Skolem difference Fibonacci mean labelling of some special class of graphs

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ABSTRACT

The concept of Skolem difference mean labelling was introduced by K. Murugan and A. Subramanian[6]. The concept of Fibonacci labelling was introduced by David W. Bange and Anthony E. Barkauskas[1] in the form Fibonacci graceful. This motivates us to introduce skolem difference Fibonacci mean labelling and is defined as follows: "A graph G with p vertices and q edges is said to have Skolem difference Fibonacci mean labelling if it is possible to label the vertices x∈ V with distinct elements f(x) from the set $\{1,2,\ldots,F_{p+q}\}$ *in such a way that the edge e = uv is labelled with* $\left|\frac{f(u)-f(v)}{2}\right|$ *if* $|f(u)-f(v)|$ *is even and* $\frac{|f(u)-f(v)|+1}{2}$ *if* $|f(u)-f(v)|$ *is odd and the resulting edge labels are distinct and are from* $\{F_1, F_2,\ldots,F_q\}$*. A graph that admits* ***Skolem difference Fibonacci mean labelling*** *is called a Skolem difference Fibonacci mean graph". In this paper, we prove that some special class of graphs are Skolem difference Fibonacci mean graphs.*

AMS Classification 05C78

KEYWORDS

Skolem difference mean labelling, Fibonacci labelling, Skolem difference Fibonacci mean labelling, Fan F_n*,* F_m *@* $2P_n$*, triangular snake graph* TS_n*,* $rP_n \cup sP_m$*,* $\bigcup_{i=2}^{n} P_i$

1. INTRODUCTION

A graph G with p vertices and q edges is said to have Skolem difference Fibonacci mean labelling if it is possible to label the vertices x∈ V with distinct elements f(x) from the set $\{1,2,\ldots,F_{p+q}\}$ in such a way that the edge e = uv is labelled with $\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting edge labels are distinct and are from $\{F_1, F_2,\ldots,F_q\}$. A graph that admits Skolem difference Fibonacci mean labelling is called a Skolem difference Fibonacci mean graph. It was found that standard graphs [7], special class of trees [8], H- class of graphs [9] and path related graphs [10] are Skolem difference Fibonacci mean graphs. The following definitions and notations are used in main results.

2. DEFINITIONS

Definition 2.1.

Let $G_1 (V_1, E_1)$ and $G_2 (V_2, E_2)$ be two graphs. Then their union $G = G_1 \cup G_2$ is a graph with vertex set $V = V_1 \cup V2$ and edge set $E = E_1 \cup E_2$.

Definition 2.2.

The join $G_1 + G_2$ of G_1 and G_2 consists of $G_1 \cup G_2$ and all lines joining V_1 with V_2. The graph $P_n + K_1$ is called a Fan.

Definition 2.3.

G_1 @ G_2 is the one point union of G_1 and G_2. One point union of G_1 and G_2 is obtained by identifying one vertex of G_1 to a vertex of G_2.

Definition 2.3.

A triangular snake is obtained from a path $v_1, v_2,\ldots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for i = 1, 2,..., n-1.

3. RESULTS

3.1 Theorem :

Every fan $F_n = P_n+K_1$ is Skolem difference Fibonacci mean graph if $n \geq 3$.

Proof:

Let G be the graph $F_n = P_n+K_1$.

Let V (G) = $\{v_i, / 0 \leq i \leq n\}$

E (G) = $\{v_0v_i /1 \le i \le n\} \cup \{ v_iv_{i+1}/ 1\le i \le n-1\}$

Then $|V(G)|$ = n+1 and $|E(G)|$ = 2n-1

Let f: V (G) $\rightarrow \{1,2,...,F_{3n}\}$ be defined as follows

$f(v_0) = 1$

$f(v_i) = 2F_{2i-1} + 1, 1\le i \le n$

$f^+(E) = \{f(v_0v_i) /1\le i \le n\} \cup \{ f(v_iv_{i+1})/ 1\le i \le n-1\}$

$= \{f(v_0v_1), f(v_0v_2),..., f(v_0v_n)\} \cup \{f(v_1v_2), f(v_2v_3), ..., f(v_{n-1}v_n)\}$

$$= \left\{ \left|\frac{f(v_0)-f(v_1)}{2}\right|, \left|\frac{f(v_0)-f(v_2)}{2}\right|,..., \left|\frac{f(v_0)-f(v_n)}{2}\right| \right\} \cup \left\{\left|\frac{f(v_1)-f(v_2)}{2}\right|, \left|\frac{f(v_2)-f(v_3)}{2}\right|,..., \left|\frac{f(v_{n-1})-f(v_n)}{2}\right|\right\}$$

$$= \left\{\left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_3-1}{2}\right|,..., \left|\frac{1-2F_{2n-1}-1}{2}\right|\right\} \cup \left\{ \left|\frac{2F_1+1-2F_3-1}{2}\right|, \left|\frac{2F_3+1-2F_5-1}{2}\right|,..., \left|\frac{2F_{2n-3}+1-2F_{2n-1}-1}{2}\right|\right\}$$

$= \{F_1, F_3,..., F_{2n-1}\} \cup \{F_2, F_4,...,F_{2n-2}\}$

$= \{F_1, F_2,...,F_{2n-1}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2,...,F_{2n-1}$.

Hence, the fan $F_n = P_n+K_1$ is skolem difference Fibonacci mean graph if $n \ge 3$.

3.2 Example:

Skolem difference Fibonacci mean labelling of the graph $F_6 = P_6+K_1$ is

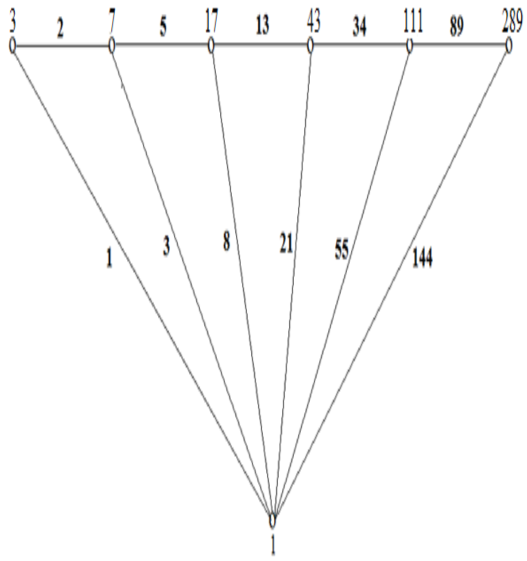

Fig. 1: F_6

3.3 Theorem :

$F_m @ 2P_n$ is Skolem difference Fibonacci mean graph

Proof:

Let G be $F_m @ 2P_n$

Let V (G) = $\{u, v_i, u_j, w_j/1 \le i \le m \text{ and } 1\le j \le n-1\}$

Let E (G) = $\{uv_i/1 \le i \le m\} \cup \{v_iv_{i+1}/ 1\le i \le m-1\} \cup \{v_mu_1, u_ju_{j+1}/1\le j \le n-2\} \cup \{uw_1, w_jw_{j+1}/1\le j \le n-2\}$

Then $|V(G)|$ = m+2n-1 and $|E(G)|$ = 2m+2n-3

Let f: V (G) $\rightarrow \{1,2,...,F_{3m+4n-4}\}$ be defined as follows

$f(u) = 1$

$f(v_i) = 2F_{2i-1} + 1, 1\le i \le m$

$f(u_j) = 2F_{2m+j-1} +f(u_{j-1}), 2\le j \le n-1$

$f(u_1) = 2F_{2m+1} + 1$

$f(w_j) = 2F_{2m+n+j-2} + f(w_{j-1}), 2\le j \le n-1$

$f(w_1) = 2F_{2m+n-1} + 1$

$f^{+}(E) = \{f(uv_i), f(v_m u_1), f(u_j u_{j+1}), f(uw_1), f(w_j w_{j+1}) /1 \le i \le m, \quad 1\le j \le n\text{-}2\} \cup \{f(v_i v_{i+1})/ 1\le i \le m\text{-}1\}$

$= \{f(uv_1), f(uv_2),..., f(uv_m), f(v_m u_1), f(u_1u_2), f(u_2u_3), ..., f(u_{n-1}u_n), f(uw_1), f(w_1w_2), f(w_2w_3), ..., f(w_{n-1}w_n)\} \cup \{f(v_1v_2), f(v_2v_3), ..., f(v_{m-1}v_m)\}$

$$= \left\{\left|\frac{f(u)-f(v_1)}{2}\right|, \left|\frac{f(u)-f(v_2)}{2}\right|,..., \left|\frac{f(u)-f(v_m)}{2}\right|, \left|\frac{f(v_m)-f(u_1)}{2}\right|, \left|\frac{f(u_1)-f(u_2)}{2}\right|, \left|\frac{f(u_2)-f(u_3)}{2}\right|,..., \left|\frac{f(u_{n-1})-f(u_n)}{2}\right|, \left|\frac{f(u)-f(w_1)}{2}\right|, \left|\frac{f(w_1)-f(w_2)}{2}\right|, \left|\frac{f(w_2)-f(w_3)}{2}\right|,..., \left|\frac{f(w_{n-1})-f(w_n)}{2}\right|\right\} \cup \left\{\left|\frac{f(v_1)-f(v_2)}{2}\right|, \left|\frac{f(v_2)-f(v_3)}{2}\right|,..., \left|\frac{f(v_{m-1})-f(v_m)}{2}\right|\right\}$$

$$= \left\{\left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_3-1}{2}\right|,..., \left|\frac{1-2F_{2m-1}-1}{2}\right|, \left|\frac{2F_{2m-1}+1-2F_{2m+1}-1}{2}\right|, \left|\frac{f(u_1)-2F_{2m+1}-f(u_1)}{2}\right|, \left|\frac{f(u_2)-2F_{2m+2}-f(u_2)}{2}\right|,..., \left|\frac{f(u_{n-2})-2F_{2m+n-2}-f(u_{n-2})}{2}\right|, \left|\frac{1-2F_{2m+n-1}-1}{2}\right|, \left|\frac{f(w_1)-2F_{2m+n}-f(w_1)}{2}\right|, \left|\frac{f(w_2)-2F_{2m+n+1}-f(w_2)}{2}\right|,..., \left|\frac{f(w_{n-2})-2F_{2m+2n-3}-f(w_{n-2})}{2}\right|\right\} \cup \left\{\left|\frac{2F_1+1-2F_3-1}{2}\right|, \left|\frac{2F_3+1-2F_5-1}{2}\right|,..., \left|\frac{2F_{2m-3}+1-2F_{2m-1}-1}{2}\right|\right\}$$

$= \{F_1, F_3,..., F_{2m-1}, F_{2m}, F_{2m+1}, F_{2m+2},..., F_{2m+n-2}, F_{2m+n-1}, F_{2m+n}, F_{2m+n+1},..., F_{2m+2n-3}\} \cup \{F_2, F_4,...,F_{2m-2}\}$

$= \{F_1, F_2, F_3, F_4,..., F_{2m-2}, F_{2m-1}, F_{2m}, F_{2m+1}, F_{2m+2},..., F_{2m+n-2}, F_{2m+n-1}, F_{2m+n}, F_{2m+n+1},..., F_{2m+2n-3}\}$

$= \{F_1, F_2,..., F_{2m+2n-3}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2,..., F_{2m+2n-3}$.

Hence, F_m @ $2P_n$ is a Skolem difference Fibonacci mean graph.

3.4 Example:

Skolem difference Fibonacci mean labelling of the graph F_4 @ $2P_4$ is

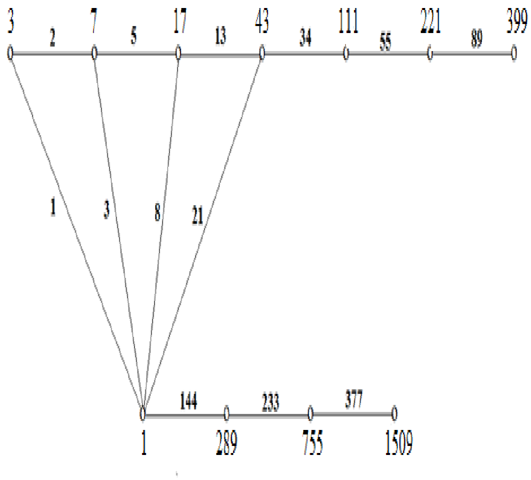

Fig. 2: F_4 @ $2P_4$

3.5 Theorem:

The triangular snake graph TS_n is a Skolem difference Fibonacci mean graph.

Proof:

Let G be TS_n

Let V (G) = $\{v_i, w_j / 1 \le i \le n+1 \text{ and } 1 \le j \le n\}$

Let E (G) = $\{v_iv_{i+1} / 1 \le i \le n\} \cup \{v_jw_j/ 1 \le j \le n\} \cup \{v_jw_{(j-1)}/ \quad 2 \le j \le n+1\}$

Then $|V(G)|$ = 2n+1 and $|E(G)|$ = 3n

Let f: V (G) $\rightarrow \{1, 2,...,F_{5n+1}\}$ be defined as follows

$f(v_1) = 3$, $f(v_2) = 7$, $f(w_1) = 1$

$f(v_{i+1}) = 2F_{3i} + f(v_i), 2 \le i \le n$

$f(w_i) = 2F_{4+3(i-2)} + f(v_i), 2 \le i \le n$

$f^{+}(E) = \{f(v_iv_{i+1}) / 1 \le i \le n\} \cup \{ f(v_jw_j)/1 \le j \le n\} \cup \{f(v_jw_{(j-1)})/ 2 \le j \le n+1\}$

$= \{f(v_1v_2), f(v_2v_3), ..., f(v_nv_{n+1})\} \cup \{f(v_1w_1), f(v_2w_2),...,f(v_nw_n)\} \cup \{f(v_2w_1), f(v_3w_2),...,f(v_{n+1}w_n)\}$

$= \{ \left|\frac{f(v_1)-f(v_2)}{2}\right|, \left|\frac{f(v_2)-f(v_3)}{2}\right|, \ldots, \left|\frac{f(v_n)-f(v_{n+1})}{2}\right| \} \cup \{ \left|\frac{f(v_1)-f(w_1)}{2}\right|, \left|\frac{f(v_2)-f(w_2)}{2}\right|, \ldots, \left|\frac{f(v_n)-f(w_n)}{2}\right| \} \cup \{ \left|\frac{f(v_2)-f(w_1)}{2}\right|, \left|\frac{f(v_3)-f(w_2)}{2}\right|, \ldots, \left|\frac{f(v_{n+1})-f(w_n)}{2}\right| \}$

$= \{ \left|\frac{3-7}{2}\right|, \left|\frac{f(v_2)-2F_6-f(v_2)}{2}\right|, \ldots, \left|\frac{f(v_n)-2F_{3n}-f(v_n)}{2}\right| \} \cup \{ \left|\frac{3-1}{2}\right|, \left|\frac{f(v_2)-2F_4-f(v_2)}{2}\right|, \ldots, \left|\frac{f(v_n)-2F_{3n-2}-f(v_n)}{2}\right| \} \cup \{ \left|\frac{7-1}{2}\right|, \left|\frac{2F_6+f(v_2)-2F_4-f(v_2)}{2}\right|, \ldots, \left|\frac{2F_{3n}+f(v_n)-2F_{3n-2}-f(v_n)}{2}\right| \}$

$= \{2, F_6, \ldots, F_{3n}\} \cup \{1, F_4, \ldots, F_{3n-2}\} \cup \{3, |F_6 - F_4|, \ldots, |F_{3n} - F_{3n-2}|\}$

$= \{F_2, F_6, \ldots, F_{3n}\} \cup \{F_1, F_4, \ldots, F_{3n-2}\} \cup \{F_3, F_5, \ldots, F_{3n-1}\}$

$= \{F_1, F_2, F_3, F_4, F_5, F_6, \ldots, F_{3n-2}, F_{3n-1}, F_{3n}\}$

$= \{F_1, F_2, \ldots, F_{3n}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2, \ldots, F_{3n}$.

Hence, the triangular snake graph TS_n is a Skolem difference Fibonacci mean graph.

3.6 Example:

Skolem difference Fibonacci mean labelling of the graph TS_5 is

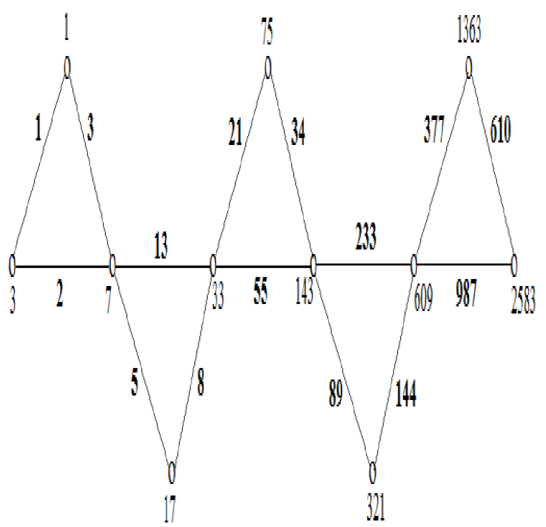

Fig. 3: TS_5

3.7 Theorem:

The graph $rP_n \cup sP_m$ is skolem difference Fibonacci mean for all r, s ≥ 1 and m, n ≥ 2.

Proof:

Let $V(rP_n \cup sP_m) = \{u_{ij} / 1 \le i \le r \text{ and } 1 \le j \le n\} \cup \{v_{ij} / 1 \le i \le s \text{ and } 1 \le j \le m\}$

$E(rP_n \cup sP_m) = \{u_{ij}u_{i(j+1)} / 1 \le i \le r \text{ and } 1 \le j \le n-1\} \cup \{v_{ij}v_{i(j+1)} / 1 \le i \le s \text{ and } 1 \le j \le m-1\}$

Then $|V(rP_n \cup sP_m)| = nr + ms$ and $|E(rP_n \cup sP_m)| = r(n-1) + s(m-1)$

Let $f: V(G) \to \{1, 2, \ldots, F_{2(nr+ms)-r-s}\}$ be defined as follows

$f(u_{1j}) = 2F_{j+1}, 1 \le j \le n$

$f(u_{i1}) = f(u_{(i-1)n})+1, 2 \le i \le r$

$f(u_{ij}) = 2F_{n(i-1)+j-i} + f(u_{i(j-1)}), 2 \le i \le r$ and $2 \le j \le n$

$f(v_{11}) = f(u_{rn})+1$

$f(v_{1j}) = 2F_{r(n-1)+j-1} + f(v_{1(j-1)}), 2 \le j \le m$

$f(v_{i1}) = f(v_{(i-1)m})+1, 2 \le i \le s$

$f(v_{ij}) = 2F_{r(n-1)+m(i-1)+j-i} + f(v_{i(j-1)}), 2 \le i \le s \;\&\; 2 \le j \le m$

$f^+(E) = \{f(u_{ij}\, u_{i(j+1)}) / 1 \le i \le r \text{ and } 1 \le j \le n-1\} \cup \{f(v_{ij}\, v_{i(j+1)}) / 1 \le i \le s \text{ and } 1 \le j \le m-1\}$

$= \{f(u_{11}, u_{12}), f(u_{12}\, u_{13}), \ldots, f(u_{1(n-1)}\, u_{1n}), f(u_{21}\, u_{22}), f(u_{22}\, u_{23}), \ldots f(u_{2(n-1)}\, u_{2n}) \ldots, f(u_{r1}\, u_{r2}), f(u_{r2}\, u_{r3}), \ldots, f(u_{r(n-1)}\, u_{rn})\} \cup \{f(v_{11}\, v_{12}), f(v_{12}\, v_{13}), \ldots, f(v_{1(m-1)}\, v_{1m}), f(v_{21}\, v_{22}), f(v_{22}\, v_{23}), \ldots, f(v_{2(m-1)}\, v_{2m}), \ldots, f(v_{s1}\, v_{s2}), f(v_{s2}\, v_{s3}), \ldots f(v_{s(m-1)}\, v_{sm})\}$

$= \{ \left|\frac{f(u_{11})-f(u_{12})}{2}\right|, \left|\frac{f(u_{12})-f(u_{13})}{2}\right|, \ldots, \left|\frac{f(u_{1(n-1)})-f(u_{1n})}{2}\right|, \left|\frac{f(u_{21})-f(u_{22})}{2}\right|, \left|\frac{f(u_{22})-f(u_{23})}{2}\right|, \ldots, \left|\frac{f(u_{2(n-1)})-f(u_{2n})}{2}\right|, \ldots, \left|\frac{f(u_{r1})-f(u_{r2})}{2}\right|, \left|\frac{f(u_{r2})-f(u_{r3})}{2}\right|, \ldots, \left|\frac{f(u_{r(n-1)})-f(u_{rn})}{2}\right| \} \cup \{ \left|\frac{f(v_{11})-f(v_{12})}{2}\right|, \left|\frac{f(v_{12})-f(v_{13})}{2}\right|, \ldots, \left|\frac{f(v_{1(m-1)})-f(v_{1m})}{2}\right|,$

$$\left|\frac{f(v_{21})-f(v_{22})}{2}\right|, \left|\frac{f(v_{22})-f(v_{23})}{2}\right|,\ldots, \left|\frac{f(v_{2(m-1)})-f(v_{2m})}{2}\right|,\ldots,$$

$$\left|\frac{f(v_{s1})-f(v_{s2})}{2}\right|, \left|\frac{f(v_{s2})-f(v_{s3})}{2}\right|,\ldots, \left|\frac{f(v_{s(m-1)})-f(v_{sm})}{2}\right|\}$$

$f^{+}(E)$ =

$$\{ \left|\frac{2F_2-2F_3}{2}\right|, \left|\frac{2F_3-2F_4}{2}\right|,\ldots, \left|\frac{2F_n-2F_{n+1}}{2}\right|, \left|\frac{f(u_{21})-2F_n-f(u_{21})}{2}\right|, \left|\frac{f(u_{22})-2F_{n+1}-f(u_{22})}{2}\right|,\ldots,$$

$$\left|\frac{f(u_{2(n-1)})-2F_{2n-2}-f(u_{2(n-1)})}{2}\right|,\ldots, \left|\frac{f(u_{r1})-2F_{n(r-1)+2-r}-f(u_{r1})}{2}\right|,\ldots.$$

$$\left|\frac{f(u_{r2})-2F_{n(r-1)+3-r}-f(u_{r2})}{2}\right|,\ldots, \left|\frac{f(u_{r(n-1)})-2F_{n(r-1)+n-r}-f(u_{rn})}{2}\right|\} \cup$$

$$\{\left|\frac{f(v_{11})-2F_{r(n-1)+1}-f(v_{11})}{2}\right|, \left|\frac{f(v_{12})-2F_{r(n-1)+2}-f(v_{12})}{2}\right|,\ldots,$$

$$\left|\frac{f(v_{1(m-1)})-2F_{r(n-1)+m-1}-f(v_{1(m-1)})}{2}\right|, \left|\frac{f(v_{21})-2F_{r(n-1)+m}-f(v_{21})}{2}\right|,$$

$$\left|\frac{f(v_{22})-2F_{r(n-1)+m+1}-f(v_{22})}{2}\right|,\ldots, \left|\frac{f(v_{2(m-1)})-2F_{r(n-1)+m+m-2}-f(v_{2(m-1)})}{2}\right|,\ldots,$$

$$\left|\frac{f(v_{s1})-2F_{r(n-1)+m(s-1)+2-s}-f(v_{s1})}{2}\right|, \left|\frac{f(v_{s2})-2F_{r(n-1)+m(s-1)+3-s}-f(v_{s2})}{2}\right|,\ldots,$$

$$\left|\frac{f(v_{s(m-1)})-2F_{r(n-1)+m(s-1)+m-s}-f(v_{s(m-1)})}{2}\right|\}$$

$f^{+}(E)$ = $\{F_1, F_2, \ldots F_{n-1}, F_n, F_{n+1}, \ldots F_{2n-2}, \ldots F_{n(r-1)+2-r}, F_{n(r-1)+3-r}, \ldots, F_{r(n-1)}\} \cup$

$\{F_{r(n-1)+1}, F_{r(n-1)+2}, \ldots, F_{r(n-1)+m-1}, F_{r(n-1)+m}, F_{r(n-1)+m+1}, \ldots F_{r(n-1)+2m-2}, \ldots, F_{r(n-1)+m(s-1)+2-s}, F_{r(n-1)+m(s-1)+3-s}, \ldots, F_{r(n-1)+s(m-1)}\}$

= $\{F_1, F_2, \ldots, F_{r(n-1)+s(m-1)}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2, \ldots, F_{r(n-1)+s(m-1)}$.

Hence, $rP_n \cup sP_m$ is a skolem difference Fibonacci mean graph for all r, s ≥ 1 and m, n ≥ 2 .

3.8 Example:

Skolem difference Fibonacci mean labelling of the graph $2P_4 \cup 3P_5$ is

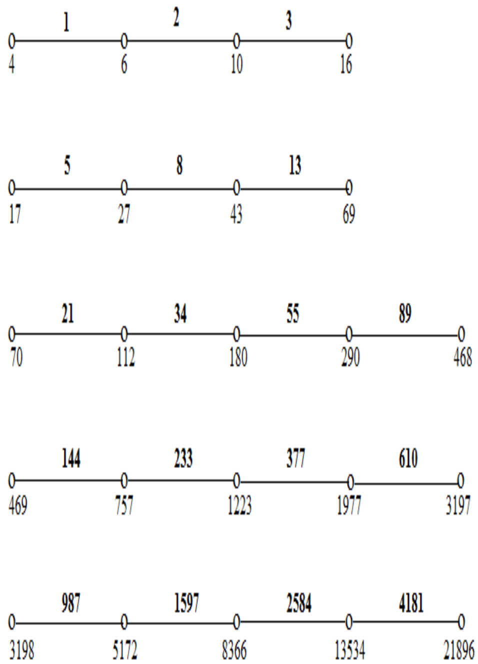

Fig. 4: $2P_4 \cup 3P_5$

3.9 Theorem:

The graph $\cup_{i=2}^{n} P_i$ is skolem difference Fibonacci mean graph for all n ≥ 2.

Proof:

Let G be the graph $\cup_{i=2}^{n} P_i$

Let V (G) = $\{v_{ij} / 2 \le i \le n \text{ and } 1 \le j \le i\}$

E (G) = $\{v_{ij}v_{i(j+1)} / 2 \le i \le n \text{ and } 1 \le j \le i-1\}$

Then $|V(G)| = \frac{n^2+n-2}{2}$ and $|E(G)| = \frac{n^2-n}{2}$

Let f: V (G) → $\{1, 2, \ldots, F_{n^2-1}\}$ be defined as follows

$f(v_{1j}) = 2F_{j+1}$, j = 1, 2

$f(v_{i1}) = f(v_{(i-1)i})+1$, $2 \le i \le n$

$f(v_{ij}) = 2F_{\sum_{k=2}^{i}(k-1)+j-1} + f(v_{i(j-1)})$, $2 \leq i \leq n$ and $2 \leq j \leq (i+1)$

It can be easily verified that the edge set labels are distinct and are $F_1, F_2, \ldots, F_{\frac{n^2-n}{2}}$.

Hence, the graph $\cup_{i=2}^{n} P_i$ is skolem difference Fibonacci mean graph for all $n \geq 2$.

3.10 Example:

Skolem difference Fibonacci mean labelling of the graph $\cup_{i=2}^{5} P_i$ is

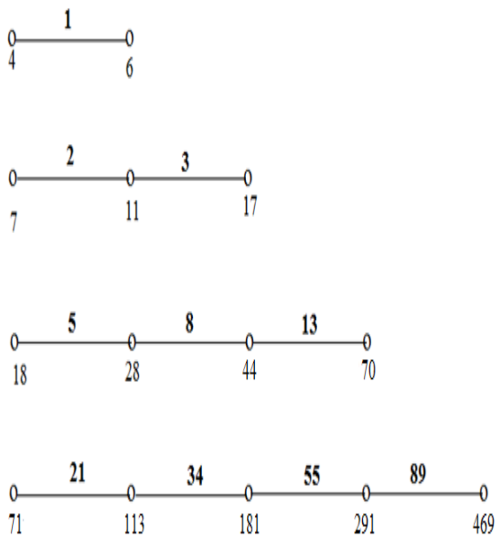

Fig. 5: $\cup_{i=2}^{5} P_i$

REFERENCES

[1] David W. Bange and Anthony E. Barkauskas, "Fibonacci graceful graphs" (1980).

[2] Harary, Graph Theory, Addison Welsley (1969).

[3] J.A.Bondy and U.S.R.Murthy, "*Graph Theory and Applications*" (North-Holland), Newyork, 1976.

[4] J.A.Gallian, A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 18(2014), #DS6.

[5] Kathiresan K. and Amutha S., Fibonacci graceful graphs, Ph.D. Thesis, Madurai Kamaraj University, (October 2006).

[6] K. Murugan and A. Subramanian, Skolem difference mean labelling of H- graphs, *International Journal of Mathematics and Soft Computing*, Vol.1, No. 1, (August 2011), p115-129.

[7] L. Meenakshi sundaram and A. Nagarajan, Skolem difference Fibonacci mean labelling of standard (communicated).

[8] L. Meenakshi sundaram and A. Nagarajan, Skolem difference Fibonacci mean labelling of special class of trees (communicated).

[9] L. Meenakshi sundaram and A. Nagarajan, Skolem difference Fibonacci mean labelling of H- class of graphs (communicated).

[10] L. Meenakshi sundaram and A. Nagarajan, Skolem difference Fibonacci mean labelling of path related graphs (communicated).