New Method for Solution of First Order and First Degree Differential Equation

Mayank Rai

B. Tech, Computer Science, M. M. M. Engg College, Gorakhpur,U.P.(India)-273010
C/O Sh. Shyamdhar Rai, Village & Post-Gomadih, Distt- Azamgarh, U.P.(India).-276202

Abstract

A new method has been devised for solution of first order and first degree differential equation.

Keywords

Constant ^{C}, Constant$^{C}{}_{1,}$ Constant$^{C}{}_{2,}$ Constant$^{C}{}_{n,}$

INTRODUCTION

An equation which involves differential co-efficient is called a differential equation. The order of a differential equation is the order of the highest differential co-efficient present in the equation. The degree of a differential equation is the degree of the highest derivative after removing the radical sign and fraction. For example , the order and degree of the differential equation dy/dx+y=x is 1 and 1 respectively. Variable Separable, Homogeneous Equation Method, Linear equation of first order and Exact differential equation method are standard methods for solution of first order and first degree differential equations . But some differential equations of first order and first degree are so complex and lengthy that none of the four methods mentioned above can be used to solve the equations easily. In such cases, the “New Method for Solution of Differential Equation of First Order and First Degree” can be used in finding the solution of differential equations.

II. ANALYSIS

New Method for Solution of First Order and First Degree Differential Equation

A first order and first degree differential equation can be represented as

$F(x, y, dy/dx) = 0.$ (1)

Whose solution can be represented as

$f(x, y) = c$ (2)

If the differential equation represented by (1) can be divided into two different differential equations and is represented as

$F_1(x, y, dy/dx) +F_2(x, y, dy/dx)=0$ (3)

Then the solution of the differential equation represented by (1) can be found using the procedure given below:

Step1. $F_1(x, y, dy/dx) =0$ (4)

And $F_2(x, y, dy/dx) =0$ (5)

Step2. If $f_1(x , y) = c_1$ (6)

And $f_2(x, y) = c_2$ (7)

be the solutions of the differential equations represented by (4) and (5) respectively.

Step3. Solution of the differential equation represented by (1) is

$f(x ,y) = f_1(x , y) + f_2(x, y) = c_1 + c_2 =c$ (8)

This method can be generalized also. If the differential equation represented by (1) can be divided into n differential equations and is represented as

$F_1 (x, y , dy/dx) + F_2 (x , y, dy/dx) + \ldots\ldots.+ F_n(x, y, dy/dx)=0$ (9)

Then the solution of the differential equation represented by (1) is

$f(x ,y) = f_1 (x , y) + f_2(x , y) + \ldots\ldots.+ f_n(x , y) = c_1 + c_2 + \ldots.+ c_n = c.$

Example

$(2x y^4 e^y + 2x y^3+y) dx + (x^2y^4e^y – x^2y^2-3x) dy =0$ (1)

Or $(2x e^y + 2x/y+1/y^3) + (x^2 e^y-x^2/y^2-3x/y^4) dy/dx =0$

Or $(e^y - 1/y^2 - 3/(xy^4))dy/dx + (2e^y/x + 2/(xy) + 1/(x^2y^3)) = 0$

Or $\{(e^y - 1/y^2)dy/dx + 2/x(e^y + 1/y)\} + \{-3/(xy^4)\, dy/dx + 1/(x^2y^3)\} = 0$ (1)

Here differential equation represented by (1) is expressed as sum of two different differential equations i.e.

$(e^y - 1/y^2)\, dy/dx + 2/x(e^y + 1/y) = 0$ (2)

And $-3/(xy^4)\, dy/dx + 1/(x^2y^3) = 0$ (3)

From (2),

Let $e^y + 1/y = z$

Or $(e^y - 1/y^2)\, dy/dx = dz/dx$ (differentiating w. r. t. x)

Or $dz/dx + (2/x)z = 0$

Or $dz/z + 2(dx/x) = 0$

Or lnz+2 lnx= lnc1 (on integration)

Or $zx^2 = c_1$

Or $(e^y + 1/y)\, x^2 = c_1$ (4)

Which is solution of differential equation represented by (2).

From (3),

$-3\, dy/dx + y/x = 0$

Or $-3\, dy/y + dx/x = 0$

Or $-3 \ln y + \ln x = \ln c_2$ (on integration)

Or $x/(y^3) = c_2$ (5)

Hence, the solution of the differential equation represented by (1) in the example is

$(e^y + 1/y)x^2 + x/(y^3) = c$

Precaution

While solving the differential equations derived from the original equation, Right Hand Side (R.H.S.) of the equation must always be equal to zero and all variable terms must be kept to the Left Hand Side (L.H.S.) of the equation. Transferring the variable terms to the R.H.S. may lead to wrong answer. For example if (3) of the above mentioned example is expressed as

$3/(xy^4)dy/dx = 1/x^2y^3$

Or $3\, dy/dx = y/x$

Or $3\, dy/y = dx/x$

Or $3\ln y - \ln x = \ln c_2$ (on integration)

Or $y^3/x = c_2$ which is wrong.

III. CONCLUSION

The "New Method for solution of First Order and First Degree Differential Equation"can be used to solve lengthy differential equation of first order and first degree as this method divides the original equation into various differential equations and facilitates solution of these derived equations independently. In other words, it reduces the complexity of the original equation. The method benefits in such cases as the differential equation can't be reduced into Exact differential equation form or is non-homogeneous or non-Linear differential equation.

IV. REFERENCES

1) E. Rukmangadachari : Differential Equations of First order and First Degree. Differential Equations.2(2.1-2.72)(2014)

2) George B. Thomas,Jr.: Further Applications of Integration.Thomas'Calculus.9(642-663)(2011)

3) H. K. Dass: Differential Equations of First Order. Introduction to Engineering Mathematics.18(455-481)(2007)