Fractional derivatives involving multivariable I-function I

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ABSTRACT
In this document, we derive three key formulas for the fractional derivatives of the multivariable I-function defined by Prasad [2] which is defined by a multiple contour integral of Mellin-Barnes type.

Keywords:Fractional derivative, multivariable I-function, Generalized Leibnitz rule.

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1.Introduction and preliminaries.

Srivastava et al [3] have obtained a number of key formulas for the fractional derivatives of the multivariable H-function defined by Srivastava et al [4].The main of this paper is obtained three formulas for the fractional derivatives of the multivariable I-function defined by Prasad [2].

The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha^{(1)}_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a^{(1)}_j,\alpha^{(1)}_j)_{1,p^{(1)}};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta^{(1)}_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b^{(1)}_j,\beta^{(1)}_j)_{1,q^{(1)}};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}}\alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}}\beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)} + \left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s}\alpha_{sk}^{(i)}-\sum_{k=n_s+1}^{p_s}\alpha_{sk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_s}\beta_{sk}^{(i)}\right) \tag{1.3}$$

where $i=1,\cdots,r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,),max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$I(z_1,\cdots,z_r)\;=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,),min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty$$

where $k=1,\cdots,r:\alpha'_k=min[Re(b_j^{(k)}/\beta_j^{(k)})],j=1,\cdots,m_k$ and

$$\beta'_k=max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})],j=1,\cdots,n_k$$

We will use these following notations in this paper:

$$U_r=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};V_r=0,n_2;0,n_3;\cdots;0,n_{r-1} \tag{1.4}$$

$$W_r=(p^{(1)},q^{(1)});\cdots;(p^{(r)},q^{(r)});X_r=(m^{(1)},n^{(1)});\cdots;(m^{(r)},n^{(r)}) \tag{1.5}$$

$$A=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)});\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)}) \tag{1.6}$$

$$B=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)});\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)}) \tag{1.7}$$

$$\mathfrak{A}=(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)}):\mathfrak{B}=(b_{rk};\beta_{rk}^{(1)},\beta_{rk}^{(2)},\cdots,\beta_{rk}^{(r)}) \tag{1.8}$$

$$A_1=(a_k^{(1)},\alpha_k^{(1)})_{1,p^{(1)}};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}};B_1=(b_k^{(1)},\beta_k^{(1)})_{1,q^{(1)}};\cdots;(b_k^{(r)},\beta_k^{(r)})_{1,q^{(r)}} \tag{1.9}$$

The multivariable I-function of r-variables write :

$$I(z_1,\cdots,z_r)=I_{U_r:p_r,q_r;W_r}^{V_r;0,n_r;X_r}\left(\begin{array}{c|c} z_1 & A\;;\mathfrak{A};A_1 \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & B;\;\mathfrak{B};\;B_1 \end{array}\right) \tag{1.10}$$

$$I(z'_1,z'_2,...z'_s)=I_{p'_2,q'_2,p'_3,q'_3;\cdots;p'_s,q'_s:p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)}}^{0,n'_2;0,n'_3;\cdots;0,n'_r:m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)}}\left(\begin{array}{c|c} z'_1 & (a'_{2j};\alpha_{2j}'^{(1)},\alpha_{2j}'^{(2)})_{1,p'_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ z'_s & (b'_{2j};\beta_{2j}'^{(1)},\beta_{2j}'^{(2)})_{1,q'_2};\cdots; \end{array}\right.$$

$$\left.\begin{matrix}(\mathrm{a'}_{sj};\alpha^{(1)\prime}{}_{sj},\cdots,\alpha'_{sj}{}^{(s)})_{1,p'_s}:(a_j'^{(1)},\alpha_j'^{(1)})_{1,p'^{(1)}};\cdots;(a'_j{}^{(s)},\alpha'_j{}^{(s)})_{1,p'^{(s)}}\\ \\ (\mathrm{b'}_{sj};\beta'^{(1)}_{sj},\cdots,\beta'_{sj}{}^{(s)})_{1,q'_s}:(b_j'^{(1)},\beta_j'^{(1)})_{1,q'^{(1)}};\cdots;(b'_j{}^{(s)},\beta'_j{}^{(s)})_{1,q'^{(s)}}\end{matrix}\right) \tag{1.11}$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\psi(t_1,\cdots,t_s)\prod_{i=1}^{s}\xi_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.12}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

where $|argz'_i|<\frac{1}{2}\Omega'_i\pi$,

$$\Omega'_i=\sum_{k=1}^{n'^{(i)}}\alpha'_k{}^{(i)}-\sum_{k=n'^{(i)}+1}^{p'^{(i)}}\alpha'_k{}^{(i)}+\sum_{k=1}^{m'^{(i)}}\beta'_k{}^{(i)}-\sum_{k=m^{(i)}+1}^{q'^{(i)}}\beta'_k{}^{(i)}+\left(\sum_{k=1}^{n'_2}\alpha'_{2k}{}^{(i)}-\sum_{k=n_2+1}^{p'_2}\alpha'_{2k}{}^{(i)}\right)$$

$$+\cdots+\left(\sum_{k=1}^{n'_s}\alpha'_{sk}{}^{(i)}-\sum_{k=n'_s+1}^{p'_s}\alpha'_{sk}{}^{(i)}\right)-\left(\sum_{k=1}^{q'_2}\beta'_{2k}{}^{(i)}+\sum_{k=1}^{q'_3}\beta'_{3k}{}^{(i)}+\cdots+\sum_{k=1}^{q'_s}\beta'_{sk}{}^{(i)}\right) \tag{1.13}$$

where $i=1,\cdots,s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z'_1,\cdots,z'_s)=0(\,|z'_1|^{\alpha'_1},\ldots,|z'_s|^{\alpha'_s}\,)\,,max(\,|z'_1|\,,\cdots,|z'_s|\,)\rightarrow 0$$

$$I(z'_1,\cdots,z'_s)\;=0(\,|z'_1|^{\beta'_1}\;,\cdots,|z'_s|^{\beta'_s}\,)\,,min(\,|z'_1|\,,\cdots,|z'_s|\,)\rightarrow\infty$$

where $k=1,\cdots,z:\alpha''_k=min[Re(b_j'^{(k)}/\beta_j'^{(k)})],j=1,\cdots,m'_k$ and

$$\beta''_k=max[Re((a_j'^{(k)}-1)/\alpha_j'^{(k)})],j=1,\cdots,n'_k$$

We will use these following notations in this paper :

$$U_s=p'_2,q'_2;p'_3,q'_3;\cdots;p'_{s-1},q'_{s-1};V_s=0,n_2;0,n_3;\cdots;0,n_{s-1} \tag{1.14}$$

$$W_s=(p'^{(1)},q'^{(1)});\cdots;(p'^{(s)},q'^{(s)});X_s=(m'^{(1)},n'^{(1)});\cdots;(m'^{(s)},n'^{(s)}) \tag{1.15}$$

$$A'=(a'_{2k};\alpha'^{(1)}_{2k},\alpha'^{(2)}_{2k});\cdots;(a'_{(s-1)k};\alpha'^{(1)}_{(s-1)k},\alpha'^{(2)}_{(s-1)k},\cdots,\alpha'^{(s-1)}_{(s-1)k}) \tag{1.16}$$

$$B' = (b'_{2k}; \beta'^{(1)}_{2k}, \beta'^{(2)}_{2k}); \cdots ; (b'_{(s-1)k}; \beta'^{(1)}_{(s-1)k}, \beta'^{(2)}_{(s-1)k}, \cdots , \beta'^{(s-1)}_{(s-1)k}) \tag{1.17}$$

$$\mathfrak{A}' = (a'_{sk}; \alpha'^{(1)}_{sk}, \alpha'^{(2)}_{sk}, \cdots , \alpha'^{(s)}_{sk}) : \mathfrak{B}' = (b'_{sk}; \beta'^{(1)}_{sk}, \beta'^{(2)}_{sk}, \cdots , \beta'^{(s)}_{sk}) \tag{1.18}$$

$$A'_1 = (a'^{(1)}_k, \alpha^{(1)}_k)_{1,p^{(1)}}; \cdots ; (a'^{(s)}_k, \alpha'^{(s)}_k)_{1,p'^{(s)}}; B'_1 = (b'^{(1)}_k, \beta'^{(1)}_k)_{1,p'^{(1)}}; \cdots ; (b'^{(s)}_k, \beta'^{(s)}_k)_{1,p'^{(s)}} \tag{1.19}$$

The multivariable I-function write :

$$I(z'_1, \cdots , z'_s) = I^{V_s;0,n'_s;X_s}_{U_s:p'_s,q'_s;W_s}\left(\begin{array}{c|c} \text{z'}_1 & \text{A' };\mathfrak{A}';\text{A'}_1 \\ \cdot & \\ \cdot & \\ \cdot & \\ \text{z'}_s & \text{B'}; \mathfrak{B}'; \text{B'}_1 \end{array}\right) \tag{1.20}$$

The Riemann-Liouville fractional derivative (or integral) of order μ is defined as follows [1 ,page49]

$$D^{\mu}_x f(x) = \begin{cases} \frac{1}{\Gamma(-u)} \int_0^x (x-t)^{-\mu-1} f(t)\mathrm{d}t, Re(\mu) < 0 \\ \quad \cdots \\ \frac{d^m}{dx^m}\left[D^{\mu-m}_x \{f(x)\}\right], 0 \leqslant m, m \in \mathbb{N} \end{cases} \tag{1.21}$$

We have the following fractional derivative formula [1,page 67, eq. (4.4.4)] is also required :

$$D^{\mu}_x (x^{\lambda}) = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\mu+1)} x^{\lambda-\mu}, Re(\lambda) > -1 \tag{1.22}$$

The generalized Leibnitz formula for fractional calculus is required in the following form [1,page 76, eq.(5.5.2)]

$$D^{\mu}_x [f(x)g(x)] = \sum_{l=0}^{\infty} \binom{\mu}{l} D^{\mu-l}_x [f(x)] D^l_x [g(x)] \tag{1.23}$$

μ is a real or complex arbitrary number.

We have the binomial formula : $(x+a)^{\lambda} = a^{\lambda} \sum_{m=0}^{\infty} \binom{\lambda}{m} \left(\frac{x}{a}\right)^m ; \left|\frac{x}{a}\right| < 1$ (1.24)

2. Main results

Formula 1

$$D^{\mu}_x \left\{ x^k (x^{\upsilon}+\zeta)^{\lambda} I\left[z_1 x^{\rho_1}(x^{\upsilon}+\zeta)^{-\sigma_1}, \cdots , z_r x^{\rho_r}(x^{\upsilon}+\zeta)^{-\sigma_r}\right]\right\} = \zeta^{\lambda} x^{k-\mu} \sum_{m=0}^{\infty} \frac{(-x^{\upsilon}/\zeta)^m}{m!}$$

$$I_{U_r:p_r+2,q_r+2;W_r}^{V_r;0,n_r+2;X_r}\left(\begin{array}{c|c} \mathrm{z}_1\zeta^{-\sigma_1}x^{\rho_1} & \mathrm{A}\ ;(1+\lambda-m;\sigma_1,\cdots,\sigma_r),(-k-\upsilon m;\rho_1,\cdots,\rho_r),\mathfrak{A};A_1 \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r\zeta^{-\sigma_r}x^{\rho_r} & \mathrm{B};\ (1+\lambda;\sigma_1,\cdots,\sigma_r),(\mu-k-\upsilon m;\rho_1,\cdots,\rho_r),\mathfrak{B};B_1 \end{array}\right) \tag{2.1}$$

Provided that :

a) $min\{\upsilon,\rho_i,\sigma_i\}>0, i=1,\cdots,r; \left|arg\left(\frac{x^\upsilon}{\zeta}\right)\right|<\pi$

b) $Re\left[k+\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m^{(i)}}\frac{b_j^{(i)}}{\beta_j^{(i)}}\right]>-1$

c) $|arg\,(z_i)|<\frac{1}{2}\Omega_i\pi\ \ (i=1,\cdots,r)$ where Ω_i is defined by (1.3)

Formula 2

$$D_x^\mu\left\{x^k(x^\upsilon+\zeta)^\lambda I\left[z_1x^{\rho_1}(x^\upsilon+\zeta)^{-\sigma_1},\cdots,z_rx^{\rho_r}(x^\upsilon+\zeta)^{-\sigma_r}\right]I^*\left[w_1x^{\lambda_1},\cdots,w_sx^{\lambda_s}\right]\right\}$$

$$=\zeta^\lambda x^{k-\mu}\sum_{l,m=0}^{\infty}\binom{\mu}{l}\frac{(-x^\upsilon/\zeta)^m}{m!}I_{U_r:p_r+2,q_r+2;W_r}^{V_r;0,n_r+2;X_r}\left(\begin{array}{c|c} \mathrm{z}_1\zeta^{-\sigma_1}x^{\rho_1} & \mathrm{A}\ ;(1+\lambda-m;\sigma_1,\cdots,\sigma_r), \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r\zeta^{-\sigma_r}x^{\rho_r} & \mathrm{B};\ (1+\lambda;\sigma_1,\cdots,\sigma_r), \end{array}\right.$$

$$\left.\begin{array}{c} \text{-}\upsilon m;\rho_1,\cdots,\rho_r),\mathfrak{A};A_1 \\ \\ (\text{l-}\upsilon m;\rho_1,\cdots,\rho_r),\mathfrak{B};B_1 \end{array}\right) I_{U_s:p_s'+1,q_s'+1;W_s}^{V_s;0,n_s'+1;X_s}\left(\begin{array}{c|c} \mathrm{w}_1x^{\lambda_1} & \mathrm{A'}\ ;(\text{-k};\lambda_1,\cdots,\lambda_s),\mathfrak{A}';A_1' \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{w}_sx^{\lambda_s} & \mathrm{B'};(\mu-k-l;\lambda_1,\cdots,\lambda_s),\mathfrak{B}';B_1' \end{array}\right) \tag{2.2}$$

where $I^*(.,\cdots,.)$ is the s-variables I-function (occuring on the right-hand side) without the additional parameters $(-k;\lambda_1,\cdots,\lambda_s)$ and $(\mu-k-l;\lambda_1,\cdots,\lambda_s)$

Provided that :

a) $min\{\upsilon,\rho_i,\sigma_i,\lambda_j\}>0, i=1,\cdots,r; j=1,\cdots,s; \left|arg\left(\frac{x^\upsilon}{\zeta}\right)\right|<\pi$

b) $\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m^{(i)}}\frac{b_j^{(i)}}{\beta_j^{(i)}}>-1;\ Re\left[k+\sum_{i=1}^{s}\lambda_i\min_{1\leqslant j\leqslant m'^{(i)}}\frac{b_j'^{(i)}}{\beta_j'^{(i)}}\right]>-1$

c) $|arg\,(z_i)| < \frac{1}{2}\Omega_i\pi \;\; (i = 1, \cdots, r)$and $\;|arg\,(w_i)| < \frac{1}{2}\Omega'_i\pi \;\; (i = 1, \cdots, s)$

where $\Omega_i^{(k)}$ and $\Omega_i'^{(k)}$ are defined respectively by (1.3) and (1.13)

Formula 3

$$D_x^{\mu} D_x^{\mu'} \left\{ x^k y^{k'} (x^{\upsilon} + \zeta)^{\lambda} (y^{\upsilon'} + \eta)^{\lambda'} I \begin{pmatrix} z_1 x^{\rho_1} y^{\lambda_1} (x^{\upsilon} + \zeta)^{-\sigma_1} (y^{\upsilon'} + \eta)^{-\upsilon_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r x^{\rho_r} y^{\lambda_r} (x^{\upsilon} + \zeta)^{-\sigma_r} (y^{\upsilon'} + \eta)^{-\upsilon_r} \end{pmatrix} \right\}$$

$$= \zeta^{\lambda} \eta^{\lambda'} x^{k-\mu} y^{k'-\mu'} \sum_{l,m=0}^{\infty} \frac{(-x^{\upsilon}/\zeta)^l (-y^{\upsilon'}/\eta)^m}{l!m!} I_{U_r:p_r+4,q_r+4;W_r}^{V_r;0,n_r+4;X_r} \left(\begin{array}{c} z_1 \zeta^{-\sigma_1} x^{\rho_1} \eta^{-\upsilon_1} x^{\lambda_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r \zeta^{-\sigma_r} x^{\rho_r} \eta^{-\upsilon_r} x^{\lambda_r} \end{array} \right|$$

A ;(1+$\lambda - l; \sigma_1, \cdots, \sigma_r$),(1+$\lambda' - m; \upsilon_1, \cdots, \upsilon_r$), (-k -$\upsilon l; \rho_1, \cdots, \rho_r$),

B; (1+$\lambda; \sigma_1, \cdots, \sigma_r$), (1+$\lambda'; \upsilon_1, \cdots, \upsilon_r$), $(\mu - k - \upsilon l; \rho_1, \cdots, \rho_r)$,

-k'-$\upsilon' m; \lambda_1, \cdots, \lambda_r), \mathfrak{A}; A_1$

$(\mu' - k' - \upsilon' m; \lambda_1, \cdots, \lambda_r), \mathfrak{B}; B_1 \Big)$ (2.3)

Provided that :

a) $min\{\upsilon, \upsilon' \rho_i, \sigma_i, \lambda_i, \upsilon_i\} > 0, i = 1, \cdots, r; \left|arg\left(\frac{x^{\upsilon}}{\zeta}\right)\right| < \pi, \left|arg\left(\frac{y^{\upsilon'}}{\eta}\right)\right| < \pi$

b) $Re\left[k + \sum_{i=1}^{r} \rho_i \min_{1 \leqslant j \leqslant m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}}\right] > -1 \,;\, Re\left[k' + \sum_{i=1}^{r} \lambda_i \min_{1 \leqslant j \leqslant m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}}\right] > -1$

Proof

To prove (2.1) we first replace the multivariable I-function occuring on the left-hand side by its Mellin-Barnes contour integral, collect the power of x and $(x^{\upsilon} + \zeta)$ and apply the binomial expansion with the help of (1.24). We can apply the formula (1.22) and interpret the resulting Mellin-Barnes contour integral as an I-function of r-variables.

To prove (2.2), we make use of the generalized Leibniz rule for fractional derivatives with the help of (1.23).

with $f(x) = I^* \left[w_1 x^{\lambda_1}, \cdots, w_s x^{\lambda_s} \right]$ and

$$g(x) = (x^{\upsilon} + \zeta)^{\lambda} I \left[z_1 x^{\rho_1} (x^{\upsilon} + \zeta)^{-\sigma_1}, \cdots, z_r x^{\rho_r} (x^{\upsilon} + \zeta)^{-\sigma_r} \right]$$

and apply two special cases of the formula (2.1) when $\mu \rightarrow \mu - l; \lambda = 0, \rho_i = \lambda_i$ and $\sigma_i \rightarrow 0; i = 1, \cdots, r$ or when

$\mu = m \ (m \in \mathbb{N}_0)$, we arrive at (2.2), the s-variables I^*involved in (2.2), being identical.

To prove (2.3) , we apply the fractional derivative (2.1) twice, first with respect to y, and then with respect to x ; here x and y are assumed to be independent variables.

3. Conclusion

In this paper we have evaluated three formulas concerning the fractional derivatives and the multivariable I-function defined by Prasad [2]. The three formulaes established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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