

Fourier expansion of generalized prolate spheroidal wave function concerning generalized polynomials ,Aleph-function and multivariable I-function

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ABSTRACT

The object of the paper is to establish an integral formula involving the generalized prolate spheroidal wave function, generalized hypergeometric function, generalized polynomials, Aleph-function of one variable and multivariable I-function. This integral formula has been employed to obtain an expansion formula for multivariable I-function, generalized hypergeometric function, a class of polynomial and Aleph-function in terms of generalized prolate spheroidal wave function. This expansion formula being of very general nature can be transformed to provide many new results involving various commonly used special functions occurring in applied mathematics, mathematics physics and mechanics. During the course of finding, we establish several particular cases.

Keywords :generalized multivariable I-function, Aleph-function, class of multivariable polynomials, generalized hypergeometric function, finite integral, generalized prolate spheroidal wave function, expansion formula, multivariable H-function.

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1.Introduction and preliminaries.

The object of the paper is to establish an integral formula involving the generalized prolate spheroidal wave function, generalized hypergeometric function, generalized polynomials, Aleph-function of one variable and multivariable I-function. We will study the case concerning the multivariable H-function.

The Aleph- function , introduced by Südland [8] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z) = \aleph_{P_i, Q_i, c_i; r}^{M, N} \left(z \left| \begin{matrix} (a_j, A_j)_{1, n}, [c_i(a_{ji}, A_{ji})]_{n+1, p_i; r} \\ (b_j, B_j)_{1, m}, [c_i(b_{ji}, B_{ji})]_{m+1, q_i; r} \end{matrix} \right. \right) = \frac{1}{2\pi\omega} \int_L \Omega_{P_i, Q_i, c_i; r}^{M, N}(s) z^{-s} ds \quad (1.1)$$

for all z different to 0 and

$$\Omega_{P_i, Q_i, c_i; r}^{M, N}(s) = \frac{\prod_{j=1}^M \Gamma(b_j + B_j s) \prod_{j=1}^N \Gamma(1 - a_j - A_j s)}{\sum_{i=1}^r c_i \prod_{j=N+1}^{P_i} \Gamma(a_{ji} + A_{ji} s) \prod_{j=M+1}^{Q_i} \Gamma(1 - b_{ji} - B_{ji} s)} \quad (1.2)$$

With :

$$|\arg z| < \frac{1}{2}\pi\Omega \quad \text{Where } \Omega = \sum_{j=1}^M \beta_j + \sum_{j=1}^N \alpha_j - c_i \left(\sum_{j=M+1}^{Q_i} \beta_{ji} + \sum_{j=N+1}^{P_i} \alpha_{ji} \right) > 0 \quad \text{with } i = 1, \dots, r$$

For convergence conditions and other details of Aleph-function , see Südland et al [8].

Serie representation of Aleph-function is given by Chaurasia et al [1].

$$\aleph_{P_i, Q_i, c_i; r}^{M, N}(z) = \sum_{G=1}^M \sum_{g=0}^{\infty} \frac{(-)^g \Omega_{P_i, Q_i, c_i, r}^{M, N}(s)}{B_G g!} z^{-s} \quad (1.3)$$

With $s = \eta_{G,g} = \frac{b_G + g}{B_G}$, $P_i < Q_i$, $|z| < 1$ and $\Omega_{P_i, Q_i, c_i; r}^{M, N}(s)$ is given in (1.2) (1.4)

The generalized polynomials of multivariables defined by Srivastava [6], is given in the following manner :

$$S_{N_1, \dots, N_s}^{M_1, \dots, M_s}[y_1, \dots, y_s] = \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1 K_1}}{K_1!} \dots \frac{(-N_s)_{M_s K_s}}{K_s!} A[N_1, K_1; \dots; N_s, K_s] y_1^{K_1} \dots y_s^{K_s} \tag{1.5}$$

where M_1, \dots, M_s are arbitrary positive integers and the coefficients are $A[N_1, K_1; \dots; N_s, K_s]$ arbitrary constants, real or complex.

The generalized hypergeometric function series is defined as follows :

$${}_p F_q(y) = \sum_{s'=0}^{\infty} \frac{[(a_{p'})]_{s'}}{[(b_{q'})]_{s'}} y^{s'} \tag{1.6}$$

Here $[(a_{p'})]_{s'} = (a_1)_{s'} \dots (a_{p'})_{s'}$; $[(b_{q'})]_{s'} = (b_1)_{s'} \dots (b_{q'})_{s'}$.

The serie (1.7) converge if $p' \leq q'$ and $|y| < 1$.

In the document , we note : $a_1 = \frac{(-N_1)_{M_1 K_1}}{K_1!} \dots \frac{(-N_s)_{M_s K_s}}{K_s!} A[N_1, K_1; \dots; N_s, K_s]$ (1.7)

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1, \dots, z_r) = I_{p_2, q_2, p_3, q_3; \dots; p_r, q_r; p', q'; \dots; p^{(r)}, q^{(r)}}^{0, n_2; 0, n_3; \dots; 0, n_r; m', n'; \dots; m^{(r)}, n^{(r)}} \left(\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} (a_{2j}; \alpha'_{2j}, \alpha''_{2j})_{1, p_2}; \dots; \\ (b_{2j}; \beta'_{2j}, \beta''_{2j})_{1, q_2}; \dots; \end{matrix} \right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \xi(t_1, \dots, t_r) \prod_{i=1}^s \phi_i(s_i) z_i^{s_i} ds_1 \dots ds_r \tag{1.9}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [4]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|arg z_i| < \frac{1}{2} \Omega_i \pi, \text{ where}$$

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) +$$

$$+ \left(\sum_{k=1}^{n_r} \alpha_{rk}^{(i)} - \sum_{k=n_r+1}^{p_r} \alpha_{rk}^{(i)} \right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \dots + \sum_{k=1}^{q_r} \beta_{rk}^{(i)} \right) \quad (1.10)$$

where $i = 1, \dots, r$

The complex numbers z_i are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the asymptotic expansion in the following convenient form :

$$I(z_1, \dots, z_r) = O(|z_1|^{\gamma'_1}, \dots, |z_r|^{\gamma'_r}), \max(|z_1|, \dots, |z_r|) \rightarrow 0$$

$$I(z_1, \dots, z_r) = O(|z_1|^{\beta'_s}, \dots, |z_r|^{\beta'_s}), \min(|z_1|, \dots, |z_r|) \rightarrow \infty$$

where $k = 1, \dots, r : \alpha'_k = \min[Re(b_j^{(k)} / \beta_j^{(k)})], j = 1, \dots, m_k$ and

$$\beta'_k = \max[Re((a_j^{(k)} - 1) / \alpha_j^{(k)})], j = 1, \dots, n_k$$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \dots; p_{r-1}, q_{r-1}; V = 0, n_2; 0, n_3; \dots; 0, n_{s-1} \quad (1.11)$$

$$W = (p', q'); \dots; (p^{(r)}, q^{(r)}); X = (m', n'); \dots; (m^{(r)}, n^{(r)}) \quad (1.12)$$

$$A = (a_{2k}, \alpha'_{2k}, \alpha''_{2k}); \dots; (a_{(r-1)k}, \alpha'_{(r-1)k}, \alpha''_{(r-1)k}, \dots, \alpha^{(r-1)}_{(r-1)k}) \quad (1.13)$$

$$B = (b_{2k}, \beta'_{2k}, \beta''_{2k}); \dots; (b_{(r-1)k}, \beta'_{(r-1)k}, \beta''_{(r-1)k}, \dots, \beta^{(r-1)}_{(r-1)k}) \quad (1.14)$$

$$\mathfrak{A} = (a_{sk}; \alpha'_{rk}, \alpha''_{rk}, \dots, \alpha^r_{rk}) : \mathfrak{B} = (b_{rk}; \beta'_{rk}, \beta''_{rk}, \dots, \beta^r_{rk}) \quad (1.15)$$

$$A' = (a'_k, \alpha'_k)_{1,p'}; \dots; (a^{(r)}_k, \alpha^{(r)}_k)_{1,p^{(r)}}; B' = (b'_k, \beta'_k)_{1,p'}; \dots; (b^{(r)}_k, \beta^{(r)}_k)_{1,p^{(r)}} \quad (1.16)$$

The multivariable I-function write :

$$I(z_1, \dots, z_r) = I_{U;p_r,q_r;W}^{V;0,n_r;X} \left(\begin{array}{c|c} z_1 & A; \mathfrak{A}; A' \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & B; \mathfrak{B}; B' \end{array} \right) \quad (1.17)$$

2. Generalized prolate spheroidal wave function

The generalized prolate spheroidal wave functions has been recently defined by Gupta [3] as the solution of the differential equation

$$(1 - x^2)y'' + [\beta - \alpha - (\alpha + \beta + 2)x]y' + [\zeta(c) - c^2x^2]y = 0 \tag{2.1}$$

in the form of an infinite sum :

$$\phi_n^{(\alpha,\beta)}(c, x) = \sum_{j=0}^{\infty} d_{j,n}^{(\alpha,\beta)}(c) P_{n+j}^{(\alpha,\beta)}(x) \tag{2.2}$$

where $\zeta(c)$ being separation constants for every value of constant parameter c and the coefficients $d_{j,n}^{(\alpha,\beta)}(c)$ can be determined by a five term recursion formula in a manner quite parallel to prolate spheroidal wave functions. More recently, Sharma [5] has developed multiple generalized prolate spheroidal wave transforms by using the orthogonality property given by Gupta [3].

$$\int_{-1}^1 (1 - x)^\alpha (1 + x)^\beta \phi_n^{(\alpha,\beta)}(c, x) \phi_m^{(\alpha,\beta)}(c, x) dx = N_{n,m}^{(\alpha,\beta)} \delta_{m,n} \text{ ,where} \tag{2.3}$$

$$N_{n,n}^{(\alpha,\beta)} = 2^{\alpha+\beta+1} \sum_{j=0}^{\infty} \left[d_{j,n}^{(\alpha,\beta)} \right]^2 \frac{\Gamma(\alpha + j + 1) \Gamma(\beta + j + 1)}{(1 + \alpha + \beta + 2n + 2j) \Gamma(n + j + 1) \Gamma((1 + \alpha + \beta + n + j))} \tag{2.4}$$

$\delta_{m,n}$ is the Kronecker symbol.

3.Required integral

We have the following integral (Erdelyi et al [2], page 284, eq.3)

$$\int_{-1}^1 (1 - x)^\rho (1 + x)^\sigma P_n^{(\alpha,\beta)}(x) dx = \frac{2^{\rho+\sigma+1} \Gamma(\rho + 1) \Gamma(\sigma + 1)}{n \Gamma(\rho + \sigma + 2)}$$

$${}_3F_2[-n, \alpha + \beta + n + 1, \rho + 1; \alpha + 1, \rho + \sigma + 2 ; 1] , Re(\rho) > 0, Re(\sigma) > 0 \tag{3.1}$$

4. Main integral

The integral formula to be established here is

$$\int_{-1}^1 (1 - x)^\rho (1 + x)^\sigma \phi_n^{(\alpha,\beta)} \{c, x\} \mathfrak{N}_{P_i, Q_i, c_i; r'}^{M, N} (t(1 - x)^e (1 + x)^f)$$

$${}_M F_N((a_M); (b_N); y(1 - x)^{e'} (1 + x)^{f'}) S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \left(\begin{matrix} t_1(1 - x)^{g_1} (1 + x)^{w_1} \\ \dots \\ t_s(1 - x)^{g_s} (1 + x)^{w_s} \end{matrix} \right)$$

$$I \left(\begin{matrix} z_1(1 - x)^{h_1} (1 + x)^{k_1} \\ \dots \\ z_r(1 - x)^{h_r} (1 + x)^{k_r} \end{matrix} \right) dx = \sum_{p,q=0}^{\infty} \sum_{G=1}^M \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_s=0}^{[N_s/M_s]} a_1 \frac{(-)^g \Omega_{P_i, Q_i, c_i, r'}^{M, N}(\eta_{G, g})}{B_G g!}$$

$$\frac{\prod_{i=1}^M (a_i)_q}{\prod_{i=1}^N (b_i)_q q!} t^{\eta_{G,g}} x_1^{K_1} \dots t_s^{K_s} y^q 2^{(\eta_{G,g}(e+f)+q(e'+f')+\sum_{i=0}^s K_i(g_i+w_i))} d_{p,n}^{(\alpha,\beta)}(c)$$

$$\sum_{m=0}^{n+p} \frac{(-n-p)_m (\alpha + \beta + n + p + 1)_m}{m! (\alpha + 1)_m} I_{U:p_r+2,q_r+1;W}^{V;0,n_r+2;X} \left(\begin{array}{c|c} 2^{h_1+k_1} z_1 & A ; \\ \dots & \dots \\ \dots & \dots \\ 2^{h_r+k_r} z_r & B ; \end{array} \right. \\ \left. \begin{array}{l} (-m-\rho - e\eta_{G,g} - e'q - \sum_{i=1}^s K_i g_i; h_1, \dots, h_r), (-\sigma - f\eta_{G,g} - f'q - \sum_{i=1}^s K_i w_i; k_1, \dots, k_r), \mathfrak{A} : A' \\ (-1-m-\rho - \sigma - (e+f)\eta_{G,g} - (e'+f')q - \sum_{i=1}^s K(g_i + w_i); h_1 + k_1, \dots, h_r + k_r), \mathfrak{B} : B' \end{array} \right) \quad (4.1)$$

Provided

a) $\min\{e, f, e', f', g_i, w_i, h_j, k_j\} > 0, i = 1, \dots, s; j = 1, \dots, r$

b) $Re[\rho + e \min_{1 \leq j \leq M} \frac{b_j}{B_j} + \sum_{i=1}^r h_i \min_{1 \leq j \leq m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}}] > -1$

c) $Re[\sigma + f \min_{1 \leq j \leq M} \frac{b_j}{B_j} + \sum_{i=1}^r k_i \min_{1 \leq j \leq m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}}] > -1$

d) $|arg z_i| < \frac{1}{2} \Omega_i \pi$, where Ω_i is defined by (1.10)

e) $|arg t| < \frac{1}{2} \pi \Omega$ where $\Omega = \sum_{j=1}^M \beta_j + \sum_{j=1}^N \alpha_j - c_i (\sum_{j=M+1}^{Q_i} \beta_{ji} + \sum_{j=N+1}^{P_i} \alpha_{ji}) > 0$

f) $\alpha > -1, \beta > -1$ and $M \leq N$ ($M = N + 1$ and $|y| < 1$)

Proof Let $M = \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \xi(s_1, \dots, s_r) \prod_{k=1}^r \phi_k(s_k) z_k^{s_k}$

To establish (4.1), express the generalized prolate spheroidal wave function as given in (2.2), the generalized hypergeometric function in serie with the help of (1.6), the Aleph-function of one variable in serie with the help of (1.3), the general polynomials with the help of (1.5) and the multivariable I-function in terms of Mellin-Barnes type contour integral with the help of (1.9), changing the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process), we obtain

$$\sum_{p,q=0}^{\infty} \sum_{G=1}^M \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_s=0}^{[N_s/M_s]} a_1 \frac{(-)^g \Omega_{P_i, Q_i, c_i, r'}^{M, N} (\eta_{G,g})}{B_G g!} \frac{\prod_{i=1}^M (a_i)_q}{\prod_{i=1}^N (b_i)_q q!} t^{\eta_{G,g}} x_1^{K_1} \dots t_s^{K_s} y^q d_{p,n}^{(\alpha,\beta)}(c)$$

$$M \left[\int_{-1}^1 (1-x)^{\rho+e\eta_{G,g}+e'q+\sum_{i=1}^s K_i g_i + \sum_{i=1}^r h_i s_i} (1+x)^{\rho+f\eta_{G,g}+f'q+\sum_{i=1}^s K_i w_i + \sum_{i=1}^r k_i s_i} P_{n+p}^{(\alpha,\beta)}(x) dx \right] ds_1 \cdots ds_r \tag{4.2}$$

Now evaluating the inner x -integral with the help of (3.1). Writing series representation for ${}_3F_2$, changing the order of integration and summation involved therein and expressing the multiple contour integral as the multivariable I-function, we obtain the right hand side of (4.1).

5. Expansion formula

We have the general formula expansion

$$(1-x)^{\rho-\alpha}(1+x)^{\sigma-\beta} \aleph_{P_i, Q_i, c_i; r'}^{M, N}(t(1-x)^e(1+x)^f) {}_M F_N((a_M); (b_N); y(1-x)^{e'}(1+x)^{f'})$$

$$S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \left(\begin{matrix} t_1(1-x)^{g_1}(1+x)^{w_1} \\ \dots \\ t_s(1-x)^{g_s}(1+x)^{w_s} \end{matrix} \right) I \left(\begin{matrix} z_1(1-x)^{h_1}(1+x)^{k_1} \\ \dots \\ z_r(1-x)^{h_r}(1+x)^{k_r} \end{matrix} \right) = \sum_{l,p,q=0}^{\infty} \sum_{G=1}^M \sum_{g=0}^{\infty}$$

$$\sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} a_1 \frac{(-)^g \Omega_{P_i, Q_i, c_i; r'}^{M, N}(\eta_{G,g}) \prod_{i=1}^M (a_i)_q}{B_G g! \prod_{i=1}^N (b_i)_q q!} t^{\eta_{G,g}} x_1^{K_1} \cdots t_s^{K_s} y^q \frac{D_{p,n}^{(\alpha,\beta)}(c)}{N_{l,l}^{(\alpha,\beta)}}$$

$$2 \left(\eta_{G,g}(e+f)+q(e'+f')+\sum_{i=0}^s K_i(g_i+w_i) \right) \sum_{m=0}^{n+p} \frac{(-n-p)_m(\alpha+\beta+n+p+1)_m}{m!(\alpha+1)_m}$$

$$I_{U:p_r+2, q_r+1; W}^{V:0, n_r+2; X} \left(\begin{matrix} 2^{h_1+k_1} z_1 \\ \dots \\ 2^{h_r+k_r} z_r \end{matrix} \middle| \begin{matrix} A ; (-m-\rho - e\eta_{G,g} - e'q - \sum_{i=1}^s K_i g_i; h_1, \dots, h_r), \\ \dots \\ B ; \end{matrix} \right) \left(\begin{matrix} (-\sigma - f\eta_{G,g} - f'q - \sum_{i=1}^s K_i w_i; k_1, \dots, k_r), \mathfrak{A} : A' \\ \dots \\ (-1-m-\rho - \sigma - (e+f)\eta_{G,g} - (e'+f')q - \sum_{i=1}^s K(g_i+w_i); h_1+k_1, \dots, h_r+k_r), \mathfrak{B} : B' \end{matrix} \right) \phi_l^{(\alpha,\beta)}(c, x) \tag{5.1}$$

Proof

To establish (5.1), let $f(x) = \aleph_{P_i, Q_i, c_i; r'}^{M, N}(t(1-x)^e(1+x)^f)(1-x)^{\rho-\alpha}(1+x)^{\sigma-\beta}$

$$MF_N((a_M); (b_N); y(1-x)^{e'}(1+x)^{f'}) S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \begin{pmatrix} t_1(1-x)^{g_1}(1+x)^{w_1} \\ \dots \\ t_s(1-x)^{g_s}(1+x)^{w_s} \end{pmatrix} I \begin{pmatrix} z_1(1-x)^{h_1}(1+x)^{k_1} \\ \dots \\ z_r(1-x)^{h_r}(1+x)^{k_r} \end{pmatrix} = \sum_{l=0}^{\infty} A_l \phi_l^{(\alpha, \beta)}(c, x) \tag{5.2}$$

The equation is valid if $f(x)$ is continuous and bounded variation in the domain $(-1, 1)$. Multiplying both sides of (5.2) by $(1-x)^\alpha(1+x)^\beta \phi_n^{(\alpha, \beta)}(c, x)$, $\alpha > -1, \beta > -1$ and integrating with respect to x from -1 to 1 , change the order and summation (which is permissible) on the right, we obtain :

$$\int_{-1}^1 (1-x)^\rho(1+x)^\sigma \phi_n^{(\alpha, \beta)} \{c, x\} \aleph_{P_i, Q_i, c_i, r'}^{M, N}(t(1-x)^e(1+x)^f) MF_N((a_M); (b_N); y(1-x)^{e'}(1+x)^{f'}) S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \begin{pmatrix} t_1(1-x)^{g_1}(1+x)^{w_1} \\ \dots \\ t_s(1-x)^{g_s}(1+x)^{w_s} \end{pmatrix} I \begin{pmatrix} z_1(1-x)^{h_1}(1+x)^{k_1} \\ \dots \\ z_r(1-x)^{h_r}(1+x)^{k_r} \end{pmatrix} dx = \sum_{l=0}^{\infty} A_l \int_{-1}^1 (1-x)^\alpha(1+x)^\beta \phi_n^{(\alpha, \beta)} \{c, x\} \phi_l^{(\alpha, \beta)}(c, x) dx \tag{5.3}$$

Using the orthogonality property of the generalized prolate spheroidal wave function (2.3) on the right hand side and the result (4.1) on the left hand side of (5.3), we obtain

$$A_l = \sum_{p, q=0}^{\infty} \sum_{G=1}^M \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_s=0}^{[N_s/M_s]} a_1 \frac{(-)^g \Omega_{P_i, Q_i, c_i, r'}^{M, N}(\eta_{G, g}) \prod_{i=1}^M (a_i)_q}{B_{Gg}! \prod_{i=1}^N (b_i)_q q!} t^{\eta_{G, g}} x_1^{K_1} \dots t_s^{K_s} y^q \frac{D_{p, n}^{(\alpha, \beta)}(c)}{N_{l, l}^{(\alpha, \beta)}} 2 \left(\eta_{G, g}(e+f)+q(e'+f')+\sum_{i=0}^s K_i(g_i+w_i) \right) \sum_{m=0}^{n+p} \frac{(-n-p)_m(\alpha+\beta+n+p+1)_m}{m!(\alpha+1)_m}$$

$$I_{U: p_r+2, q_r+1; W}^{V: 0, n_r+2; X} \left(\begin{matrix} 2^{h_1+k_1} z_1 \\ \dots \\ 2^{h_r+k_r} z_r \end{matrix} \middle| \begin{matrix} A ; (-m-\rho - e\eta_{G, g} - e'q - \sum_{i=1}^s K_i g_i; h_1, \dots, h_r), \\ \dots \\ B ; \end{matrix} \right. \tag{5.4}$$

$$\left. \begin{matrix} (-\sigma - f\eta_{G, g} - f'q - \sum_{i=1}^s K_i w_i; k_1, \dots, k_r), \mathfrak{A} : A' \\ (-1-m-\rho - \sigma - (e+f)\eta_{G, g} - (e'+f')q - \sum_{i=1}^s K(g_i + w_i); h_1 + k_1, \dots, h_r + k_r), \mathfrak{B} : B' \end{matrix} \right)$$

on substituting the value of A_l from (5.4) in (5.2) and using the lemma([4],page 57, eq.2), we obtain the desired result.

6. Multivariable H-function

If $U = V = A = B = 0$, the multivariable I-function defined by Prasad degenerate in multivariable H-function defined by Srivastava et al [7]. We have the following result.

$$(1 - x)^{\rho - \alpha} (1 + x)^{\sigma - \beta} \mathfrak{N}_{P_i, Q_i, c_i, r'}^{M, N} (t(1 - x)^e (1 + x)^f) {}_M F_N((a_M); (b_N); y(1 - x)^{e'} (1 + x)^{f'})$$

$$S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \left(\begin{matrix} t_1(1 - x)^{g_1} (1 + x)^{w_1} \\ \dots \\ t_s(1 - x)^{g_s} (1 + x)^{w_s} \end{matrix} \right) H \left(\begin{matrix} z_1(1 - x)^{h_1} (1 + x)^{k_1} \\ \dots \\ z_r(1 - x)^{h_r} (1 + x)^{k_r} \end{matrix} \right) = \sum_{l, p, q=0}^{\infty} \sum_{G=1}^M \sum_{g=0}^{\infty}$$

$$\sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_s=0}^{[N_s/M_s]} a_1 \frac{(-)^g \Omega_{P_i, Q_i, c_i, r'}^{M, N} (\eta_{G, g}) \prod_{i=1}^M (a_i)_q}{B_G g!} t^{\eta_{G, g}} x_1^{K_1} \dots t_s^{K_s} y^q \frac{D_{p, n}^{(\alpha, \beta)}(c)}{N_{l, l}^{(\alpha, \beta)}}$$

$$2 \left(\eta_{G, g} (e + f) + q(e' + f') + \sum_{i=0}^s K_i (g_i + w_i) \right) \sum_{m=0}^{n+p} \frac{(-n - p)_m (\alpha + \beta + n + p + 1)_m}{m! (\alpha + 1)_m}$$

$$H_{p_r + 2, q_r + 1; W}^{0, n_r + 2; X} \left(\begin{matrix} 2^{h_1 + k_1} z_1 \\ \dots \\ 2^{h_r + k_r} z_r \end{matrix} \middle| \begin{matrix} (-m - \rho - e \eta_{G, g} - e' q - \sum_{i=1}^s K_i g_i; h_1, \dots, h_r), \\ \dots \\ \dots \end{matrix} \right) \left(\begin{matrix} (-\sigma - f \eta_{G, g} - f' q - \sum_{i=1}^s K_i w_i; k_1, \dots, k_r), \mathfrak{A} : A' \\ \dots \\ (-1 - m - \rho - \sigma - (e + f) \eta_{G, g} - (e' + f') q - \sum_{i=1}^s K(g_i + w_i); h_1 + k_1, \dots, h_r + k_r), \mathfrak{B} : B' \end{matrix} \right) \phi_l^{(\alpha, \beta)}(c, x) \tag{6.1}$$

9. Conclusion

The I-function of several variables defined by Prasad [4] presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions o several variables such as m ,multivariable Fox's H-function, Fox's H-function , Meijer's G-function, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modied Bessel function, Whittaker function, exponential function , binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

REFERENCES

- [1] Chaurasia V.B.L and Singh Y. New generalization of integral equations of fredholm type using Aleph-function
Int. J. of Modern Math. Sci. 9(3), 2014, p 208-220.
- [2] Erdelyi A. et al Tables of integral transforms, Vol II. Mc. Graw-Hill, New York (1954).
- [3] Gupta R.K. Generalized prolate spheroidal wave functions. Proc. India. Acad. Sci 85A (2), (1977),page 104-114.
- [4] Y.N. Prasad , Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , page 231-237.
- [5] Sharma S.D. Multiple generalized prolate spheroidal wave transforms and its applications . Kyungpook Mathematical Journal, Vol 20 (1) ,(1980), page 121-127.
- [6] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), page183-191.
- [7] H.M. Srivastava And R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.
- [8] Südland N.; Baumann, B. and Nonnenmacher T.F. , Open problem : who knows about the Aleph-functions?
Fract. Calc. Appl. Anal., 1(4) (1998): 401-402.

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