EOQ model for controllable deterioration rate and time dependent demand and Inventory holding cost

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Abstract - *In this paper, we have developed a deterministic inventory model for deteriorating items in which demand rate and holding cost are quadratic and linear function of time. During deterioration period, deterioration rate can be controlled using preservation technology (PT). An exponential distribution is used to represent the distribution of time to deterioration. The derived model is illustrated by a numerical example.*

Keywords - *Inventory, deteriorating items, preservation technology, exponential distribution, quadratic demand, time varying holding cost*.

I. INTRODUCTION

Inventory may be considered as an accumulation of a product that would be used to satisfy future demands for that product. An optimal replenishment policy is dependent on ordering cost, inventory carrying cost and shortage cost. An important problem confronting a supply manager in any modern organization is the control and maintenance of inventories of deteriorating items. Fortunately, the rate of deterioration is too small for items like steel, toys, glassware, hardware, etc. There is little requirement for considering deterioration in the determination of economic lot size. So in this paper, an inventory model is developed for deteriorating items by considering the fact that using the preservation technology the retailer can reduce the deterioration rate by which he can reduce the economic losses, improve the customer service level and increase business competitiveness.

In reality, the demand and holding cost for physical goods may be time dependent. Time also plays and important role in the inventory system. So, in this paper we consider that demand and holding cost are time dependent.

Recently, Mishra and Singh [9] developed a deteriorating inventory model with partial backlogging when demand and deterioration rate is constant. Vinod kumar Mishra [12] developed an inventory model of instantaneous deteriorating items with controllable deterioration rate for time dependent demand and holding cost. Vinod kumar Mishra [13] developed deteriorating inventory model with controllable deterioration rate for time-dependent demand and time-varying holding cost. Parmar Kirtan and U. B. Gothi [10] developed EOQ model with constant deterioration rate and time dependent demand and IHC.

Leea and Dye [8] formulated a deteriorating inventory model with stock-dependent demand by allowing preservation technology cost as a decision variable in conjunction with replacement policy. Dye and Hsieh [2] presented an extended model of Hsu et al. [4] by assuming that the preservation technology cost is a function of the length of replenishment cycle.

J. Jagadeeswari and P. K. Chenniappan [5] developed an order level inventory model for deteriorating items with time-quadratic demand and partial backlogging. Sarala Pareek and Garima Sharma [11] developed an inventory model with Weibull distribution deteriorating item with exponential declining demand and partial backlogging. R. Amutha and Dr. E. Chandrasekaran [1] developed an inventory model for deteriorating Products with Weibull Distribution Deterioration, Time-Varying Demand and Partial Backlogging. Kirtan Parmar and U. B. Gothi [6] developed a deterministic inventory model for deteriorating items where time to deterioration has Exponential distribution and with time-dependent quadratic demand. Also, U. B. Gothi and Kirtan Parmar [3] have extended above deterministic inventory model by taking two parameter Weibull distributions to represent the distribution of time to deterioration and shortages are allowed and partially backlogged. Kirtan Parmar and U. B. Gothi [7] developed an economic production model for deteriorating items using three parameter Weibull distributions with constant production rate and time varying holding cost.

The consideration of PT is important due to rapid social changes, and the fact that PT can reduce the deterioration rate significantly. By the efforts of investing in preservation technology, we can reduce the deterioration rate. So in this paper, we made the model of Mishra and Singh [9] more realistic by considering the fact that use of preservation technology can reduce the deterioration rate significantly, which help the retailers to reduce their economic losses.

In this paper, we have analyzed an inventory system for deteriorating items under quadratic demand using preservation technology and time dependent IHC. The assumptions and notations of the model are introduced in the next section. The mathematical model and Analysis is derived in section 3, algorithm is derived in section 4 and numerical illustration is presented in section 5. The article ends with some concluding remarks and scope of a future research.

II. Assumptions and Notations

The mathematical model is based on the following notations and assumptions.

A. Notations

$R(t)$: Quadratic demand rate.

A : Ordering cost per order.

C_h : Inventory holding cost per unit per unit of time.

C_d : Deterioration cost per unit per unit time.

$m(\xi)$: Reduced deterioration rate due to use of preservation technology.

θ : Deterioration rate.

τ_p : Resultant deterioration rate,

$\tau_p = (\theta - m(\xi))$.

Q : Order quantity in one cycle.

P_c : Purchase cost per unit.

t_d :The time from which the deterioration start in the inventory.

t_1 : Length of cycle time (decision variable).

TC : Total cost per unit time.

$Q(t)$: The instantaneous state of the positive inventory level at time t.

B. Assumptions

The model is derived under the following assumptions.

1. The inventory system deals with single item.
2. The annual demand rate is a function of time and it is $R(t) = a + bt + ct^2$ $(a, b, c > 0)$.
3. Preservation technology is used for controlling the deterioration rate.
4. Holding cost is linear function of time and it is $C_h = h + rt$ $(h, r > 0)$.
5. The lead time is zero.
6. Time horizon is finite.
7. The deterioration rate is constant.
8. No repair or replacement of the deteriorated items takes place during a given cycle.
9. Total inventory cost is a real, continuous function which is convex to the origin.

III. Mathematical Model and Analysis

Here, the replenishment policy of a deteriorating item with partial backlogging is considered. The objective of the inventory problem is to determine the optimal order quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible, where the preservation technology is used to control the deterioration rate. The behaviour of inventory system at any time is shown in Fig. 1.

Replenishment is made at time $t = 0$ and the inventory level is at its maximum level Q. During the period $[0, t_d]$ the inventory level is decreasing and at time t_1 the inventory reaches zero level.

The pictorial representation is shown in the Fig. 1.

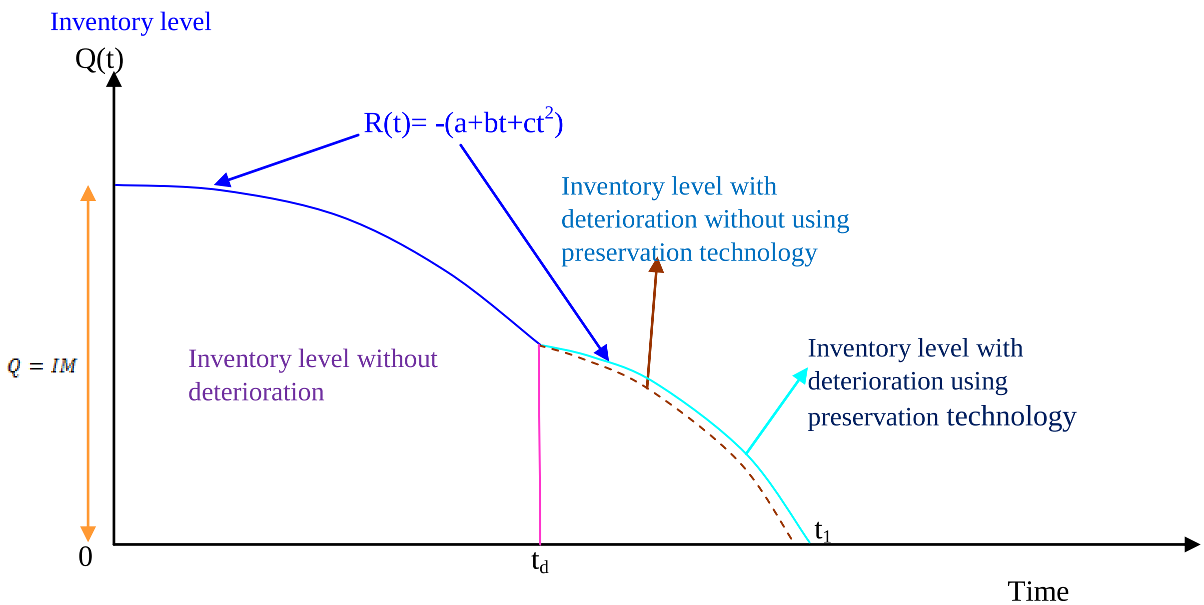

Fig. 1 Graphical representation of the inventory system

As described above, the inventory level decreases owing to demand rate as well as deterioration during [0, t_1]. Hence, the differential equation representing the inventory status is given by

$$\frac{dQ(t)}{dt} = -(a + bt + ct^2) \qquad (0 \le t \le t_d) \qquad (1)$$

$$\frac{dQ(t)}{dt} + \tau_p Q(t) = -(a + bt + ct^2) \qquad (t_d \le t \le t_1) \qquad (2)$$

The boundary conditions are $Q(0) = Q$ and $Q(t_1) = 0$ (3)

Using the boundary condition $Q(0) = Q$ the solution of the equation (1) is

$$\Rightarrow Q(t) = Q - \left(at + \frac{bt^2}{2} + \frac{ct^3}{3}\right) \qquad (0 \le t \le t_d) \qquad (4)$$

Similarly, the solution of equation (2) is given by

$$e^{(\theta - m(\xi))t} Q(t) = -\int (a + bt + ct^2) e^{(\theta - m(\xi))t} dt$$

$$\Rightarrow e^{(\theta - m(\xi))t} Q(t) = \left\{k - \left[at + (a\theta + b)\frac{t^2}{2} + (b\theta + c)\frac{t^3}{3} + c\theta\frac{t^4}{4} - m(\xi)\left\{a\frac{t^2}{2} + b\frac{t^3}{3} + c\frac{t^4}{4}\right\}\right]\right\}$$

(Neglecting higher powers of θ) (where $k = at_1 + (a\theta + b)\frac{t_1^2}{2} + (b\theta + c)\frac{t_1^3}{3} + c\theta\frac{t_1^4}{4} - m(\xi)\left\{a\frac{t_1^2}{2} + b\frac{t_1^3}{3} + c\frac{t_1^4}{4}\right\}$ which is obtained using $Q(t_1) = 0$

$$Q(t) = k - k(\theta - m(\xi))t + a(\theta - m(\xi))\frac{t^2}{2} + b(\theta - m(\xi))\frac{t^3}{6} + c(\theta - m(\xi))\frac{t^4}{12} + c(\theta - m(\xi))\frac{t^5}{4} - at - b\frac{t^2}{2} - c\frac{t^3}{3} - m(\xi)\left\{a(2\theta - m(\xi))\frac{t^3}{2} + b(2\theta - m(\xi))\frac{t^4}{3} + c(2\theta - m(\xi))\frac{t^5}{4}\right\} \qquad (t_d \le t \le t_1) \qquad (5)$$

In equations (4) and (5) values of $Q(t)$ and $Q(t)$ should coincide at $= t_d$, which implies that

$$Q-\left(at_d+\frac{bt_d^2}{2}+\frac{ct_d^3}{3}\right)$$
$$=\Bigg[k-k\big(\theta-m(\xi)\big)t_d+a\big(\theta-m(\xi)\big)\frac{t_d^2}{2}+b\big(\theta-m(\xi)\big)\frac{t_d^3}{6}\quad+c\big(\theta-m(\xi)\big)\frac{t_d^4}{12}$$
$$+c\big(\theta-m(\xi)\big)\frac{t_d^5}{4}-at_d-b\frac{t_d^2}{2}-c\frac{t_d^3}{3}$$
$$-m(\xi)\left\{a\big(2\theta-m(\xi)\big)\frac{t_d^3}{2}+b\big(2\theta-m(\xi)\big)\frac{t_d^4}{3}+c\big(2\theta-m(\xi)\big)\frac{t_d^5}{4}\right\}\Bigg]$$

$$Q=IM=\Bigg[k-k\big(\theta-m(\xi)\big)t_d+a\big(\theta-m(\xi)\big)\tfrac{t_d^2}{2}+b\big(\theta-m(\xi)\big)\tfrac{t_d^3}{6}+c\big(\theta-m(\xi)\big)\tfrac{t_d^4}{12}+c\big(\theta-m(\xi)\big)\tfrac{t_d^5}{4}-m(\xi)\left\{a\big(2\theta-m(\xi)\big)\tfrac{t_d^3}{2}+b\big(2\theta-m(\xi)\big)\tfrac{t_d^4}{3}+c\big(2\theta-m(\xi)\big)\tfrac{t_d^5}{4}\right\}\Bigg] \tag{6}$$

The total cost comprises of following costs

1) The ordering cost $OC = A$ (7)

2) The deterioration cost during the period $[t_d, t_1]$

$$DC=C_d\left\{Q-\int_{t_d}^{t_1}R(t)dt\right\}$$
$$=C_d\left\{Q-\left[a(t_1-t_d)+\tfrac{b}{2}(t_1^2-t_d^2)+\tfrac{c}{3}(t_1^3-t_d^3)\right]\right\} \tag{8}$$

3) The inventory holding cost during the period $[0, t_1]$

$$IHC=\int_0^{t_d}(h+rt)\,Q(t)dt+\int_{t_d}^{t_1}(h+rt)Q(t)dt$$
$$=\int_0^{t_d}\left\{(h+rt)\left[Q-\left(at+\frac{bt^2}{2}+\frac{ct^3}{3}\right)\right]\right\}dt$$
$$+\int_{t_d}^{t_1}\Bigg\{(h+rt)\Bigg[k-k\big(\theta-m(\xi)\big)t+a\big(\theta-m(\xi)\big)\frac{t^2}{2}+b\big(\theta-m(\xi)\big)\frac{t^3}{6}+c\big(\theta-m(\xi)\big)\frac{t^4}{12}$$
$$+c\big(\theta-m(\xi)\big)\frac{t^5}{4}-at-b\frac{t^2}{2}-c\frac{t^3}{3}$$
$$-m(\xi)\left\{a\big(2\theta-m(\xi)\big)\frac{t^3}{2}+b\big(2\theta-m(\xi)\big)\frac{t^4}{3}+c\big(2\theta-m(\xi)\big)\frac{t^5}{4}\right\}\Bigg]\Bigg\}dt$$

$$\Rightarrow IHC = \left\{ \begin{array}{c} h\left[Qt_d - \left(\frac{at_d^2}{2} + \frac{bt_d^3}{6} + \frac{ct_d^4}{12}\right)\right] + r\left[\frac{Qt_d^2}{2} - \left(\frac{at_d^3}{3} + \frac{bt_d^4}{8} + \frac{ct_d^5}{15}\right)\right] \\ +hk(t_1 - t_d) + [rk - hk(\theta - m(\xi))]\left[\frac{t_1^2 - t_d^2}{2}\right] + ha(\theta - m(\xi))\left[\frac{t_1^3 - t_d^3}{6}\right] + hb(\theta - m(\xi))\left[\frac{t_1^4 - t_d^4}{24}\right] \\ +hc(\theta - m(\xi))\left[\frac{t_1^5 - t_d^5}{60}\right] + hc(\theta - m(\xi))\left[\frac{t_1^6 - t_d^6}{24}\right] - m(\xi)\left\{ \begin{array}{l} ah(2\theta - m(\xi))\left[\frac{t_1^4 - t_d^4}{8}\right] \\ +bh(2\theta - m(\xi))\left[\frac{t_1^5 - t_d^5}{15}\right] \\ +ch(2\theta - m(\xi))\left[\frac{t_1^6 - t_d^6}{24}\right] \end{array} \right\} \\ +ra(\theta - m(\xi))\left[\frac{t_1^4 - t_d^4}{8}\right] + rb(\theta - m(\xi))\left[\frac{t_1^5 - t_d^5}{30}\right] \\ +rc(\theta - m(\xi))\left[\frac{t_1^6 - t_d^6}{72}\right] + rc(\theta - m(\xi))\left[\frac{t_1^7 - t_d^7}{28}\right] - m(\xi)\left\{ \begin{array}{l} ra(2\theta - m(\xi))\left[\frac{t_1^5 - t_d^5}{10}\right] \\ +rb(2\theta - m(\xi))\left[\frac{t_1^6 - t_d^6}{18}\right] \\ +rc(2\theta - m(\xi))\left[\frac{t_1^7 - t_d^7}{28}\right] \end{array} \right\} \\ -\left\{ha\left(\frac{t_1^2 - t_d^2}{2}\right) + hb\left(\frac{t_1^3 - t_d^3}{6}\right) + hc\left(\frac{t_1^4 - t_d^4}{12}\right) + ra\left(\frac{t_1^3 - t_d^3}{3}\right) + rb\left(\frac{t_1^4 - t_d^4}{8}\right) + rc\left(\frac{t_1^5 - t_d^5}{15}\right)\right\} \end{array} \right\} \tag{9}$$

Thus, the order size during total interval $[0, t_1]$ is given by

$$Q = IM$$

4) Purchase cost per cycle

$$PC = P_c Q$$

$$PC = P_c\left\{\left[k - k(\theta - m(\xi))t_d + a(\theta - m(\xi))\frac{t_d^2}{2} + b(\theta - m(\xi))\frac{t_d^3}{6} + c(\theta - m(\xi))\frac{t_d^4}{12} + c(\theta - m(\xi))\frac{t_d^5}{4} - m(\xi)\left\{a(2\theta - m(\xi))\frac{t_d^3}{2} + b(2\theta - m(\xi))\frac{t_d^4}{3} + c(2\theta - m(\xi))\frac{t_d^5}{4}\right\}\right]\right\} \tag{10}$$

Hence the total cost per unit time is given by

$$TC = \frac{1}{t_1}(OC + DC + IHC + PC)$$

$$TC=\frac{1}{t_1}\left\{\begin{array}{c}
A+C_d\left\{Q-\left[a(t_1-t_d)+\frac{b}{2}(t_1^2-t_d^2)+\frac{c}{3}(t_1^3-t_d^3)\right]\right\} \\
+ \\
+\left\{\begin{array}{c}
h\left[Qt_d-\left(\frac{at_d^2}{2}+\frac{bt_d^3}{6}+\frac{ct_d^4}{12}\right)\right]+r\left[\frac{Qt_d^2}{2}-\left(\frac{at_d^3}{3}+\frac{bt_d^4}{8}+\frac{ct_d^5}{15}\right)\right] \\
+hk(t_1-t_d)+\left[rk-hk\big(\theta-m(\xi)\big)\right]\left[\frac{t_1^2-t_d^2}{2}\right]+ha\big(\theta-m(\xi)\big)\left[\frac{t_1^3-t_d^3}{6}\right] \\
+hb\big(\theta-m(\xi)\big)\left[\frac{t_1^4-t_d^4}{24}\right] \\
+hc\big(\theta-m(\xi)\big)\left[\frac{t_1^5-t_d^5}{60}\right]+hc\big(\theta-m(\xi)\big)\left[\frac{t_1^6-t_d^6}{24}\right] \\
-m(\xi)\left\{\begin{array}{l}
ah\big(2\theta-m(\xi)\big)\left[\frac{t_1^4-t_d^4}{8}\right] \\
+bh\big(2\theta-m(\xi)\big)\left[\frac{t_1^5-t_d^5}{15}\right] \\
+ch\big(2\theta-m(\xi)\big)\left[\frac{t_1^6-t_d^6}{24}\right]
\end{array}\right\} \\
+ra\big(\theta-m(\xi)\big)\left[\frac{t_1^4-t_d^4}{8}\right]+rb\big(\theta-m(\xi)\big)\left[\frac{t_1^5-t_d^5}{30}\right] \\
+rc\big(\theta-m(\xi)\big)\left[\frac{t_1^6-t_d^6}{72}\right]+rc\big(\theta-m(\xi)\big)\left[\frac{t_1^7-t_d^7}{28}\right] \\
-m(\xi)\left\{\begin{array}{l}
ra\big(2\theta-m(\xi)\big)\left[\frac{t_1^5-t_d^5}{10}\right] \\
+rb\big(2\theta-m(\xi)\big)\left[\frac{t_1^6-t_d^6}{18}\right] \\
+rc\big(2\theta-m(\xi)\big)\left[\frac{t_1^7-t_d^7}{28}\right]
\end{array}\right\} \\
-\left\{\begin{array}{c}
ha\left(\frac{t_1^2-t_d^2}{2}\right)+hb\left(\frac{t_1^3-t_d^3}{6}\right)+hc\left(\frac{t_1^4-t_d^4}{12}\right)+ra\left(\frac{t_1^3-t_d^3}{3}\right) \\
+rb\left(\frac{t_1^4-t_d^4}{8}\right)+rc\left(\frac{t_1^5-t_d^5}{15}\right)
\end{array}\right\}
\end{array}\right\} \\
+P_c\left\{\begin{array}{c}
[k-k\big(\theta-m(\xi)\big)t_d+a\big(\theta-m(\xi)\big)\frac{t_d^2}{2}+b\big(\theta-m(\xi)\big)\frac{t_d^3}{6}+c\big(\theta-m(\xi)\big)\frac{t_d^4}{12} \\
+c\big(\theta-m(\xi)\big)\frac{t_d^5}{4}]-m(\xi)\left\{a\big(2\theta-m(\xi)\big)\frac{t_d^3}{2}+b\big(2\theta-m(\xi)\big)\frac{t_d^4}{3}+c\big(2\theta-m(\xi)\big)\frac{t_d^5}{4}\right\}
\end{array}\right\}
\end{array}\right\} \tag{11}$$

Our objective is to determine optimum value of t_1 to minimize TC. The values of t_1 for which

$\frac{dTC}{dt_1}=0$ satisfying the condition

$\left(\frac{d^2TC}{dt_1^2}\right)>0$

The optimal solution of the equation (11) is obtained by using Mathematica software. This has been illustrated by the following numerical example.

IV. NUMERICAL EXAMPLE

We consider the following parametric values for $A=300,\ a=10,\ b=8,\ c=5,\ h=1,\ r=0.5,$ $t_d=1,\ \theta=1.27,\ m(\xi)=0.05,\ C_d=5,\ P_c=15.$

We obtain the optimal value of $t_1 = 1.13636$ units, Q = 2.81 and minimum total cost (TC) = 214.001.

V. Conclusions

The products with high deterioration rate are always crucible to the retailer's business. In real markets, the retailer can reduce the deterioration rate of a product by making effective capital investment in storehouse equipment. In this study, to reduce the deterioration rate during deterioration period of deteriorating items, we use the preservation technology. A solution procedure is given to find an optimal replenishment cycle and order quantity so that the total inventory cost per unit time is minimum. A numerical example has been presented to illustrate the model. This model can further be extended by taking more realistic assumptions such as probabilistic demand rate, a model allowing shortages etc.

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