
1/60	418.84	124.45	500	II	500
1/60	418.84	124.45	600	II	600

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