A double integral involving I-function of several variables

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ABSTRACT
In this document, we obtain an double integral involving the multivariable I-function, the general class of polynomials of several variables and Aleph-function of one variable which are the sufficiently general in nature and are capable of yielding a large number of simpler and useful results merely by specializing the parameters in them. Further we establish some special cases.

KEYWORDS : I-function of several variables, integrals, general class of polynomials,,Aleph-function of one variable, multivariable H-function.

1.Introduction and preliminaries.

In this document, we obtain an double integral involving the multivariable I-function defined by Prasad [2], the general class of polynomials of several variables and Aleph-function of one variable. We will study the case concerning the multivariable H-function defined by Srivastava et al [4].

The Aleph- function , introduced by Südland [5] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z)=\aleph_{P_i,Q_i,c_i;r}^{M,N}\left(z\ \middle|\ \begin{matrix}(\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r}\\(\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right)=\frac{1}{2\pi\omega}\int_L \Omega_{P_i,Q_i,c_i;r}^{M,N}(s)z^{-s}\mathrm{d}s \quad (1.1)$$

for all z different to 0 and

$$\Omega_{P_i,Q_i,c_i;r}^{M,N}(s)=\frac{\prod_{j=1}^M\Gamma(b_j+B_js)\prod_{j=1}^N\Gamma(1-a_j-A_js)}{\sum_{i=1}^r c_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \quad (1.2)$$

With :$|argz|<\frac{1}{2}\pi\Omega$ Where $\Omega=\sum_{j=1}^M\beta_j+\sum_{j=1}^N\alpha_j-c_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0\,;i=1,\cdots,r$

For convergence conditions and other details of Aleph-function , see Südland et al [5].the serie representation of Aleph-function is given by Chaurasia et al [1].

$$\aleph_{P_i,Q_i,c_i;r}^{M,N}(z)=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(s)}{B_G g!}z^{-s} \quad (1.3)$$

With $s=\eta_{G,g}=\frac{b_G+g}{B_G}$, $P_i<Q_i$, $|z|<1$ and $\Omega_{P_i,Q_i,c_i;r}^{M,N}(s)$ is given in (1.2) (1.4)

The generalized polynomials defined by Srivastava [3], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s]=\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \quad (1.5)$$

Where $M_1, \cdots, M_s$ are arbitrary positive integers and the coefficients $A[N_1, K_1; \cdots; N_s, K_s]$ are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1 = \frac{(-N_1)_{M_1 K_1}}{K_1!} \cdots \frac{(-N_s)_{M_s K_s}}{K_s!} A[N_1, K_1; \cdots; N_s, K_s] \tag{1.6}$$

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1, \cdots, z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}} \left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array} \middle| \begin{array}{c} (\mathrm{a}_{2j}; \alpha'_{2j}, \alpha''_{2j})_{1,p_2}; \cdots; \\ \\ (\mathrm{b}_{2j}; \beta'_{2j}, \beta''_{2j})_{1,q_2}; \cdots; \end{array} \right.$$

$$\left. \begin{array}{c} (\mathrm{a}_{rj}; \alpha'_{rj}, \cdots, \alpha^{(r)}_{rj})_{1,p_r} : (a'_j, \alpha'_j)_{1,p'}; \cdots; (a^{(r)}_j, \alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj}; \beta'_{rj}, \cdots, \beta^{(r)}_{rj})_{1,q_r} : (b'_j, \beta'_j)_{1,q'}; \cdots; (b^{(r)}_j, \beta^{(r)}_j)_{1,q^{(r)}} \end{array} \right) \tag{1.7}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \xi(t_1, \cdots, t_r) \prod_{i=1}^{s} \phi_i(s_i) z_i^{s_i} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.8}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i \pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) +$$

$$+ \left(\sum_{k=1}^{n_r} \alpha_{rk}^{(i)} - \sum_{k=n_r+1}^{p_r} \alpha_{rk}^{(i)} \right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_r} \beta_{rk}^{(i)} \right) \tag{1.9}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$I(z_1, \cdots, z_r) = 0(|z_1|^{\gamma'_1}, \cdots, |z_r|^{\gamma'_r}), max(|z_1|, \cdots, |z_r|) \rightarrow 0$

$I(z_1, \cdots, z_r) = 0(|z_1|, \cdots, |z_r|^{\beta'_s}), min(|z_1|, \cdots, |z_r|) \rightarrow \infty$

where $k=1,\cdots,z:\alpha'_k=min[Re(b_j^{(k)}/\beta_j^{(k)})], j=1,\cdots,m_k$ and

$\beta'_k=max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})], j=1,\cdots,n_k$

We will use these following notations in this paper :

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};V=0,n_2;0,n_3;\cdots;0,n_{s-1} \tag{1.10}$$

$$W=(p',q');\cdots;(p^{(r)},q^{(r)});X=(m',n');\cdots;(m^{(r)},n^{(r)}) \tag{1.11}$$

$$A=(a_{2k},\alpha'_{2k},\alpha''_{2k});\cdots;(a_{(r-1)k},\alpha'_{(r-1)k},\alpha''_{(r-1)k},\cdots,\alpha^{(r-1)}_{(r-1)k}) \tag{1.12}$$

$$B=(b_{2k},\beta'_{2k},\beta''_{2k});\cdots;(b_{(r-1)k},\beta'_{(r-1)k},\beta''_{(r-1)k},\cdots,\beta^{(r-1)}_{(r-1)k}) \tag{1.13}$$

$$\mathfrak{A}=(a_{sk};\alpha'_{rk},\alpha''_{rk},\cdots,\alpha^{r}_{rk}):\mathfrak{B}=(b_{rk};\beta'_{rk},\beta''_{rk},\cdots,\beta^{r}_{rk}) \tag{1.14}$$

$$A'=(a'_k,\alpha'_k)_{1,p'};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}};B'=(b'_k,\beta'_k)_{1,q'};\cdots;(b_k^{(r)},\beta_k^{(r)})_{1,q^{(r)}} \tag{1.15}$$

The multivariable I-function write :

$$I(z_1,\cdots,z_r)=I^{V;0,n_r;X}_{U:p_r,q_r;W}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}\ ;\mathfrak{A};\mathrm{A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B};\ \mathfrak{B};\ \mathrm{B'} \end{array}\right) \tag{1.16}$$

2. Required integrals

We have the two following integrals :

$$1)\int_0^{\pi/2} e^{i(\alpha+\beta)\theta}(sin\theta)^{\alpha-1}(cos\theta)^{\beta-1}\mathrm{d}\theta=e^{i\pi\alpha/2}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)},Re(\alpha)>0,Re(\beta)>0 \tag{2.1}$$

$$2)\int_0^{\infty} x^{r-1/2}\left[(x+a)(x+b)\right]^{-r}\mathrm{d}x=\sqrt{\pi}(\sqrt{a}+\sqrt{b})^{1-2r}\frac{\Gamma\left(r-\frac{1}{2}\right)}{\Gamma(r)},Re(r)>\frac{1}{2} \tag{2.2}$$

3. Main integral

Let $$g(r,\theta,\alpha,\beta,\gamma)=e^{i(\alpha+\beta)\theta}(sin\theta)^{\alpha}(cos\theta)^{\beta}\left[\frac{r(\sqrt{a}+\sqrt{b})^2}{(r+a)(r+b)}\right]^{\gamma} \tag{3.1}$$

In this section, we will evaluate the general double integral with the helps of results (2.1) and (2.2). We have the general relation:

$$\int_0^\infty \int_0^{\pi/2} \frac{g(r,\theta,\alpha,\beta,\gamma)}{\sqrt{r}sin\theta cos\theta} \aleph^{M,N}_{P_i,Q_i,c_i;r'}(zg(r,\theta,c,d,e)) S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c} \mathrm{y}_1 g[r,\theta,c_1,d_1:e_1] \\ \cdot\cdot\cdot \\ \mathrm{y}_s g[r,\theta,c_s,d_s:e_s] \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1 g[r,\theta,\delta_1,\mu_1:\lambda_1] \\ \cdot\cdot\cdot \\ \mathrm{z}_R g[r,\theta,\delta_R,\mu_R:\lambda_R] \end{array}\right) \mathrm{d}r\mathrm{d}\theta = \sqrt{\pi}e^{i\pi\alpha/2}(\sqrt{a}+\sqrt{b})\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} a_1$$

$$\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!} z^{\eta_{G,g}} y_1^{K_1}\cdots y_s^{K_s} e^{i\pi/2(c\eta_{G,g}+\sum_{i=1}^s K_i c_i)}\left(\sqrt{a}+\sqrt{b}\right)^{-2(e\eta_{G,g}+\sum_{i=1}^s K_i e_i)}$$

$$I^{V;0,n_R+3;X}_{U:p_R+3,q_R+2;W}\left(\begin{array}{c} \mathrm{z}_1\frac{e^{i\pi\delta_1/2}}{(\sqrt{a}+\sqrt{b})^{2\lambda_1}} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_R\frac{e^{i\pi\delta_R/2}}{(\sqrt{a}+\sqrt{b})^{2\lambda_R}} \end{array}\middle| \begin{array}{l} \mathrm{A}\ ;\ (\frac{3}{2}\text{-}\gamma - e\eta_{G,g} - \sum_{i=1}^s K_i e_i;\lambda_1,\cdots,\lambda_R), \\ \quad\quad\cdot\cdot\cdot \\ \mathrm{B}\ ;\ (1\text{-}\gamma - e\eta_{G,g} - \sum_{i=1}^s K_i e_i;\lambda_1,\cdots,\lambda_R), \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\alpha - d\eta_{G,g} - \sum_{i=1}^s K_i d_i;\mu_1,\cdots,\mu_R),(1-\beta-c\eta_{G,g}-\sum_{i=1}^s K_i c_i;\delta_1,\cdots,\delta_R),\mathfrak{A}:A' \\ \cdot\cdot\cdot \\ (1\text{-}\alpha-\beta-(c+d)\eta_{G,g}-\sum_{i=1}^s K_i(c_i+d_i);\mu_1+\delta_1,\cdots,\mu_R+\delta_R),\mathfrak{B}:B' \end{array}\right) \quad (3.2)$$

Provided that

a)$min(c,d,e,c_i,d_,e_i,\delta_j,\mu_j,\lambda_j) > 0, i=1,\cdots,s; j=1,\cdots,R$

b) $Re[\beta + c\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j} + \sum_{i=1}^R \mu_i \min_{1\leqslant j\leqslant m^{(i)}}\frac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$

c) $Re[\alpha + d\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j} + \sum_{i=1}^R \delta_i \min_{1\leqslant j\leqslant m^{(i)}}\frac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$

d) $Re[\gamma + e\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j} + \sum_{i=1}^R \lambda_i \min_{1\leqslant j\leqslant m^{(i)}}\frac{b_j^{(i)}}{\beta_j^{(i)}}] > \frac{1}{2}$

e)$|argz_i| < \frac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.9) ; $i=1,\cdots,R$

f) $|argz| < \frac{1}{2}\pi\Omega$ where $\Omega = \sum_{j=1}^M \beta_j + \sum_{j=1}^N \alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0$

Proof of (3.2). Let $M = \frac{1}{(2\pi\omega)^R}\int_{L_1}\cdots\int_{L_R}\xi(s_1,\cdots,s_R)\prod_{k=1}^R \phi_k(s_k)$

To obtain (3.2), express a general class of polynomials of several variables occurring in the integrand of (3.2) as defined in (1.5),series representation of the Aleph-function by (1.3) and the multivariable I-function defined by Prasad [2] by its Mellin-Barnes contour integral with the help of (1.8). Now we interchange the order of summation and integrations (which is permissible under the conditions stated above), we obtain :

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} a_1\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!} z^{\eta_{G,g}} y_1^{K_1}\cdots y_s^{K_s}\int_0^{\infty}\int_0^{\pi/2}\frac{g(r,\theta,\alpha,\beta,\gamma)}{\sqrt{r}sin\theta cos\theta}$$

$$(g(r,\theta,c,d,e))^{\eta_{G,g}}\prod_{i=1}^{s}(g(r,\theta,c_i,d_i,e_i))^{K_i}\ M\left\{\prod_{k=1}^{R}(g(r,\theta,\delta_k,\mu_k,\lambda_k))^{s_k}\right\}\mathrm{d}s_1\cdots\mathrm{d}s_R\mathrm{d}r\mathrm{d}\theta \tag{3.3}$$

Assuming the inversion of order of integrations in (3.2) to be permissible by absolute convergence of the integrals involved, we have :

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} a_1\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!} z^{\eta_{G,g}} y_1^{K_1}\cdots y_s^{K_s}$$

$$M\left(\left(\int_0^{\infty}\left[\frac{r(\sqrt{a}+\sqrt{b})^2}{(r+a)(r+b)}\right]^{\gamma+e\eta_{G,g}+\sum_{i=1}^{s}K_ie_i+\sum_{k=1}^{R}\lambda_k s_k}\mathrm{d}r\right)\right.$$

$$\left(\int_0^{\pi/2}e^{i(\alpha+\beta+(c+d)\eta_{G,g}+\sum_{i=1}^{s}K_i(c_i+d_i)+\sum_{k=1}^{R}s_i(\delta_i+\mu_i))\theta}\left(cos\theta\right)^{\alpha+c\eta_{G,g}+\sum_{i=1}^{s}K_ic_i+\sum_{k=1}^{R}\delta_i s_i}\right.$$

$$\left.\left(sin\theta\right)^{\beta+d\eta_{G,g}+\sum_{i=1}^{s}K_id_i+\sum_{k=1}^{R}\mu_i s_i}\mathrm{d}\theta\right)\mathrm{d}s_1\cdots\mathrm{d}s_R \tag{3.4}$$

We evaluate the inner integrals with the help of (2.1) and (2.2), we get

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} a_1\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!} z^{\eta_{G,g}} y_1^{K_1}\cdots y_s^{K_s}\sqrt{\pi}e^{i\pi\alpha/2}(\sqrt{a}+\sqrt{b})$$

$$(\sqrt{a}+\sqrt{b})^{-2(e\eta_{G,g}+\sum_{i=1}^{s}K_ie_i+\sum_{k=1}^{R}s_k\lambda_i)}e^{i\pi/2(\eta_{G,g}c+\sum_{i=1}^{s}K_ic_i+\sum_{k=1}^{R}s_k\delta_k)}$$

$$M\left(\frac{\Gamma(\alpha+c\eta_{G,g}+\sum_{i=1}^{s}K_ic_i+\sum_{k=1}^{R}\delta_k s_k)\Gamma(\beta+d\eta_{G,g}+\sum_{i=1}^{s}K_id_i+\sum_{k=1}^{R}\mu_k s_k)}{\Gamma(\alpha+\beta+(c+d)\eta_{G,g}+\sum_{i=1}^{s}K_i(c_i+d_i)+\sum_{k=1}^{R}(\delta_k+\mu_k)s_k)}\right.$$

$$\left.\frac{\Gamma(\gamma+e\eta_{G,g}+\sum_{i=1}^{s}K_is_i+\sum_{k=1}^{R}s_k\lambda_k+\frac{1}{2})}{\Gamma(\gamma+e\eta_{G,g}+\sum_{i=1}^{s}K_is_i+\sum_{k=1}^{R}s_k\lambda_k)}\right)\mathrm{d}s_1\cdots\mathrm{d}s_R \tag{3.5}$$

Finally interpreting the resulting Mellin-Barnes contour integral as a multivariable I-function, we obtain the desired result (3.2).

4. Multivariable H-function

If $U = V = A = B = 0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava et al [4]. We have the following result.

$$\int_0^\infty \int_0^{\pi/2} \frac{g(r,\theta,\alpha,\beta,\gamma)}{\sqrt{r}sin\theta cos\theta} \aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(zg(r,\theta,c,d,e)\big) S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix} \mathrm{y}_1 g[r,\theta,c_1,d_1:e_1] \\ \cdots \\ \mathrm{y}_s g[r,\theta,c_s,d_s:e_s] \end{pmatrix}$$

$$H\begin{pmatrix} \mathrm{z}_1 g[r,\theta,\delta_1,\mu_1:\lambda_1] \\ \cdots \\ \mathrm{z}_R g[r,\theta,\delta_R,\mu_R:\lambda_R] \end{pmatrix} \mathrm{d}r\mathrm{d}\theta = \sqrt{\pi}e^{i\pi\alpha/2}(\sqrt{a}+\sqrt{b})\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} a_1$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!} z^{\eta_{G,g}} y_1^{K_1}\cdots y_s^{K_s} e^{i\pi/2(c\eta_{G,g}+\sum_{i=1}^s K_i c_i)}\big(\sqrt{a}+\sqrt{b}\big)^{-2(e\eta_{G,g}+\sum_{i=1}^s K_i e_i)}$$

$$H_{p_R+3,q_R+2;W}^{0,n_R+3;X}\left(\begin{array}{c} \mathrm{z}_1\frac{e^{i\pi\delta_1/2}}{(\sqrt{a}+\sqrt{b})^{2\lambda_1}} \\ \cdots \\ \cdots \\ \mathrm{z}_R\frac{e^{i\pi\delta_R/2}}{(\sqrt{a}+\sqrt{b})^{2\lambda_R}} \end{array}\left|\begin{array}{c} (\tfrac{3}{2}\text{-}\gamma - e\eta_{G,g} - \sum_{i=1}^s K_i e_i;\lambda_1,\cdots,\lambda_R), \\ \cdots \\ (1\text{-}\gamma - e\eta_{G,g} - \sum_{i=1}^s K_i e_i;\lambda_1,\cdots,\lambda_R), \end{array}\right.\right.$$

$$\left.\begin{array}{c} (1\text{-}\alpha - d\eta_{G,g} - \sum_{i=1}^s K_i d_i;\mu_1,\cdots,\mu_R),(1-\beta-c\eta_{G,g}-\sum_{i=1}^s K_i c_i;\delta_1,\cdots,\delta_R),\mathfrak{A}:A' \\ \cdots \\ (1\text{-}\alpha-\beta-(c+d)\eta_{G,g}-\sum_{i=1}^s K_i(c_i+d_i);\mu_1+\delta_1,\cdots,\mu_R+\delta_R),\mathfrak{B}:B' \end{array}\right) \quad (4.1)$$

which holds true under the same conditions as needed in (3.2) with $U = V = A = B = 0$

5. Conclusion

Due to general nature of the multivariable I-function defined by Prasad [2] and the double integral involving here, our formulas are capable to be reduced into many known and news integrals involving the special functions of one and several variables and polynomials of one and several variables.

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