Generalized Common Fixed Point Result in Cone Metric Spaces

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Abstract

In this paper, we study the existence of coincidence points and generalized common fixed point theorem for three self mappings in cone metric spaces and relaxing the completeness of the space. This improves the results of K. Purdhvi[1].

Keywords: Fixed point, Cone Metric Space, Coincidence Point.

1 Introduction and preliminaries

Ordered Banach spaces, normal cones and topical functions have applications in optimization theory. This motivate research in ordered linear metric spaces (see, e.g. [7],[8]). In 2007, cone metric space was introduced by Huang and Zhang [6] who generalized metric space into cone metric space replacing the set of real numbers by an ordered Banach space and obtained some fixed point theorems in this cone metric space. Later on, many authors are inspired with this cone metric space and studied Huang and Zhang [6] fixed point theorems and extended this idea to different contractive conditions (see, e.g. [1-4,8]). Recently K. Prudhvi [1] obtained a Common Fixed Point Result in Cone Metric Spaces for three self maps in cone metric spaces and

relaxing the completeness conditions. The aim of this paper, is to study the existence of coincidence points and generalized common fixed point theorem for three self-maps in cone metric spaces, which is an extension of the results of K. Prudhvi[1] who proved fixed point theorems for three self-mappings without assuming commutative and completeness conditions with in cone metric spaces. We need the following definitions and results, consistent with [1,3,6], in the sequel.

Definition 1.1. *Let E be a real Banach spaces and P a subset of E. The set P is called a cone if and only if:*
(a). P is closed, non-empty and $P \neq \{0\}$;
(b). $a, b \in R$, $x, y \in P$ implies $ax + by \in P$;
(c). $P \cap -P = \{0\}$.

Definition 1.2. *Let P be a cone in a Banach space E, define partial ordering $' \leq '$ with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$ to indicate $x \leq y$ but $x \neq y$ while $x \ll y$ will stand for $y - x \in intP$, where $intP$ denotes the interior of the set P. This cone P is called an order cone.*

Definition 1.3. *Let E be a Banach space and $P \subset E$ be an order cone. The order cone P is called normal if there exists $L > 0$ such that for all $x, y \in E$,*

$$0 \leqslant x \leqslant y \qquad implies \qquad \|x\| \leqslant L\|y\|.$$

The least positive number L satisfies the above inequality is called the normal constant of P.

Definition 1.4. *Let X be a non-empty set of E. Suppose that the map $d : X \times X \to E$ satisfies:*
(a) $0 \leq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;
(b) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(c) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.
Then d is called a cone metric on X and (X, d) is called a cone metric space.

It is clear that the concept of a cone metric space is more general than that of a metric space.

Example 1.1. [6] *Let $E = R^2, P = \{(x, y) \in E$ such that: $x, y \geq 0\} \subset R^2, X = R$ and $d : X \times X \to E$ such that*

$$d(x, y) = (|x - y|, \alpha|x - y|),$$

where $\alpha \geq 0$ is a constant. Then (X, d) is a metric space.

Clearly, this example shows that cone metric spaces generalize metric spaces.

Definition 1.5. *Let (X, d) be a cone metric space. We say that $\{x_n\}$ is a Cauchy sequence if for every c in E with $c \gg 0$, there is a natural number N such that for all $n, m > N, d(x_n, x_m) \ll c$.*

It is shown in [6] that a convergent sequence in a cone metric space (X, d) is a Cauchy sequence.

Definition 1.6. *Let (X, d) be a metric space. We say that $\{x_n\}$ is convergent sequence if for any $c \gg 0$ there is an N such that for all $n > N$, $d(x_n, x) \ll c$, for some fixed x in X. We denote this $x_n \to x$ (as $n \to \infty$). The space (X, d) is called a complete cone metric space if every Cauchy sequence is convergent ([6]).*

Definition 1.7. [2] *Let $f, g : X \to X$ be mappings. If $w = f(z) = g(z)$ for some $z \in X$, then z is called a coincidence point of f and g, and w is called a point of coincidence of f and g.*

Definition 1.8. [4] *The mappings $f, g : X \to X$ are said to be weakly compatible if for every $x \in X$, holds:*

$$f(g(x)) = g(f(x)) \qquad whenever \qquad f(x) = g(x).$$

Lemma 1.2. *Let X be a non-empty and the mappings f, g and h have a unique point of coincidence point w in X. If (f, g) and (g, h) are weakly compatible self-maps of X, then f, g and h have a unique common fixed point.*

2 Main results

In this section, we obtain coincidence points and generalized common fixed point theorem for three self-maps in cone metric spaces.
We adopted the technique which is used in [4].
Let (X, d) be a cone metric space and f, g and h be self-mappings of X such that $f(X) \cup g(X) \subseteq h(X)$. Suppose $x_0 \in X$ and $x_1 \in X$ is chosen such that $hx_1 = fx_0$ and $x_2 \in X$ is chosen such that $hx_2 = gx_1$. Continuing in this way, the sequence hx_n such that

$$y_{2n} = hx_{2n+1} = fx_{2n},$$
$$y_{2n+1} = hx_{2n+2} = gx_{2n+1}, n = 0, 1, 2, ...$$

is called a (f,g)-self sequence with initial point x_0.

We start with a proposition that will be required in the sequel.

Proposition 2.1. *Let (X,d) be a cone metric space, and P be a normal cone with normal constant L. Suppose that the mappings f, g and h are three self-maps of X such that $f(X) \bigcup g(X) \subseteq h(X)$ satisfying*

$$d(fx,gy) \le \alpha d(hx,hy)+\beta max\{d(hx,fx),d(hy,gy)\}+\gamma[d(hx,gy)+d(hy,fx)] \tag{1}$$

for all $x, y \in X$, where $\alpha, \beta, \gamma \in [0,1)$ and $\alpha + \beta + 2\gamma < 1$.
Then every $(f-g)$ sequence with initial point $x_0 \in X$ is a Cauchy sequence.

Proof. Suppose hx_n is a $(f-g)$ sequence with initial point x_0.
Assume $hx_n \ne hx_{n+1}$ for all $n \in N$, then for all n.
Using (1) we have

$$\begin{aligned}
d(y_{2n}, y_{2n+1}) &= d(hx_{2n+1}, hx_{2n+2}) \\
&= d(fx_{2n}, gx_{2n+1}) \\
&\le \alpha d(hx_{2n}, hx_{2n+1}) \\
&\quad + \beta max\{d(hx_{2n}, fx_{2n+1}), d(hx_{2n+1}, gx_{2n+1})\} \\
&\quad + \gamma[d(hx_{2n}, gx_{2n+1}) + d(hx_{2n+1}, fx_{2n+2})] \\
&\le \alpha d(hx_{2n}, hx_{2n+1}) \\
&\quad + \beta max\{d(hx_{2n}, hx_{2n+1}), d(hx_{2n+1}, hx_{2n+2})\} \\
&\quad + \gamma[d(hx_{2n}, hx_{2n+2}) + d(hx_{2n+1}, hx_{2n+1})] \\
&\le \alpha d(hx_{2n}, hx_{2n+1}) + \beta M_1 \\
&\quad + \gamma[d(hx_{2n}, hx_{2n+2}) + d(hx_{2n+1}, hx_{2n+1})]
\end{aligned}$$

Where $M_1 = max\{d(hx_{2n}, hx_{2n+1}), d(hx_{2n+1}, hx_{2n+2})\}$
Now two cases arises,
Case I: If suppose that $M_1 = d(hx_{2n}, hx_{2n+1})$ we have,

$$\begin{aligned}
d(y_{2n}, y_{2n+1}) &\le \alpha d(hx_{2n}, hx_{2n+1}) + \beta d(hx_{2n}, hx_{2n+1}) \\
&\quad + \gamma[d(hx_{2n}, hx_{2n+2}) + d(hx_{2n+1}, hx_{2n+1})] \\
&\le (\alpha + \beta) d(hx_{2n}, hx_{2n+1}) \\
&\quad + \gamma[d(hx_{2n}, hx_{2n+1}) + d(hx_{2n+1}, hx_{2n+1})] \\
&\le (\alpha + \beta + \gamma) d(hx_{2n}, hx_{2n+1}) + \gamma(hx_{2n+1}, hx_{2n+2}) \\
&\le (\alpha + \beta + \gamma) d(y_{2n-1}, y_{2n}) + \gamma(y_{2n}, y_{2n+1}) \\
d(y_{2n}, y_{2n+1}) &\le \frac{(\alpha + \beta + \gamma)}{1 - \gamma} d(y_{2n}, y_{2n-1})
\end{aligned}$$

Let $\lambda_1 = \frac{(\alpha+\beta+\gamma)}{(1-\gamma)}$, where $\lambda_1 < 1$.

Hence
$$d(y_{2n}, y_{2n+1}) \leq \lambda_1 d(y_{2n}, y_{2n-1}).$$

Case II: If suppose that $M_1 = d(hx_{2n+1}, hx_{2n+2})$.

$$
\begin{aligned}
d(y_{2n}, y_{2n+1}) &\leq \alpha d(hx_{2n}, hx_{2n+1}) + \beta d(hx_{2n+1}, hx_{2n+2}) \\
&\quad + \gamma[d(hx_{2n}, hx_{2n+2}) + d(hx_{2n+1}, hx_{2n+1})] \\
&\leq \alpha d(hx_{2n}, hx_{2n+1}) + (\beta + \gamma)d(hx_{2n+1}, hx_{2n+2}) \\
&\quad + \gamma d(hx_{2n}, hx_{2n+2}) \\
&\leq \alpha d(hx_{2n}, hx_{2n+1}) + (\beta + \gamma)d(hx_{2n+1}, hx_{2n+2}) \\
&\quad + \gamma d(hx_{2n}, hx_{2n+1}) \\
&\leq (\alpha + \gamma)d(hx_{2n}, hx_{2n+1}) + (\beta + \gamma)d(hx_{2n+1}, hx_{2n+2}) \\
&\leq (\alpha + \gamma)d(hx_{2n-1}, hx_{2n}) + (\beta + \gamma)d(hx_{2n}, hx_{2n+1}) \\
d(y_{2n}, y_{2n+1}) &\leq \frac{(\alpha + \gamma)}{1 - (\beta + \gamma)} d(hx_{2n}, hx_{2n-1})
\end{aligned}
$$

Let $\lambda_2 = \frac{(\alpha+\gamma)}{1-(\beta+\gamma)}$, where $\lambda_2 < 1$.

Hence
$$d(y_{2n}, y_{2n+1}) \leq \lambda_2 d(y_{2n}, y_{2n-1}).$$

Two cases shows that

$$d(y_{2n}, y_{2n+1}) \leq \lambda d(y_{2n}, y_{2n-1}) \qquad (2)$$

where, $\lambda = \lambda_1 = \lambda_2$.
Similarly, it can be shown that

$$d(y_{2n+1}, y_{2n+2}) \leq \lambda d(y_{2n}, y_{2n+1}).$$

Therefore for all n,

$$d(y_{n+1}, y_{n+2}) \leq \lambda d(y_n, y_{n+1}) \leq \lambda^2 d(y_{n-1}, y_n) \leq \ldots \leq \lambda^{n+1} d(y_0, y_1)$$

Now for any $m > n$

$$
\begin{aligned}
d(y_n, y_m) &\leq \lambda d(y_n, y_{n+1}) \leq \lambda^2 d(y_{n+1}, y_{n+2}) \leq \ldots \leq \lambda^{m-1} d(y_0, y_m) \\
&\leq (\lambda^n + \lambda^{n+1} + \lambda^{n+2} + \ldots + \lambda^{m-1}) d(y_1, y_0) \\
d(y_n, y_m) &\leq \frac{\lambda^n}{1 - \lambda} d(y_1, y_0).
\end{aligned}
$$

From (1.2), we have

$$\|d(y_n, y_m)\| \leq \frac{\lambda^n}{1-\lambda} \|d(y_1, y_0)\|.$$

Since $\lambda < 1$, $\frac{\lambda^n}{1-\lambda} \to 0$ as $n \to \infty$.

Which implies that

$$d(y_n, y_m) \to 0 \text{ as } n, m \to \infty.$$

Hence y_n is a Cauchy sequence, where $y_n = hx_n$. $\qquad\qquad\square$

Here, we further improve Proposition 2.1 as follows.

Theorem 2.2. *Let (X,d) be a cone metric space, and P be a normal cone with normal constant K. Suppose that the mappings f, g and h are three self-maps of X such that $f(X) \cup g(X) \subseteq h(X)$ satisfying*

$$d(fx, gy) \leq \alpha d(hx, hy) + \beta max\{d(hx, fx), d(hy, gy)\} + \gamma[d(hx, gy) + d(hy, fx)] \tag{3}$$

for all $x, y \in X$, where $\alpha, \beta, \gamma \in [0, 1)$ and $\alpha + \beta + 2\gamma < 1$.
If $f(X) \cup g(X)$ or $h(X)$ is a complete subsequence of X, then f, g and h have unique point of coincidence. Moreover, if (f,h) and (g,h) are weakly compatible, then f, g and h have a unique common fixed point.

Proof. Since $h(X)$ is complete subspace of X. And since, by the proposition (2.1) a $(f-g)$ sequence (hx_n) with the initial point x_0 is a Cauchy sequence, there exists $u, v \in X$ such that $hx_n \to v = hu$.
The same argument holds if $f(X) \cup g(X)$ is a complete subsequence of X with $v \in f(X) \cup g(X)$.
From (3) and triangle inequality

$$\begin{aligned}
d(hu, fu) &= d(hu, hx_{2n} + d(hx_{2n}, fu)) \\
&= d(hu, hx_{2n}) + d(fu, gx_{2n-1}) \\
&\leq d(v, hx_{2n}) + \alpha d(hu, hx_{2n-1}) \\
&\quad + \beta max\{d(hu, fu), d(hx_{2n-1}, gx_{2n-1})\} \\
&\quad + \gamma[d(hu, gx_{2n-1}) + d(hx_{2n-1}, fu)] \\
d(hu, fu) &\leq d(v, hx_{2n}) + \alpha d(hu, hx_{2n-1}) \\
&\quad + \beta M_1 + \gamma[d(hu, gx_{2n-1}) + d(hx_{2n-1}, fu)]
\end{aligned}$$

Where $M_1 = max\{d(hu, fu), d(hx_{2n-1}, gx_{2n-1})\}$

Now two cases arises,

Case I: If suppose that $M_1 = d(hu, fu)$ we have,

$$\begin{aligned}
d(hu, fu) &\leq d(v, hx_{2n}) + \alpha d(hu, hx_{2n-1}) + \beta d(hu, fu) \\
&\quad + \gamma[d(hu, gx_{2n-1}) + d(hx_{2n-1}, fu)] \\
&\leq d(v, hx_{2n}) + \alpha d(v, hx_{2n-1}) + \beta d(hu, fu) \\
&\quad + \gamma[d(v, hx_{2n}) + d(hx_{2n-1}, v) + d(hu, fu)] \\
&\leq (1 + \gamma)d(v, hx_{2n}) + (\alpha + \gamma)d(v, hx_{2n-1}) + (\beta + \gamma)d(hu, fu) \\
&\leq \frac{1 + \gamma}{1 - (\beta + \gamma)}d(v, hx_{2n}) + \frac{\alpha + \gamma}{1 - (\beta + \gamma)}d(v, hx_{2n-1}) \\
d(hu, fu) &\leq \lambda_1 d(v, hx_{2n}) + \lambda_2 d(v, hx_{2n-1}).
\end{aligned}$$

Let $\lambda_1 = \frac{1+\gamma}{1-(\beta+\gamma)}$ and $\lambda_2 = \frac{\alpha+\gamma}{1-(\beta+\gamma)}$.

Which from (1.2), implies that

$$\|d(hu, fu)\| \leq L\{\lambda_1\|d(v, hx_{2n})\| + \lambda_2\|d(v, hx_{2n-1})\|\}.$$

Case II: If suppose that $M_1 = d(hx_{2n-1}, gx_{2n-1})$

$$\begin{aligned}
d(hu, fu) &\leq d(v, hx_{2n}) + \alpha d(hu, hx_{2n-1}) + \beta d(hx_{2n-1}, gx_{2n-1}) \\
&\quad + \gamma[d(hu, gx_{2n-1}) + d(hx_{2n-1}, fu)] \\
&\leq d(v, hx_{2n}) + \alpha d(v, hx_{2n-1}) + \beta[d(hx_{n-1}, hu) + d(hu, gx_{2n-1})] \\
&\quad + \gamma[d(hu, hx_{2n}) + d(hx_{2n-1}, hu) + d(hu, fu)] \\
&\leq d(v, hx_{2n}) + \alpha d(v, hx_{2n-1}) + \beta[d(hx_{n-1}, v) + d(v, gx_{2n})] \\
&\quad + \gamma[d(v, hx_{2n}) + d(hx_{2n-1}, v) + d(hu, fu)] \\
&\leq (1 + \beta + \gamma)d(v, hx_{2n}) + (\alpha + \beta + \gamma)d(v, hx_{2n-1}) + \gamma d(hu, fu) \\
d(hu, fu) &\leq \frac{1 + \beta + \gamma}{1 - \gamma}d(v, hx_{2n}) + \frac{\alpha + \beta + \gamma}{1 - \gamma}d(v, hx_{2n-1}).
\end{aligned}$$

Let $\lambda_1 = \frac{1+\beta+\gamma}{1-\gamma}$ and $\lambda_2 = \frac{\alpha+\beta+\gamma}{1-\gamma}$.

Which from (1.2) implies that

$$\|d(hu, fu)\| \leq L\{\lambda_1\|d(v, hx_{2n})\| + \lambda_2\|d(v, hx_{2n-1})\|\}.$$

Now the right hand side of the above approaches to zero as $n \rightarrow 0$.
Hence,

$$\|d(hu, fu)\| = 0 \qquad \text{and} \qquad fu = hu(= v). \tag{3}$$

Similarly, by using the inequality

$$d(hu, gu) \leq d(hu, hx_{2n+1}) + d(hx_{2n+1}, gu).$$

We can show that

$$hu = gu(= v).$$

Thus, $v = hu = fu = gu$ and hence we conclude that v is a point of coincidence of f, g and h.

Now we show that the point of coincidence is unique.

Assume that there is another point of coincidence v in X such that

$$v^* = fu^* = gu^* = hu^* \text{ for some } u^* \in X.$$

It is easy to see that using (3) that

$$
\begin{aligned}
d(v, v^*) &= d(fu, gu^*) \\
&\leq \alpha d(hu, hu^*) + \beta max[d(hu, fu), d(hu^*, gu^*)] \\
&\quad + \gamma[d(hu, gu^*) + d(hu^*, fu)] \\
&\leq \alpha d(hu, hu^*) + \beta M_1 + \gamma[d(hu, gu^*) + d(hu^*, fu)].
\end{aligned}
$$

Now two cases arises,

Where $M_1 = max[d(hu, fu), d(hu^*, gu^*)]$.

Case I: If suppose that $M_1 = d(hu, fu)$ we have,

$$
\begin{aligned}
d(v, v^*) &\leq \alpha d(hu, hu^*) + \beta d(hu, fu) + \gamma[d(hu, gu^*) + d(hu^*, fu)] \\
&\leq \alpha d(v, v^*) + \beta d(v, v) + \gamma[d(u, u^*) + d(u^*, u)] \\
&\leq \frac{2\gamma}{1-\alpha} d(u, u^*) \\
d(v, v^*) &\leq bd(u, u^*).
\end{aligned}
$$

Case II: If suppose that $M_1 = d(hu^*, gu^*)$ we have,

$$
\begin{aligned}
d(v, v^*) &\leq \alpha d(hu, hu^*) + \beta d(hu^*, gu^*) + \gamma[d(hu, gu^*) + d(hu^*, fu)] \\
&\leq \alpha d(v, v^*) + \beta d(v^*, v^*) + \gamma[d(u, u^*) + d(u^*, u)] \\
&\leq \frac{2\gamma}{1-\alpha} d(u, u^*) \\
d(v, v^*) &\leq bd(u, u^*).
\end{aligned}
$$

Putting $b = \frac{2\gamma}{1-\alpha} < 1$.

Hence two cases shows that

$$d(v, v^*) \leq bd(u, u^*).$$

Since $\alpha + 2\gamma < 1$, then $v = v^*$.

Since (f, g) and (g, h) are weakly compatible by assumption v is the unique point of coincidence of f, g and h, then by the Lemma (1.2) we get that v is the unique common fixed point of f, g and h. $\qquad\square$

Remark 2.3. *If we choose $h = I_x$ is an identity map in the above Theorem 2.2.,then we deduce the following Theorem.*

Theorem 2.4. *Let (X,d) be a cone metric space, and P be a normal cone with normal constant K. Suppose that the mappings f and g are two self-maps of X satisfying*

$$d(fx, gy) \le \alpha d(x,y) + \beta max[d(x, fx), d(y, gy)] + \gamma[d(x, gy) + d(y, fx)] \quad (4)$$

for all $x, y \in X$, where $\alpha, \beta, \gamma \in [0, 1)$ and $\alpha + \beta + 2\gamma < 1$.
If f(X) or g(X), where is a complete subspace of X, then f and g have a unique point of coincidence. Moreover, if f and g are weakly compatible, then f and g have a unique common fixed point.

Example 2.5. *Let $X = \{1, 2, 3\}, E = R^2$ and $P = \{(x, y) \in E/x, y \ge 0\}$. Define $d, g : X \times X \to E$ as follows:*

$$d(x, y) = \begin{cases} (0,0) & if & x = y & \\ (\frac{7}{9}, 7) & if & x \ne y & x, y \in X - \{2\} \\ (1, 9) & if & x \ne y & x, y \in X - \{3\} \\ (\frac{1}{3}, 3) & if & x \ne y & x, y \in X - \{1\} \end{cases}$$

$$and \qquad g(x) = \begin{cases} 2 & if & x \ne 2 \\ 1 & if & x = 2 \end{cases}$$

And now Define a constant mappings $h, f : X \to X$ by $hx = fx = 1$, for all $x \in X$. Then

$$d(fx, gy) = \begin{cases} (0,0) & if & y \ne 2 \\ (\frac{7}{9}, 7) & if & y = 2. \end{cases}$$

and

$$\alpha d(hx, hy) + \beta max\{d(hx, fx), d(hy, gy)\} + \gamma[d(hx, gy) + d(hy, fx)] = (\tfrac{7}{9}, 7).$$

if $y = 2, \alpha = 0 = \gamma, \beta = \frac{7}{9}$.
It follows that all conditions of Theorem 2.2 are satisfied for $\alpha = 0 = \gamma, \beta = \frac{7}{9}$ and so f, g and h have unique point of coincidence and a unique common fixed point 1.

Conclusion 2.6. *Our results generalized proposition 3.1 and theorems 3.2 and 3.4 in [1].*

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