Multiple Integral Involving I-Function and Bessel-Maitland Functions.

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Abstract:

The I-Function introduced by Saxena [6] is most generalized form of hyper geometric functions including Meijer [4] G-function and Fox [1] H-function. In this paper we have obtained certain new integrals of I-Function. Some special cases of these results have also been worked out. The deviations make use of certain classical functions.

Keywords: Generalised hypergeometric function, Fox's H-function,Miejer's G-function Saxena's I-function,Bessel-Maitland function.

Introduction:

Solution of various generalised sets of dual and triple integral equations have been given by Fox ([1] and [2]), Saxena and Pogany [5] and Saxena [6]. In each case the responsibility of final solution goes to integral equation having H-Function as the kernel of the type corresponding to the set. But due to non-availability of much inversion formula the kernels are more or less restricted to some standard forms.

In this paper we shall give solution to the integral equation having considerably more generalized kernel. We shall use Laplace and Generalized Laplace transforms of Meijer [4] and operators of fractional integration. The generalization of Fox's H-Functions, called, I-Function is given by saxena [7], and its definition is given as:

$$I(z) = I^{m,n}_{p_i,q_i:r}\left[z\left|\frac{(a_j,\alpha_j)_{:1,n},(a_{ji},\alpha_{ji})_{:n+1,p}}{(b_j,\beta_j)_{:1,m},(b_{ji},\beta_{ji})_{:m+1,q}}\right.\right] = \frac{1}{2\pi i}\int_L \phi(\xi) z^{\xi}\, d\xi \quad \ldots\ldots\ldots..(1)$$

where

$$\phi(\xi) = \frac{\left\{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi)\right\}}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi)\ \prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi)\right\}}$$

with the conditions of existence given below.

(a) $\lambda > 0, |\arg z| < \pi\lambda/2$

(b) $\lambda \geq 0, |\arg z| \geq \frac{\pi\lambda}{2}, Re(\mu+1) < 0$

where

$$\lambda = \sum_{j=1}^{m}\alpha_j + \sum_{j=1}^{n}\beta_j - \max_{1\leq i\leq r}\left[\sum_{j=n+1}^{p_i}\alpha_{ji} + \sum_{j=m+1}^{q_i}\beta_{ji}\right]$$

$$\mu = \sum_{j=1}^{m}(b_j-\beta_j\sigma) - \sum_{j=1}^{n}(a_j-\alpha_j\sigma) - \min_{1\leq i\leq r}\left[\sum_{j=n+1}^{p_i}(a_j-\alpha_j\sigma) - \sum_{j=m+1}^{q_i}(b_j-\beta_j\sigma) + \frac{p_i}{2} - \frac{q_i}{2}\right]$$

If $\sigma = 0$, then

$$\mu = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j - \min_{1\le i\le r}\left[\sum_{j=n+1}^{p_i}\left(a_j - \sum_{j=m+1}^{q_i} b_j + \frac{p_i}{2} - \frac{q_i}{2}\right)\right]$$

(i) If we put r =1 in (1), it reduces to Fox's H-function

$$I_{p_i,q_i;1}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n},;(a_{ji},\alpha_{ji})_{n+1,pi}\\(b_j,\beta_j)_{1,m};(a_{ji},\beta_{ji})_{m+1,qi}\end{matrix}\right.\right] = H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n},;(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m};(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]$$

where $p_1 = p, q_1 = q, a_{ji} = a_j, b_{ji} = b_j, \alpha_{ji} = \alpha_j$ *and* $\beta_{ji} = \beta_j$.

(ii) If we put $\alpha_j = \beta_j = \alpha_{ji} = \beta_{ji} = 1$ in (1), it reduces to a function I_G defined as $I_G = \frac{1}{2\pi i}\int_L \varphi(\xi) z^{\xi} d\xi,$

Where $\phi(\xi) = \frac{\{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi)\}}{\sum_{i=1}^{r}\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi)\ \prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi)\}}$

(iii) If we put $r = 1, \alpha_j = \beta_j = \alpha_{ji} = \beta_{ji} = \alpha$ in (1), it reduces to a Meijer's G-function, i.e.

$$I_{p_i,q_i;1}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha)_{1,n},;(a_{ji},\alpha)_{n+1,pi}\\(b_j,\beta)_{1,m};(b_{ji},\alpha)_{m+1,qi}\end{matrix}\right.\right] = \frac{1}{\alpha}G_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,1)_{1,n};(a_j,1)_{n+1,p}\\(b_j,1)_{1,m};(b_j,1)_{m+1,q}\end{matrix}\right.\right]$$

we define Bessel Maitland function [9] as

$$\boldsymbol{J_\nu^\tau(x) = \sum_{r=0}^{\infty}\frac{(-x)^r}{r!\ \Gamma(1+\nu+\tau r)}} \qquad \ldots.(2)$$

We make use of following integral [7]

$$\boldsymbol{\int_0^\infty x^l\ J_\nu^\tau(x) = \frac{\Gamma(l+1)}{\Gamma(1+\nu-\tau-\tau l)}} \qquad \ldots(3)$$

Provided Re $(l) > -1, 0 < \tau < 1$.

Theorem 1:-

$$\int_{(1,N)}^{-}\prod_{k=1}^{N} x_k^{l_k}\prod_{k=1}^{N} J_{\nu_k}^{\tau_k}(ux_k)\ I_{p_i,q_i;r}^{m,n}\left[z\prod_{k=1}^{N} x_k^{\gamma_k}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i,}\\(b_j,\beta_j)_{1,m}(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]dz =$$

$$\frac{1}{u^{L+N}}\ I_{p_{i+N},q_i;r}^{m,n+N}\left[zu^{-\gamma}\left|\begin{matrix}(-l_k,\gamma_k)_{1,N},(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(1+\nu_k-\tau_k-\tau_k l_k,\nu_k\tau_k)_{1,N}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]$$

where

$$\int_{(1,N)}^{-} = \int_0^\infty \int_0^\infty \int_0^\infty \ldots \ldots \int_0^\infty N\ times \ , \quad dX = dx_1\, dx_2 \ldots\ldots dx_N \ , L = \sum_{i=1}^{N} l_i \ ,$$

……… (4)

provided

(a) $\lambda > 0|\arg z| < \frac{1}{2}\pi\lambda$
(b) $\lambda \geq 0, |\arg z| < \frac{1}{2}\pi\lambda, Re\,(\mu+1) < 0, Re(\mu + l - \nu + \tau + \tau l + 2) < 0$
(c) $\gamma - \tau\gamma > 0, \gamma > 0$
(d) $0<\tau < 1, Re\,(l+1) > 0$

$$\lambda = \sum_{j=1}^{m} \alpha_j + \sum_{j=1}^{n} \beta_j - \max_{1\leq i\leq r}\left[\sum_{j=n+1}^{p_i} \alpha_{ji} + \sum_{j=m+1}^{q_i} \beta_{ji}\right]$$

$$\mu = \sum_{j=1}^{m}(b_j - \beta_j\sigma) - \sum_{j=1}^{n}(a_j - \alpha_j\sigma) - \min_{1\leq i\leq r}\left[\sum_{j=n+1}^{p_i}(a_j - \alpha_j\sigma) - \sum_{j=m+1}^{q_i}(b_j - \beta_j\sigma) + \frac{p_i}{2} - \frac{q_i}{2}\right]$$

If $\sigma = 0$, then

$$\mu = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j - \min_{1\leq i\leq r}\left[\sum_{j=n+1}^{p_i}\left(a_j - \sum_{j=m+1}^{q_i} b_j + \frac{p_i}{2} - \frac{q_i}{2}\right)\right]$$

PROOF: First we will evaluate the following integral

$$\int_0^\infty x^l J_\nu^\tau(ux)\ I_{p_i,q_i;r}^{m,n}\left[zx^\gamma\Big|_{(b_j,\beta_j)_{1,m}(b_{ji},\beta_{ji})_{m+1,q_i}}^{(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},}\right]dx$$

We replace ux by t, then it is reduced to

$$\frac{1}{u^{l+1}}\int_0^\infty t^l J_\nu^\tau(t)\left[\frac{1}{2\pi i}\int \phi(\xi)(zu^{-\gamma}t^\gamma)^\xi d\xi\right]dt$$

Where $\phi(\xi) = \frac{\left\{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi)\right\}}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi)\right\}}$

$$= \frac{1}{u^{l+1}}\frac{1}{2\pi i}\int \frac{\Gamma(1+l+\xi\gamma)\left\{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi)\right\}}{\Gamma(1+\nu-\tau-\tau l-\tau l\xi)\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi)\right\}}(zu^{-\gamma})^\xi\, dt$$

$$= \frac{1}{u^{l+1}}\ I_{p_{i+2},q_i;r}^{m,n+1}\left[zu^{-\gamma}\Big|_{(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}^{(-l\ \ ,\gamma\ \ )\ \ ,(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(1+\nu\ \ -\tau\ \ -\tau\ \ l\ \ ,\nu\ \ \tau\ \ )}\right] \ldots\ldots(5)$$

we extend the above result to find the new result

$$\int_0^\infty \int_0^\infty x_1^{l_1} x_2^{l_2} J_{\upsilon_1}^{\tau_1}(ux_1)\ J_{\upsilon_2}^{\tau_2}(ux_2)\ I\left[zx_1^{\Upsilon_1}x_2^{\Upsilon_1}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i,}\\(b_j,\beta_j)_{1,m}(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]dx_1\ dx_2$$

=

$$\frac{1}{u^{l_1+l_2+2}}\ I_{p_{i+3},q_i;r}^{m,n+2}[zu^{-(\gamma_1+\gamma_2)}|^{(-l_1,\gamma_1),(-l_2,\gamma_2),(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(1+\nu_1-\tau_1-\tau_1 l_1,\nu_1\tau_1)\ \ (1+\nu_2-\tau_2-\tau_2 l_2,\nu_2\tau_2)}_{(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}]$$

.......(6)

Now we generalize above result by taking N-products of Bessel-Maitland functions

$$\int_{(1,N)}^{-}\prod_{k=1}^{N}x_k^{l_k}\prod_{k=1}^{N}J_{\nu_k}^{\tau_k}(ux_k)\ I_{p_i,q_i;r}^{m,n}\left[z\prod_{k=1}^{N}x_k^{\gamma_k}\,|^{(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i,}}_{(b_j,\beta_j)_{1,m}(b_{ji},\beta_{ji})_{m+1,q_i}}\right]dX$$

$$=\frac{1}{u^{L+N}}\ I_{p_{i+N},q_i;r}^{m,n+N}[zu^{-\sum_{k=1}^{N}\gamma_k}|^{(-l_k,\gamma_k)_{1,N},(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(1+\nu_k-\tau_k-\tau_k l_k,\nu_k\tau_k)_{1,N}}_{(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}]$$

Which proves (4).

Special cases:-

(i) If r = 1, Then result (3) reduces to H-function

$$\int_{(1,N)}^{-}\prod_{k=1}^{N}x_k^{l_k}\prod_{k=1}^{N}J_{\nu_k}^{\tau_k}(ux_k)\ I_{p_i,q_i;1}^{m,n}\left[z\prod_{k=1}^{N}x_k^{\gamma_k}\,|^{(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i,}}_{(b_j,\beta_j)_{1,m}(b_{ji},\beta_{ji})_{m+1,q_i}}\right]dX$$

$$=\frac{1}{u^{L+N}}\ H_{p_{i+N},q_i;1}^{m,n+N}[zu^{-\sum_{k=1}^{N}\gamma_k}|^{(-l_k,\gamma_k)_{1,N},(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(1+\nu_k-\tau_k-\tau_k l_k,\nu_k\tau_k)_{1,N}}_{(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}]$$

(ii) If $\alpha_j = \beta_j = \alpha_{ji} = \beta_{ji} = 1$,Then the result (4) reduces to interesting I_G-function

$$\int_{(1,N)}^{-}\prod_{k=1}^{N}x_k^{l_k}\prod_{k=1}^{N}J_{\nu_k}^{\tau_k}(ux_k)\ I_{p_i,q_i;1}^{m,n}\left[z\prod_{k=1}^{N}x_k^{\gamma_k}\,|^{(a_j,1)_{1,n};(a_{ji},1)_{n+1,p_i,}}_{(b_j,1)_{1,m}(b_{ji},1)_{m+1,q_i}}\right]dX$$

$$=\frac{1}{u^{L+N}}\ I_{G\,p_{i+N},q_i;1}^{\ m,n+N}\left[zu^{-\sum_{k=1}^{N}\gamma_k}|^{(-l_k,\gamma_k)_{1,N},(a_j,1)_{1,n};(a_{ji},1)_{n+1,p_i}(1+\nu_k-\tau_k-\tau_k l_k,\nu_k\tau_k)_{1,N}}_{(b_j,1)_{1,m};(b_{ji},1)_{m+1,q_i}}\right]$$

(iii) If $\alpha_j = \beta_j = \alpha_{ji} = \beta_{ji} = 1, r = 1$ *Then* $G-function$

$$\int_{(1,N)}^{-}\prod_{k=1}^{N}x_k^{l_k}\prod_{k=1}^{N}J_{\nu_k}^{\tau_k}(ux_k)\ I_{p_i,q_i;1}^{m,n}\left[z\prod_{k=1}^{N}x_k^{\gamma_k}\,|^{(a_j,1)_{1,n};(a_{ji},1)_{n+1,p_i,}}_{(b_j,1)_{1,m}(b_{ji},1)_{m+1,q_i}}\right]dX$$

=

$$\frac{1}{u^{L+N}}\, G^{m,n+N}_{p_{i+N},q_i;1}\left[zu^{-\sum_{k=1}^{N}\gamma_k}\Big|^{(-l_k,\gamma_k)_{1,N},(a_j,1)_{1,n};(a_{ji},1)_{n+1,p_i}(1+\nu_k-\tau_k-\tau_k l_k,\nu_k\tau_k)_{1,N}}_{(b_j,1)_{1,m};(b_{ji},1)_{m+1,q_i}}\right]$$

In this section *we derive following interesting recurrence relation of* I−functions

Theorem 2 :-

$$\frac{1}{u^{l+1}}\, I^{m,n+1}_{p_{i+2},q_i;r}\left[zu^{-\gamma}\Big|^{(-l\ ,\gamma\ )\ ,(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(\nu\ -\tau\ -\tau\ l\ ,\nu\ \tau\ )}_{(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}\right]-$$

$$\frac{1}{u^{l+1}}\, I^{m,n+1}_{p_{i+2},q_i;r}\left[zu^{-\gamma}\Big|^{(-l\ ,\gamma\ )\ ,(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(1+\nu\ -\tau\ -\tau\ l\ ,\nu\ \tau\ )}_{(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}\right]$$

=

$$\frac{1}{u^{l+1}}\, I^{m,n+1}_{p_{i+2},q_i;r}\left[z\,u^{-\gamma}\Big|^{(-l\ ,\gamma\ ),(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(1+\nu\ -\tau\ -\tau\ l\ ,\nu\ \tau\ ),\Gamma(1+\tau-\tau l,\tau l)}_{\Gamma(2+\tau-\tau l,\tau l)\,(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}\right]$$

...........(7)

Proof :-

Here we use the following recurrence relation of Bessel-Maitland functions

$$J^{\tau}_{\nu-1}(x)-\nu J^{\tau}_{\nu}(x)=-\tau x J^{\tau}_{\nu+\tau}(x) \qquad \dots\dots(8)$$

We know that by equation (5)

$$\int_0^\infty x^l J^{\tau}_{\nu}(ux)\ I^{m,n}_{p_i,q_i;r}\left[zx^{\gamma}\Big|^{(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},}_{(b_j,\beta_j)_{1,m}(b_{ji},\beta_{ji})_{m+1,q_i}}\right]dx\ =$$

$$\frac{1}{u^{l+1}}\, I^{m,n+1}_{p_{i+2},q_i;r}\left[zu^{-\gamma}\Big|^{(-l\ ,\gamma\ )\ ,(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(1+\nu\ -\tau\ -\tau\ l\ ,\nu\ \tau\ )}_{(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}\right]$$

similarly

$$\int_0^\infty x^l J^{\tau}_{\nu-1}(ux)\ I^{m,n}_{p_i,q_i;r}\left[zx^{\gamma}\Big|^{(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},}_{(b_j,\beta_j)_{1,m}(b_{ji},\beta_{ji})_{m+1,q_i}}\right]dx =$$

$$\frac{1}{u^{l+1}}\, I^{m,n+1}_{p_{i+2},q_i;r}\left[zu^{-\gamma}\Big|^{(-l\ ,\gamma\ )\ ,(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(\nu\ -\tau\ -\tau\ l\ ,\nu\ \tau\ )}_{(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}\right] \dots.(9)$$

$$\int_0^\infty x^l J^{\tau}_{\nu-1}(ux)\ I^{m,n}_{p_i,q_i;r}\left[zx^{\gamma}\Big|^{(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},}_{(b_j,\beta_j)_{1,m}(b_{ji},\beta_{ji})_{m+1,q_i}}\right]dx\quad -\int_0^\infty x^l J^{\tau}_{\nu}(ux)\ I^{m,n}_{p_i,q_i;r}\left[zx^{\gamma}\Big|^{(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},}_{(b_j,\beta_j)_{1,m}(b_{ji},\beta_{ji})_{m+1,q_i}}\right]dx$$

$$= -\tau \int_0^\infty x^{l+1} J_{\nu+\tau}^{\tau}(ux)\ I_{p_i,q_i;r}^{m,n}\left[zx^{\gamma}\Big|_{(b_j,\beta_j)_{1,m}(b_{ji},\beta_{ji})_{m+1,q_i}}^{(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},}\right]dx \quad \ldots\ldots(10)$$

From (5) & (9)

$$\frac{1}{u^{l+1}} I_{p_{i+2},q_i;r}^{m,n+1}\left[zu^{-\gamma}\Big|_{(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}^{(-l\ ,\gamma\ )\ ,(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(\nu\ -\tau\ -\tau\ l\ ,\nu\ \tau\ )}\right] - \frac{1}{u^{l+1}} I_{p_{i+2},q_i;r}^{m,n+1}\left[zu^{-\gamma}\Big|_{(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}^{(-l\ ,\gamma\ )\ ,(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(1+\nu\ -\tau\ -\tau\ l\ ,\nu\ \tau\ )}\right]$$

$$= \frac{1}{u^{l+1}}\frac{1}{2\pi i}\int \phi(\xi)(zu^{-\gamma})^{\xi}\left[\frac{\Gamma(l+\gamma\xi+1)}{\Gamma(\nu-\tau-\tau l-\tau l\xi)} - \frac{\nu\Gamma(l+\gamma\xi+1)}{\Gamma(1+\nu-\tau-\tau l-\tau l\xi)}\right]d\xi$$

$$= \frac{1}{u^{l+1}}\frac{1}{2\pi i}\int \phi(\xi)(zu^{-\gamma})^{\xi}\frac{\Gamma(1+l+\gamma\xi)\ \Gamma(2+\tau-\tau l-\tau l\xi)}{\Gamma(1+\nu-\tau-\tau l-\tau l\xi)\ \Gamma(1+\tau-\tau l-\tau l\xi)}\ d\xi$$

$$= \frac{1}{u^{l+1}} I_{p_{i+2},q_i;r}^{m,n+1}\left[z\ u^{-\gamma}\Big|_{\Gamma(2+\tau-\tau l,\tau l)\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}}^{(-l\ ,\gamma\ ),(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}(1+\nu\ -\tau\ -\tau\ l\ ,\nu\ \tau\ )\,\Gamma(1+\tau-\tau l,\tau l)}\right]$$

which proves (7).

Similarly we can also obtain special cases of H-function and G-function.

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