Prime Labeling Of Grotzch Graph

S.Gowri [1], S.Ruckmangadhan [1] , V .Ganesan [2]

[1] PG Department Of Mathematics, Sri Vinayaga College of Arts and Science, Ulunderpet.
[2] PG and Research Department Of Mathematics, Thiru Kolanjiappar Government Arts College, vriddhachalam.

Abstract:

A graph G with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers 1, 2, 3, … $|V|$ Such that for each edge xy the labels assigned to x and y are relatively prime. A graph which admits prime labeling is called a prime graph. In this paper, we investigate prime labeling of Grotzsch graph. We also discuss prime labeling in the context of some graph operations namely Fusion, Duplication, and Switching in Grotzsch graph.

Key Words: Grotzsch graph, Prime Labeling, Fusion, Duplication, Switching

1.Introduction:

In this paper, we consider only finite simple undirected graph. The graph G has vertex set V = V(G) and the edge set E =E(G). The set of vertices adjacent to a vertex u of G is denoted by N(u). For notations and terminology, we refer to J.A.Bondy and U.S.R. Murthy [1]. In the present work, G_2 denotes the Grotzsch graph with 11 vertices and 20 edges. Enough literatures available in printed as well as electronics form on different types of graph labeling and more than 1000 research papers have been published so far in past four decades. A current survey of various graphs labeling problem can be found in [7] (Gallian J, 2015)

Following are the common features of any graph labeling problem.

A set of numbers from which vertex labels are assigned.

A rule that assigns value to each edge.

A condition that these values must satisfy.

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by A. Tout (1982) [2]. Many researchers have studied prime graph, for example in H.C. Fu (1994 P 181-186) [5] have proved that path P_n on n vertices is a prime graph.

T.O Dertsky (1991) [4] have proved that the cycle C_n on n vertices is a prime graph. S.M. Lee (1998) [3] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Entringer conjectured that all tress have prime labeling, which is not settled till today. The prime labeling for planner grid is investigated by M. Sundaram (2006) [6]. In [8] S. K. Vaidhya and K. Kanmani have proved that the prime labeling for some cycle related graphs. In [9] S. Meena and K.Vaithilingam investigated Prime Labeling for some Helm related graphs. We will provide brief summary of definitions and other information which are necessary for the present investigations.

Definition: 1.1 If the vertices of the graph are assigned values subject to certain conditions then it is known as (vertex) graph labeling.

Definition: 1.2 Let G = (V, E) be a graph with n vertices. A bijection f: V (G) →{ 1, 2, …,n} is called a Prime labeling if for each edge $e = uv$, gcd (f(u) , f(v)) = 1. A graph which admits prime labeling is called a prime graph.

Definition: 1.3 An independent set of vertices in a graph G is a set of mutually nonadjacent vertices.

Definition: 1.4 Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by fusing (identifying) two vertices u and v by a single vertex x in G_1 such that every edge which was incident with either u (or) v in G now incident with x in G_1 .

Definition: 1.5 Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex $v_k{}'$ with $N(V_K) = N(V_K)$. In other words, a vertex $v_k{}'$ is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to $v_k{}'$.

Definition : 1.6 A vertex switching G_v of a graph G is obtained by taking a vertex v of G, by removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition: 1.7 The Grotzsch graph G_Z is a triangle – free graph with 11 vertices and 20 edges. It contains a star $K_{1,5}$ in which each pendant vertex of $K_{1,5}$ is connected with two rim vertices of the cycle C_5 whose vertex set , $V(G_Z) = \{ w, v_1, v_2, v_3, v_4, v_5; u_1, u_2, u_3, u_4, u_5\}$ and the edge set,
$E(G_Z) =$
$\{ wv_i \ /1 \le i \le 5\} \cup \{ u_i u_{i+1}/1 \le i \le 4, u_5 u_1 \} \cup$
$\{ v_1u_2, v_1v_5, v_2u_1, v_2u_3, v_3u_2, v_3u_4, v_4u_3, v_4u_5, v_5u_4, v_5u_1 \}$

Illustration: 1.8

Figure 1.1 depicts the specimen copy of Grotzsch graph

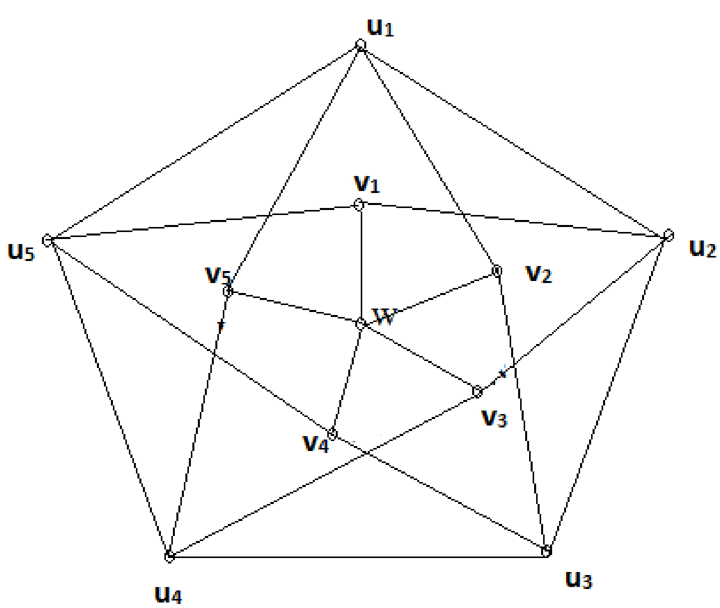

Figure 1.1 The Grotzsch graph

2 Main results:

Theorem: 2.1

The Grotzsch graph admits prime labeling

Proof:

Let G_Z be the Grotzsch graph with 11 vertices and 20 edges

V (G_Z) = { $w, v_1, v_2, v_3, v_4, v_5; u_1, u_2, u_3, u_4, u_5$}

Then | V (G_Z) |= 11

Define a function $f : V(G_Z) \to \{1,2,\ldots,11\}$ defined as follows.

$f(w)$= 1

$f(v_1)$= 2

$f(v_2)$= 4

$f(v_3)$ = 8

$f(v_4)$= 6

$f(v_5)$= 10

and $f(u_i)$=2i +1 , for $1 \leq i \leq 5$

For verify the relative prime of adjacent vertices, we consider the following three type of edges.

i. For $wv_i \in G_2$,$\gcd(f(w), f(v_i)) = \gcd(1, 2i) = 1$

ii. For $u_i u_{i+1} \in G_Z$,$\gcd(f(u_i), f(u_{i+1})) = \gcd(2i+1, 2i+3) = 1$ and $u_5 u_1 \in G_Z$,$\gcd(f(u_5), f(u_1)) = \gcd(11, 3) = 1$

iii. For $v_1 u_2 \in G_Z$,$\gcd(f(v_1), f(u_2)) = \gcd(2, 5) = 1$

$v_1 u_5 \in G_Z$,$\gcd(f(v_1), f(u_5)) = \gcd(2, 11) = 1$

$v_2 u_1 \in G_Z$,$\gcd(f(v_2), f(u_1)) = \gcd(4,3) = 1$

$v_2 u_3 \in G_Z$,$\gcd(f(v_2), f(u_3)) = \gcd(4,7) = 1$

$v_3 u_2 \in G_Z$,$\gcd(f(v_3), f(u_2)) = \gcd(8,5) = 1$

$v_3 u_4 \in G_Z$,$\gcd(f(v_3), f(u_4)) = \gcd(8,9) = 1$

$v_4 u_3 \in G_Z$,$\gcd(f(v_4), f(u_3)) = \gcd(6,7) = 1$

$v_4 u_5 \in G_Z$,$\gcd(f(v_4), f(u_5)) = \gcd(6,11) = 1$

$v_5 u_1 \in G_Z$,$\gcd(f(v_5), f(u_1)) = \gcd(10,3) = 1$

$v_5 u_4 \in G_Z$,$\gcd(f(v_5), f(u_4)) = \gcd(10,9) = 1$

Therefore, f satisfy the condition of prime labeling.

G_Z admits prime labeling.

Hence G_Z is a prime graph □

Illustration : 2.2

Figure 2.2 depicts the prime labeling of Grotzsch graph.

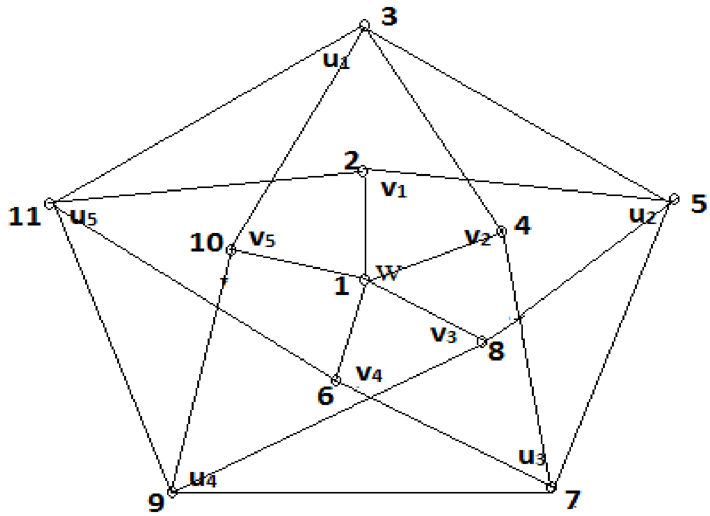

Figure 2.2 the Grotzsch graph and its prime labeling.

Theorem: 2.2

The duplication of any vertex of degree 3 in Grotzsch graph admits prime labeling.

Proof:

Let G_Z be Grotzsch graph with 11 vertices and 20 edges.

Let G be the graph obtained from G_Z by duplicating any vertex of degree 3 in Grotzsch graph.

Without loss of generality, we may take the vertex v_1 to be the duplicating vertex and let $v_1{}'$ be the duplication vertex of v_1 .

Then $|V(G)| = 12$

Define a function $f : V(G) \to \{1,2,\dots,12\}$ defined as follows

Let $f(v_1{}')=12$

$f(v_1)= 2i$ for $1 \le i \le 5$ and $f(w)= 11$

and $f(u_1) = 3$

$f(u_2) = 5$

$f(u_3) = 9$

$f(u_4) = 7$

$f(u_5) = 1$

In view of above defined labeling pattern , f satisfy the condition of the prime labeling

Therefore, G admits prime labeling

Hence, G is a prime graph. □

Illustration: 2.3

Figure 2.2 shows the prime labeling of the duplication of the vertex v_1 in G_Z .

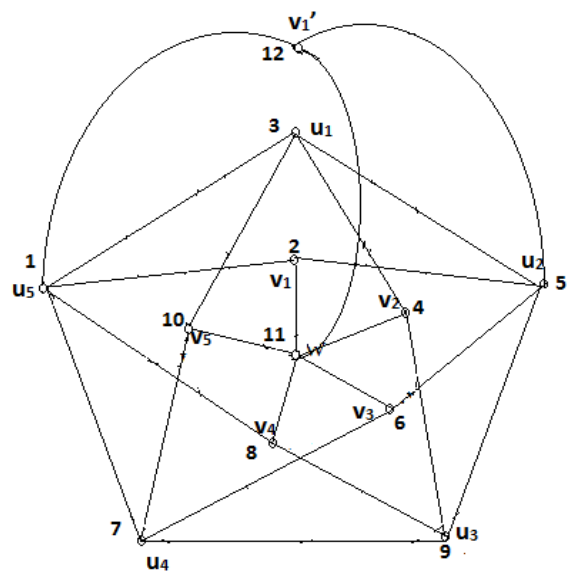

Figure 2.2 Duplication of the vertex v_1 in Grotzsch graph and its prime labeling.

Theorem: 2.3

The fusion of any two vertices of degree 4 in Grotzsch graph admits prime labeling.

Proof:

Let G_Z be the Grotzsch graph with 11 vertices and 20 edges

Let $V(G_Z) = \{w, v_i /1 \leq i \leq 5 ; u_i /1 \leq i \leq 5\}$

Let G be the graph obtained from G_Z by fusion of any two vertex of degree 4 in Grotzsch graph.

Without loss of generality, we may take the vertex u_1 and u_2 are the vertices may be fussed to the new vertex u. (i.e u $=u_1 u_2$)

Then |V(G)| = 10

Define a function $f : V(G) \rightarrow \{1,2,\ldots,10\}$ defined as follows

$f(w) = 7$

$f(v_i) = 10 - 2(i-1)$, for $1 \leq i \leq 5$

$f(v_1) = 10$

$f(v_2) = 8$

$f(v_3) = 6$

$f(v_4) = 4$

$f(v_5) = 2$

and $f(\mathrm{u} = u_1 u_2) = 1$

$f(u_3) = 3$

$f(u_4) = 5$

$f(u_5) = 9$

In view of above defined labeling pattern , f satisfy the condition of the prime labeling

Therefore , G admits prime labeling

Hence, G is a prime graph.

Illustration: 2.4

Figure 2.3 shows the prime labeling of fusion of vertices u_1 and u_2 in G_Z

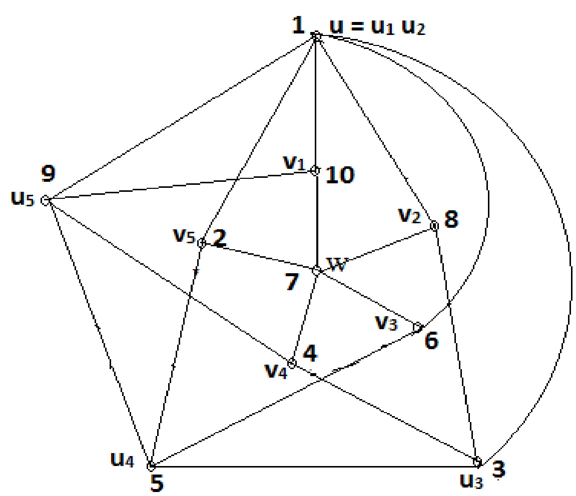

Figure 2.3 fusion of the vertex u_1 and u_2 in G_Z and its prime labeling.

Theorem: 2.4

Switching the apex vertex in Grotzsch graph admits prime labeling.

Proof:

Let G_Z be the Grotzsch graph with 11 vertices and 20 edges

Let $V(G_Z) = \{w, v_i /1 \leq i \leq 5 ; u_i /1 \leq i \leq 5\}$

Then $|V(G)| = 11$

Let G be the graph obtained from G_2 by switching the apex vertex w in G_Z .

Then $| V(G)| = 11$

Define a function f: V(G) → {1,2,…,11} defined as follows.

Let f(w) = 11

$f(v_i)$= 2i , for $1 \leq i \leq 5$

and $f(u_1)$= 1

$f(u_2)$= 5

$f(u_3)$= 3

$f(u_4)$= 7

$f(u_5)$= 9

In view of, the above defined labeling pattern, f satisfy the condition of the prime labeling

Therefore, G admits prime labeling

Hence, G is a prime graph. □

Illustration: 2.5

Figure 2.4 show the prime labeling of switching the vertex w in Grotzsch graph.

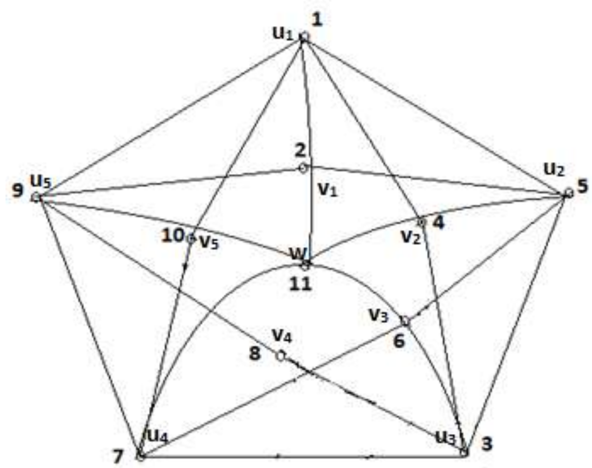

Figure 2.4 Switching the vertex w in Grotzsch graph and its prime labeling .

Conclusion:

As all the graphs are not prime graph it is very interesting to investigate graph (or) graph families which admit prime labeling. In this paper we have investigated that the grotzsch graph is a prime graph and graph operations namely, fusion, duplication and switching on grotzsch graph admits prime labeling.

To investigate similar results for other graph families and in the context of different labeling techniques is an open area of research.

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