A study of unified results involving a product of generalized Legendre's function, a general class of polynomials, Aleph-function with the multivariable I-function

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ABSTRACT
In this paper an integral involving general class of polynomials, Legendre's associated function, Aleph-function and I-function of several variables has been evaluated and an expansion formula for product of the general class of polynomials, Legendre's associated function, Aleph-function and I-function of several variables has been established with the application of this integral. The result established in this paper are of general nature and hence encompass several particular cases.

Keywords :Multivariable I-function, Aleph-function, Legendre's associated function, general class of polynomials, expansion formula,multivariable H-function

2010 Mathematics Subject Classification. 33C60, 82C31

1.Introduction and preliminaries.

In this paper an integral involving general class of polynomials, Legendre's associated function, Aleph-function and I-function of several variablesdefined by Prasad [3] has been evaluated and an expansion formula for product of the general class of polynomials, Legendre's associated function. We will study the case concerning the multivariable H-function

The Aleph- function , introduced by Südland [6] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z)=\aleph_{P_i,Q_i,c_i;r}^{M,N}\left(z \;\middle|\; \begin{matrix}(\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r}\\(\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right)=\frac{1}{2\pi\omega}\int_L \Omega_{P_i,Q_i,c_i;r}^{M,N}(s)z^{-s}\mathrm{d}s \quad (1.1)$$

for all z different to 0 and

$$\Omega_{P_i,Q_i,c_i;r}^{M,N}(s)=\frac{\prod_{j=1}^{M}\Gamma(b_j+B_js)\prod_{j=1}^{N}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r}c_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \quad (1.2)$$

With $|argz|<\frac{1}{2}\pi\Omega$ where $\Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0, i=1,\cdots,r$

For convergence conditions and other details of Aleph-function , see Südland et al [6]. The serie representation of Aleph-function is given by Chaurasia et al [1].

$$\aleph_{P_i,Q_i,c_i;r}^{M,N}(z)=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(s)}{B_Gg!}z^{-s} \quad (1.3)$$

With $s=\eta_{G,g}=\frac{b_G+g}{B_G}, P_i<Q_i, |z|<1$ and $\Omega_{P_i,Q_i,c_i;r}^{M,N}(s)$ is given in (1.2) (1.4)

The generalized polynomials defined by Srivastava [4], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s] = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.5}$$

Where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_s,K_s]$ are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1 = \frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s] \tag{1.6}$$

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}}^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha'_{rj},\cdots,\alpha_{rj}^{(r)})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta'_{rj},\cdots,\beta_{rj}^{(r)})_{1,q_r}:(b'_j,\beta'_j)_{1,q'};\cdots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}} \end{array}\right) \tag{1.7}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\xi(t_1,\cdots,t_r)\prod_{i=1}^{s}\phi_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.8}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [3]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}}\alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}}\beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)} + \left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right) +$$

$$+\left(\sum_{k=1}^{n_r}\alpha_{rk}^{(i)} - \sum_{k=n_r+1}^{p_r}\alpha_{rk}^{(i)}\right) - \left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)} + \sum_{k=1}^{q_3}\beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_r}\beta_{rk}^{(i)}\right) \tag{1.9}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$I(z_1, \cdots, z_r) = 0(|z_1|^{\gamma'_1}, \cdots, |z_r|^{\gamma'_r}), max(|z_1|, \cdots, |z_r|) \rightarrow 0$

$I(z_1, \cdots, z_r) = 0(|z_1|, \cdots, |z_r|^{\beta'_s}), min(|z_1|, \cdots, |z_r|) \rightarrow \infty$

where $k = 1, \cdots, z : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots; p_{r-1}, q_{r-1}; V = 0, n_2; 0, n_3; \cdots; 0, n_{s-1} \tag{1.10}$$

$$W = (p', q'); \cdots; (p^{(r)}, q^{(r)}); X = (m', n'); \cdots; (m^{(r)}, n^{(r)}) \tag{1.11}$$

$$A = (a_{2k}, \alpha'_{2k}, \alpha''_{2k}); \cdots; (a_{(r-1)k}, \alpha'_{(r-1)k}, \alpha''_{(r-1)k}, \cdots, \alpha^{(r-1)}_{(r-1)k}) \tag{1.12}$$

$$B = (b_{2k}, \beta'_{2k}, \beta''_{2k}); \cdots; (b_{(r-1)k}, \beta'_{(r-1)k}, \beta''_{(r-1)k}, \cdots, \beta^{(r-1)}_{(r-1)k}) \tag{1.13}$$

$$\mathfrak{A} = (a_{sk}; \alpha'_{rk}, \alpha''_{rk}, \cdots, \alpha^{r}_{rk}) : \mathfrak{B} = (b_{rk}; \beta'_{rk}, \beta''_{rk}, \cdots, \beta^{r}_{rk}) \tag{1.14}$$

$$A' = (a'_k, \alpha'_k)_{1,p'}; \cdots; (a_k^{(r)}, \alpha_k^{(r)})_{1,p^{(r)}}; B' = (b'_k, \beta'_k)_{1,q'}; \cdots; (b_k^{(r)}, \beta_k^{(r)})_{1,q^{(r)}} \tag{1.15}$$

The multivariable I-function write :

$$I(z_1, \cdots, z_r) = I^{V;0,n_r;X}_{U:p_r,q_r;W}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}\ ;\mathfrak{A};\mathrm{A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B};\ \mathfrak{B};\ \mathrm{B'} \end{array}\right) \tag{1.16}$$

2. Generalized Legendre's associated function

Formula 1

The following results are required in our investigation ([2],p.343, Eq(38); [2], p.340, Eq(26) and eq (27))

$$\int_{-1}^{1} (1-x)^{\rho}(1+x)^{\sigma}\, P^{m,n}_{k-\frac{m-n}{2}}(x)\, \mathrm{d}x = \frac{2^{\rho+\sigma-\frac{m-n}{2}}\Gamma(\rho-\frac{m}{2}+1)\Gamma(\sigma+\frac{n}{2}+1)}{\Gamma(1-m)\Gamma(\rho+\sigma-\frac{m-n}{2}+2)}$$

$$\times\, {}_3F_2\left(-k, n-m+k+1, \rho-\frac{m}{2}+1; 1-m, \rho-\sigma-\frac{m-n}{2}+2; 1\right) \tag{2.1}$$

Provided that $Re(\rho-\frac{m}{2})>-1; Re(\sigma+\frac{n}{2})>-1$

Formula 2

$$\int_{-1}^{1} P^{u,v}_{k-\frac{u-v}{2}}(x) P^{u,v}_{t-\frac{u-v}{2}}(x)\mathrm{d}x = \frac{2^{-u+v+1}k!\Gamma(k+v+1+1)}{(2k-u+v+1)\Gamma(k-u+1)\Gamma(k-u+v+1)}\delta_{k,t} \tag{2.2}$$

Where $\delta_{k,t}=1$ if $k=t, 0$ else. Provided $Re(u)<1, Re(v)>-1$

3. Main integral formula

$$\int_{-1}^{1}(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}P^{u,v}_{k-\frac{u-v}{2}}(x)\aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(y(1-x)^{\mathfrak{g}}(1+x)^{w}\,\middle|\, \begin{matrix} (\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r} \\ (\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right)$$

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix} \mathrm{y}_1(1-x)^{g_1}(1+x)^{w_1} \\ \cdots \\ \mathrm{y}_s(1-x)^{g_s}(1+x)^{w_s}\end{pmatrix} I\begin{pmatrix} \mathrm{z}_1(1-x)^{h_1}(1+x)^{k_1} \\ \cdots \\ \mathrm{z}_r(1-x)^{h_r}(1+x)^{k_r}\end{pmatrix}\mathrm{d}x$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{l=0}^{\infty} a\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!}\frac{(-k)_l l^{v-u+k+1}}{l!\Gamma(1-u+l)}2^{\rho-u+v+\sigma+(\mathfrak{g}+w)\eta_{G,g}+1}y^{\eta_{G,g}}$$

$$\prod_{j=1}^{s} y_j^{K_j}2^{(g_j+w_j)K_j} I^{V;0,n_r+2;X}_{U:p_r+2,q_r+1;W}\left(\begin{matrix}\mathrm{z}_1 2^{h_1+k_1}\\ \cdot \\ \cdot \\ \mathrm{z}_r 2^{h_r+k_r}\end{matrix}\,\middle|\,\begin{matrix} \mathrm{A}\ ;(-\sigma-v-w\eta_{G,g}-\sum_{i=1}^{s}w_iK_i:k_1,\cdots,k_r), \\ \cdot\ \ \cdot \\ \mathrm{B};\end{matrix}\right.$$

$$\left.\begin{matrix}(-\rho-m-\mathfrak{g}\eta_{G,g}+u-\sum_{i=1}^{s}g_iK_i:h_1,\cdots,h_r),\mathfrak{A}:A' \\ \cdots \\ (\mathrm{u-v}-\rho-\sigma-m-(\mathfrak{g}+w)\eta_{G,g}-\sum_{i=1}^{s}(g_i+w_i)K_i:h_1+k_1,\cdots,h_r+k_r),\mathfrak{B}:B'\end{matrix}\right) \tag{3.1}$$

Provided that

a) $Re(u)>-1, Re(v)<1$

b) $g,w>0; g_i,w_i>0, i=1,\cdots,s; h_i,k_i>0, i=1,\cdots,r; k, k-(u-v)/2$ positive integer

c) $Re[\rho-u+\mathfrak{g}\min\limits_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum\limits_{i=1}^{r}h_i\min\limits_{1\leqslant j\leqslant m^{(i)}}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>-1$

d) $Re[\sigma+v+w\min\limits_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum\limits_{i=1}^{r}h_i\min\limits_{1\leqslant j\leqslant m^{(i)}}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>-1$

c) $|argz_i| < \frac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.9)

f) $|argy| < \frac{1}{2}\pi\Omega$ Where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0$

Proof

To prove the formula(3.1), we express the general polynomials, the Aleph-function of one variable with the help of equation (1.5) and (1.3) respectively and the multivariable I-function defined by Prasad [3] in terms of Mellin-Barnes type contour integrals with the help of equation (1.7). Now interchanging the order of summation and integration (which permissible under the conditions stated), and then evaluate the inner x-integral by using the formula (2.1); we arrive at the desired result.

4. Expansion formula

In this section, we evaluate an expansion formula for product of the general class of polynomials, Legendre's associated function, Aleph-function and I-function of several variables. We have.

$$(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(y(1-x)^{\mathfrak{g}}(1+x)^{w}\ \middle|\ \begin{matrix}(\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r}\\(\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right)$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{matrix}\mathrm{y}_1(1-x)^{g_1}(1+x)^{w_1}\\\cdots\\\mathrm{y}_s(1-x)^{g_s}(1+x)^{w_s}\end{matrix}\right)I\left(\begin{matrix}\mathrm{z}_1(1-x)^{h_1}(1+x)^{k_1}\\\cdots\\\mathrm{z}_r(1-x)^{h_r}(1+x)^{k_r}\end{matrix}\right)$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{k=0}^{\infty}\sum_{l=0}^{k}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})(-k)_l\Gamma(v-u+k+l+1)}{B_G g!\qquad\qquad l!\Gamma(1-u+l)}a$$

$$\frac{(2k-u+v+1)\Gamma(k-u+1)}{k!\Gamma(k+v+1)}P_{k-\frac{u-v}{2}}^{u,v}(x)2^{\rho+\sigma+(\mathfrak{g}+w)\eta_{G,g}}\prod_{j=1}^{s}y_j^{K_j}2^{(g_j+w_j)K_j}y^{\eta_{G,g}}$$

$$I_{U:p_r+2,q_r+1;W}^{V;0,n_r+2;X}\left(\begin{matrix}\mathrm{z}_1 2^{h_1+k_1}\\\cdot\\\cdot\\\mathrm{z}_r 2^{h_r+k_r}\end{matrix}\ \middle|\ \begin{matrix}\mathrm{A}\ ;\ (-\sigma-v-w\eta_{G,g}-\sum_{i=1}^{s}w_iK_i:k_1,\cdots,k_r),\\\cdots\\\mathrm{B};\end{matrix}\right.$$

$$\left.\begin{matrix}(-\rho-m-\mathfrak{g}\eta_{G,g}+u-\sum_{i=1}^{s}g_iK_i:h_1,\cdots,h_r),\mathfrak{A}:A'\\\cdots\\(\text{u-v-}\rho-\sigma-m-(\mathfrak{g}+w)\eta_{G,g}-\sum_{i=1}^{s}(g_i+w_i)K_i:h_1+k_1,\cdots,h_r+k_r),\mathfrak{B}:B'\end{matrix}\right)\tag{4.1}$$

which holds true under the same conditions as needed in (3.1)

Proof

Let $f(x) = (1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}} \aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(y(1-x)^{\mathfrak{g}}(1+x)^{w} \,\middle|\, \begin{matrix} (\mathrm{a}_j, A_j)_{1,\mathfrak{n}}, [c_i(a_{ji}, A_{ji})]_{\mathfrak{n}+1,p_i;r} \\ (\mathrm{b}_j, B_j)_{1,m}, [c_i(b_{ji}, B_{ji})]_{m+1,q_i;r} \end{matrix}\right)$

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix} \mathrm{y}_1(1-x)^{g_1}(1+x)^{w_1} \\ \cdots \\ \mathrm{y}_s(1-x)^{g_s}(1+x)^{w_s} \end{pmatrix} I \begin{pmatrix} \mathrm{z}_1(1-x)^{h_1}(1+x)^{k_1} \\ \cdots \\ \mathrm{z}_r(1-x)^{h_r}(1+x)^{k_r} \end{pmatrix} = \sum_{k=0}^{\infty} c_k P^{u,v}_{k-\frac{u-v}{2}}(x) \tag{4.2}$$

The equation (4.2) is valid since $f(x)$ is continuous and bounded variation in the interval (-1,1). Now , multiplying both the sides of (4.2) by $P^{m,n}_{t-\frac{m-n}{2}}(x)$ and integrating with respect to x from -1 to 1 ; change the order of integration and summation (which is permissible) on the right,

$$\int_{-1}^{1}(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}} \aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(y(1-x)^{\mathfrak{g}}(1+x)^{w} \,\middle|\, \begin{matrix} (\mathrm{a}_j, A_j)_{1,\mathfrak{n}}, [c_i(a_{ji}, A_{ji})]_{\mathfrak{n}+1,p_i;r} \\ (\mathrm{b}_j, B_j)_{1,m}, [c_i(b_{ji}, B_{ji})]_{m+1,q_i;r} \end{matrix}\right)$$

$$P^{u,v}_{t-\frac{u-v}{2}}(x) S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix} \mathrm{y}_1(1-x)^{g_1}(1+x)^{w_1} \\ \cdots \\ \mathrm{y}_s(1-x)^{g_s}(1+x)^{w_s} \end{pmatrix} I \begin{pmatrix} \mathrm{z}_1(1-x)^{h_1}(1+x)^{k_1} \\ \cdots \\ \mathrm{z}_r(1-x)^{h_r}(1+x)^{k_r} \end{pmatrix} \mathrm{d}x$$

$$= \sum_{k=0}^{\infty} c_k \int_{-1}^{1} P^{u,v}_{k-\frac{u-v}{2}}(x) P^{u,v}_{t-\frac{u-v}{2}}(x) \mathrm{d}x \tag{4.3}$$

using the orthogonality property for the generalized Legendre's associated function on the right (2.2) on the right-hand side and the result (3.1) on the left hand side of (4.3) ; we obtain

$$c_k = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{l=0}^{k} \frac{(-)^g \Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!} \frac{(-k)_l \Gamma(v-u+k+l+1)}{l!\Gamma(1-u+l)} a$$

$$\frac{(2k-u+v+1)\Gamma(k-u+1)}{k!\Gamma(k+v+1)} P^{u,v}_{k-\frac{u-v}{2}}(x) 2^{\rho+\sigma+(\mathfrak{g}+w)\eta_{G,g}} \prod_{j=1}^{s} y_j^{K_j} 2^{(g_j+w_j)K_j} y^{\eta_{G,g}}$$

$$I^{V;0,n_r+2;X}_{U:p_r+2,q_r+1;W}\left(\begin{matrix} \mathrm{z}_1 2^{h_1+k_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r 2^{h_r+k_r} \end{matrix} \,\middle|\, \begin{matrix} \mathrm{A} \,;\, (-\sigma - v - w\eta_{G,g} - \sum_{i=1}^{s} w_i K_i : k_1, \cdots, k_r), \\ \cdots \\ \mathrm{B}; \end{matrix}\right.$$

$$\left.\begin{matrix} (-\rho - m - \mathfrak{g}\eta_{G,g} + u - \sum_{i=1}^{s} g_i K_i : h_1, \cdots, h_r), \mathfrak{A} : A' \\ \cdots \\ (\text{u-v-}\rho - \sigma - m - (\mathfrak{g}+w)\eta_{G,g} - \sum_{i=1}^{s}(g_i+w_i)K_i : h_1+k_1, \cdots, h_r+k_r), \mathfrak{B} : B' \end{matrix}\right) \tag{4.4}$$

Now on substituting the value of c_k in (4.2), the result (4.1) follows.

5. Multivariable H-function

If $U = V = A = B = 0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava et al [5]. We have the following result.

$$(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}\aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(y(1-x)^{\mathfrak{g}}(1+x)^{w}\left|\begin{matrix}(\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r}\\(\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right)$$

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{matrix}\mathrm{y}_1(1-x)^{g_1}(1+x)^{w_1}\\\cdots\\\mathrm{y}_s(1-x)^{g_s}(1+x)^{w_s}\end{matrix}\right)H\left(\begin{matrix}\mathrm{z}_1(1-x)^{h_1}(1+x)^{k_1}\\\cdots\\\mathrm{z}_r(1-x)^{h_r}(1+x)^{k_r}\end{matrix}\right)$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{k=0}^{\infty}\sum_{l=0}^{k}\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})(-k)_l\Gamma(v-u+k+l+1)}{B_G g!\qquad l!\Gamma(1-u+l)}a$$

$$\frac{(2k-u+v+1)\Gamma(k-u+1)}{k!\Gamma(k+v+1)}P^{u,v}_{k-\frac{u-v}{2}}(x)2^{\rho+\sigma+(\mathfrak{g}+w)\eta_{G,g}}\prod_{j=1}^{s}y_j^{K_j}2^{(g_j+w_j)K_j}y^{\eta_{G,g}}$$

$$H^{0,n_r+2;X}_{p_r+2,q_r+1;W}\left(\begin{matrix}\mathrm{z}_1 2^{h_1+k_1}\\\cdot\\\cdot\\\mathrm{z}_r 2^{h_r+k_r}\end{matrix}\left|\begin{matrix}(\text{-}\sigma-v-w\eta_{G,g}-\sum_{i=1}^{s}w_iK_i:k_1,\cdots,k_r),\\\cdots\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(\text{-}\rho-m-\mathfrak{g}\eta_{G,g}+u-\sum_{i=1}^{s}g_iK_i:h_1,\cdots,h_r),\mathfrak{A}:A'\\\cdots\\(\text{u-v-}\rho-\sigma-m-(\mathfrak{g}+w)\eta_{G,g}-\sum_{i=1}^{s}(g_i+w_i)K_i:h_1+k_1,\cdots,h_r+k_r),\mathfrak{B}:B'\end{matrix}\right)\tag{5.1}$$

which holds true under the same conditions as needed in (3.1) with $U = V = A = B = 0$

6. Conclusion

The I-function of several variables definef by Prasad [3] presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as , multivariable H-function , defined by Srivastava et al [5].

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