Semipre Generalized Closed Mappings in Intuitionistic Fuzzy Topological Spaces

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***Abstract*—In this paper we introduce intuitionistic fuzzy semipre generalized closed mappings, intuitionistic fuzzy semipre generalized open mappings and intuitionistic fuzzy M-semipre generalized closed mappings and we study some of their properties. We provide the relation between intuitionistic fuzzy M-semipre generalized closed mappings and intuitionistic fuzzy semipre generalized closed mappings.**

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I. INTRODUCTION

In 1965, Zadeh [12] introduced fuzzy sets and in 1968, Chang [3] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of this notion. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. 1997, Coker [4] introduced the concept of intuitionistic fuzzy topological spaces. In 2000, Seok Jong Lee and Eun Pyo Lee [8] investigated the properties of continuous, open and closed maps in the intuitionistic fuzzy topological spaces. In this direction we introduce the notions of intuitionistic fuzzy semipre generalized closed mappings, intuitionistic fuzzy semipre generalized open mappings and intuitionistic fuzzy M-semipre generalized closed mappings and study some of their properties.

II. PRELIMINARIES

Definition 2.1: [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$ where the functions $\mu_A: X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x)\rangle / x \in X \}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \{ \langle x, \nu_A(x), \mu_A(x)\rangle / x \in X \}$,
(iv) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X \}$,
(v) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X \}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$. The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1\rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0\rangle / x \in X \}$ are respectively the empty set and the whole set of X.

Definition 2.3: [4] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short)in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [4] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then

(i) $int(A) = \cup\{ G / G$ is an IFOS in X and $G \subseteq A \}$,
(ii) $cl(A) = \cap\{ K / K$ is an IFCS in X and $A \subseteq K \}$,
(iii) $cl(A^c) = (int(A))^c$,
(iv) $int(A^c) = (cl(A))^c$.

Definition 2.5: [5] An IFS A of an IFTS (X, τ) is an

(i) intuitionistic fuzzy semiclosed set (IFSCS in short) if $int(cl(A)) \subseteq A$,
(ii) intuitionistic fuzzy semiopen set (IFSOS in short) if $A \subseteq cl(int(A))$.

Definition 2.6: [5] An IFS A of an IFTS (X, τ) is an

(i) intuitionistic fuzzy preclosed set (IFPCS in short) if cl(int(A)) ⊆ A,
(ii) intuitionistic fuzzy preopen set (IFPOS in short) if A ⊆ int(cl(A)).
Note that every IFOS in (X, τ) is an IFPOS in X.

Definition 2.7: [5] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy α-closed set (IFαCS in short) if cl(int(cl(A))) ⊆ A,
(ii) intuitionistic fuzzy α-open set (IFαOS in short) if A ⊆ int(cl(int(A))),

Definition 2.8: [11] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy semipre closed set (IFSPCS for short) if there exists an IFPCS B such that int(B) ⊆ A ⊆ B,
(ii) intuitionistic fuzzy semipre open set (IFSPOS for short) if there exists an IFPOS B such that B ⊆ A ⊆ cl(B).
Note that IFS A is an IFSPCS if and only if int(cl(int(A))) ⊆ A. [7]

Definition 2.9: [8] An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy W-closed set (IFWCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ). An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy W-open set (IFWOS in short) if A^c is an IFWCS in (X, τ).

Definition 2.10: [10] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy semipre generalized closed set (IFSPGCS for short) if spcl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ).
Every IFCS, IFSCS, IFWCS, IFαCS, IFPCS, IFSPCS is an IFSPGCS but the converses are not true in general.

Definition 2.11: [9] The complement A^c of an IFSPGCS A in an IFTS (X, τ) is called an intuitionistic fuzzy semipre generalized open set (IFSPGOS for short) in X.
The family of all IFSPGOSs of an IFTS (X, τ) is denoted by IFSPGO(X). Every IFOS, IFSOS, IFWOS, IFαOS, IFPOS, IFSPOS is an IFSPGOS but the converses are not true in general.

Definition 2.12: [6] Let A be an IFS in an IFTS (X, τ). Then
(i) spint (A) = ∪{ G / G is an IFSPOS in X and G ⊆A },
(ii) spcl (A) = ∩{ K / K is an IFSPCS in X and A ⊆K }.
Note that for any IFS A in (X, τ), we have $spcl(A^c) = (spint(A))^c$ and $spint(A^c) = (spcl(A))^c$.

Result 2.13: [2] For an IFS A in an IFTS(X, τ), we have spcl(A) ⊇ A ∪ (int(cl(int(A)))).

Definition 2.14: [9] If every IFSPGCS in (X, τ) is an IFSPCS in (X, τ), then the space can be called as an intuitionistic fuzzy semipre $T_{1/2}$ (IFSP$T_{1/2}$ for short) space.

Definition 2.15: [7] A map f : (X, τ) → (Y, σ) is called an intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for each IFCS A in X.

Definition 2.16: [7] A map f : (X, τ) → (Y, σ) is called an
(i) intuitionistic fuzzy semiopen mapping (IFSOM for short) if f(A) is an IFSOS in Y for each IFOS A in X.
(ii) intuitionistic fuzzy α-open mapping (IFαOM for short) if f(A) is an IFαOS in Y for each IFOS A in X.
(iii) intuitionistic fuzzy preopen mapping (IFPOM for short) if f(A) is an IFPOS in Y for each IFOS A in X.

III. INTUITIONISTIC FUZZY SEMIPRE GENERALIZED CLOSED MAPPINGS

In this paper we introduce intuitionistic fuzzy semipre generalized closed mappings and intuitionistic M-semipre generalized closed mappings and study some of their properties.

Definition 3.1: A mapping f : (X, τ) → (Y, σ) is called an intuitionistic fuzzy semipre generalized closed mapping (IFSPGCM for short) if f(A) is an IFSPGCS in Y for each IFCS A in X.

For the sake of simplicity, we shall use the notation A= ⟨x, (μ, μ), (ν, ν)⟩ instead of A = ⟨x,(a/μ_a, b/μ_b), (a/ν_a, b/ν_b)⟩ in all the examples used in this paper. Similarly we shall use the notation B = ⟨x, (μ, μ), (ν, ν)⟩ instead of B = ⟨x,(u/μ_u, v/μ_v), (u/ν_u, v/ν_v)⟩ in the following examples.

Example 3.2: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.4, 0.3), (0.6, 0.4)⟩, G_2 = ⟨y, (0.7, 0.8), (0.3, 0.2)⟩, G_3 = ⟨y, (0.6, 0.7), (0.4, 0.3)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFSPGCM.

Theorem 3.3: Every IFCM is an IFSPGCM but not conversely.

Proof: Assume that f : (X, τ) → (Y, σ) is an IFCM. Let A be an IFCS in X. Then f(A) is an IFCS in Y. This implies that f(A) is an IFSPGCS in Y. Hence f is an IFSPGCM.

Example 3.4: In Example 3.2, f : (X, τ) → (Y, σ) is an IFSPGCM but not an IFCM.

Theorem 3.5: Every IFαCM is an IFSPGCM but not conversely.

Proof: Assume that f : (X, τ) → (Y, σ) is an IFαCM. Let A be an IFCS in X. Then f(A) is an IFαCS in Y. This implies that f(A) is an IFSPGCS in Y. Hence f is an IFSPGCM.

Example 3.6: In Example 3.2, f : (X, τ) → (Y, σ) is an IFSPGCM but not an IFαCM.

Theorem 3.7: Every IFSCM is an IFSPGCM but not conversely.

Proof: Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFSCM. Let A be an IFCS in X. Then f(A) is an IFSCS in Y. This implies that f(A) is an IFSPGCS in Y. Hence f is an IFSPGCM.

Example 3.8: In Example 3.2, $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFSPGCM but not an IFSCM.

Theorem 3.9: Every IFWCM is an IFSPGCM but not conversely.

Proof: Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFWCM. Let A be an IFCS in X. Then f(A) is an IFWCS in Y. This implies that f(A) is an IFSPGCS in Y. Hence f is an IFSPGCM.

Example 3.10: In Example 3.2, $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFSPGCM but not an IFWCM.

Theorem 3.11: Every IFPCM is an IFSPGCM but not conversely.

Proof: Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFPCM. Let A be an IFCS in X. Then f(A) is an IFPCS in Y. This implies that f(A) is an IFSPGCS in Y. Hence f is an IFSPGCM.

Example 3.12: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$, $G_2 = \langle y, (0.3, 0.2), (0.6, 0.6)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFSPGCM but not an IFPCM.

Definition 3.13: A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be an intuitionistic fuzzy semipre generalized open mapping (IFSPGOM for short) if f(A) is an IFSPGOS in Y for each IFOS in X.

Definition 3.14: A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be an intuitionistic fuzzy M-semipre generalized closed mapping (IFMSPGCM for short) if f(A) is an IFSPGCS in Y for every IFSPGCS A in X.

Definition 3.15: A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be an intuitionistic fuzzy M-semipre generalized open mapping (IFMSPGOM for short) if f(A) is an IFSPGOS in Y for every IFSPGOS A in X.

Example 3.16: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$, $G_2 = \langle y, (0.3, 0.2), (0.7, 0.8)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFMSPGCM.

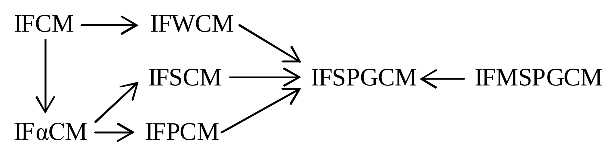
Example 3.17: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.3, 0.2), (0.7, 0.8)\rangle$, $G_2 = \langle y, (0.5, 0.6), (0.5, 0.4)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFMSPGOM.

Theorem 3.18: Every IFMSPGCM is an IFSPGCM but not conversely.

Proof: Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFMSPGCM. Let A be an IFCS in X. Then A is an IFSPGCS in X. By hypothesis f(A) is an IFSPGCS in Y. Hence f is an IFSPGCM.

Example 3.19: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$, $G_2 = \langle y, (0.6, 0.7), (0.4, 0.3)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFSPGCM but not an IFMSPGCM.

The relation between various types of intuitionistic fuzzy closed mappings is given by



The reverse implications are not true in general in the above diagram.

Theorem 3.20: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping. Then the following statements are equivalent if Y is an $IFSPT_{1/2}$ space:

(i) f is an IFSPGCM,
(ii) $spcl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.

Proof: (i) ⇒ (ii) Let A be an IFS in X. Then cl(A) is an IFCS in X. (i) implies that f(cl(A)) is an IFSPGCS in Y. Since Y is an $IFSPT_{1/2}$ space, f(cl(A)) is an IFSPCS in Y. Therefore spcl(f(cl(A))) = f(cl(A)). Now $spcl(f(A)) \subseteq spcl(f(cl(A))) = f(cl(A))$. Hence $spcl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.
(ii) ⇒ (i) Let A be any IFCS in X. Then cl(A) = A. (ii) implies that $spcl(f(A)) \subseteq f(cl(A)) = f(A)$. But $f(A) \subseteq spcl(f(A))$. Therefore spcl(f(A)) = f(A). This implies f(A) is an IFSPCS in Y. Since every IFSPCS is an IFSPGCS, f(A) is an IFSPGCS in Y. Hence f is an IFSPGCM.

Theorem 3.21: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijection. Then the following statements are equivalent if Y is an $IFSPT_{1/2}$ space:

(i) f is an IFSPGCM,
(ii) $spcl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.
(iii) $f^{-1}(spcl(B)) \subseteq cl(f^{-1}(B))$ for every IFS B of Y.

Proof: (i) ⇔ (ii) is obvious from Theorem 3.20.
(ii) ⇒ (iii) Let B be an IFS in Y. Then $f^{-1}(B)$ is an IFS in X. Since f is onto, $spcl(B) = spcl(f(f^{-1}(B)))$ and (ii) implies $spcl(f(f^{-1}(B))) \subseteq f(cl(f^{-1}(B)))$. Therefore $spcl(B) \subseteq f(cl(f^{-1}(B)))$. Now $f^{-1}(spcl(B)) \subseteq f^{-1}(f(cl(f^{-1}(B)))) = cl(f^{-1}(B))$, since f is one to one. Hence $f^{-1}(spcl(B)) \subseteq cl(f^{-1}(B))$.

(iii) ⇒ (ii) Let A be an IFS in X. Then f(A) is an IFS of Y. Since f is one to one. (iii) implies that f^{-1}(spcl(f(A))) ⊆ cl(f^{-1}(f(A))) = cl(A). Therefore f(f^{-1}(spcl(f(A)))) ⊆ f(cl(A)). Since f is onto spcl(f(A)) = f(f^{-1}(spcl(f(A)))) ⊆ f(cl(A)).

Theorem 3.22: If f : (X, τ) → (Y, σ) is an IFCM and g : (Y, σ) → (Z, η) is an IFSPGCM then g ∘ f : (X, τ) →(Z, η) is an IFSPGCM.

Proof: Let A be any IFCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since g is an IFSPGCM, g(f(A)) is an IFSPGCS in Z. Therefore g ∘ f is an IFSPGCM.

Theorem 3.23: Let f : (X, τ) → (Y, σ) be a mapping where Y is an $IFSPT_{1/2}$ space. Then the following statements are equivalent.

(i) f is an IFSPGCM,
(ii) f(B) is an IFSPGOS in Y for every IFOS B in X,
(iii) f(int(B)) ⊆ cl(int(cl(f(B)))) for every IFS B in X.

Proof: (i) ⇔ (ii) is obvious.
(ii) ⇒ (iii) Let B be an IFS in X. Then int(B) is an IFOS in X. By hypothesis f(int(B)) is an IFSPGOS in Y. Since Y is an $IFSPT_{1/2}$ space, f(int(B)) is an IFSPOS in Y. Therefore f(int(B)) ⊆ cl(int(cl(f(int(B))))) ⊆ cl(int(cl(f(B)))).
(iii) ⇒ (i) Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis, f(int(A^c)) = f(A^c) ⊆ cl(int(cl(f(A^c)))). That is int(cl(int(f(A)))) ⊆ f(A). This implies f(A) is an IFβCS in Y and hence an IFSPGCS in Y. Therefore f is an IFSPGCM.

Theorem 3.24: If f : (X, τ) → (Y, σ) is a mapping. Then the following are equivalent if Y is an $IFSPT_{1/2}$ space
(i) f is an IFSPGOM
(ii) f(int(A)) ⊆ spint(f(A)) for each IFS A of X
(iii) int(f^{-1}(B)) ⊆ f^{-1}(spint(B)) for every IFS B of Y.

Proof: (i) ⇒ (ii) Let f be an IFSPGOM. Let A be any IFS in X. Then int(A) is an IFOS in X. (i) implies that f(int(A)) is an IFSPGOS in Y. Since Y is an $IFSPT_{1/2}$ space, f(int(A)) is an IFSPOS in Y. Therefore spint(f(int(A))) = f(int(A)). Now f(int(A)) = spint(f(int(A))) ⊆ spint(f(A)).
(ii) ⇒ (iii) Let B be an IFS in Y. Then f^{-1}(B) is an IFS in X. By (ii) f(int(f^{-1}(B))) ⊆ spint(f(f^{-1}(B))) ⊆ spint(B). Now int(f^{-1}(B)) ⊆ f^{-1}(f(int(f^{-1}(B)))) ⊆ f^{-1}(spint(B)).
(iii) ⇒ (i) Let A be an IFOS in X. Then int(A) = A and f(A) is an IFS in Y. By (iii) int(f^{-1}(f(A))) ⊆ f^{-1}(spint(f(A))). Now A = int(A) ⊆ int(f^{-1}(f(A))) ⊆ f^{-1}(spint(f(A))). Therefore f(A) ⊆ f(f^{-1}(spint(f(A)))) ⊆ spint(f(A)) ⊆ f(A). This implies spint(f(A)) = f(A) is an IFSPOS in Y and hence an IFSPGOS in Y. Thus f is an IFSPGOM.

Theorem 3.25: A mapping f : (X, τ) → (Y, σ) is an IFSPGOM if f(spint(A)) ⊆ spint(f(A)) for every A ⊆ X.

Proof: Let A be an IFOS in X. Then int(A) = A. Now f(A) = f(int(A)) ⊆ f(spint(A)) ⊆ spint(f(A)), by hypothesis. But spint(f(A)) ⊆ f(A). Therefore f(A) is an IFSPOS in Y. Then f(A) is an IFSPGOS in Y. Hence f is an IFSPGOM.

Theorem 3.26: A mapping f : (X, τ) → (Y, σ) is an IFSPGOM if and only if int(f^{-1}(B)) ⊆ f^{-1}(spint(B)) for every IFS B ⊆ Y, where Y is an $IFSPT_{1/2}$ space.

Proof: Necessity: Let B be an IFS in Y. Then f^{-1}(B) ⊆ X and int(f^{-1}(B)) is an IFOS in X. By hypothesis, f(int(f^{-1}(B))) is an IFSPGOS in Y. Since Y is an $IFSPT_{1/2}$ space, f(int(f^{-1}(B))) is an IFSPOS in Y. Therefore f(int(f^{-1}(B))) = spint(f(int(f^{-1}(B)))) ⊆ spint(B). This implies int(f^{-1}(B)) ⊆ f^{-1}(spint(B)).

Sufficiency: Let A be an IFOS in X. Therefore int(A) = A. Then f(A) ⊆ Y. By hypothesis int(f^{-1}(f(A))) ⊆ f^{-1}(spint(f(A))). That is int(A) ⊆ int(f^{-1}(f(A))) ⊆ f^{-1}(spint(f(A))). Therefore A ⊆ f^{-1}(spint(f(A))). This implies f(A) ⊆ spint(f(A)) ⊆ f(A). Hence f(A) is an IFSPOS in Y and hence an IFSPGOS in Y. Thus f is an IFSPGOM.

Theorem 3.27: Let f : (X, τ) → (Y, σ) be a mapping where Y is an $IFSPT_{1/2}$ space. Then the following statements are equivalent.

(i) f is an IFSPGCM,
(ii) f(int(A)) ⊆ spint(f(A)) for each IFS A of X,
(iii) int(f^{-1}(B)) ⊆ f^{-1}(spint(B)) for every IFS B of Y.

Proof: (i) ⇒ (ii) Let f be an IFSPGCM. Let A be any IFS in X. Then int(A) is an IFOS in X. Now f(int(A)) is an IFSPGOS in Y, by Theorem 3.23. Since Y is an $IFSPT_{1/2}$ space, f(int(A)) is an IFSPOS in Y. Therefore spint(f(int(A))) = f(int(A)). Now f(int(A)) = spint(f(int(A))) ⊆ spint(f(A)).
(ii) ⇒ (iii) Let B be an IFS in Y. Then f^{-1}(B) is an IFS in X. By (ii) f(int(f^{-1}(B))) ⊆ spint(f(f^{-1}(B))) ⊆ spint(B). Now int(f^{-1}(B)) ⊆ f^{-1}(f(int(f^{-1}(B)))) ⊆ f^{-1}(spint(B)).
(iii) ⇒ (i) Let A be an IFOS in X. Then int(A) = A and f(A) is an IFS in Y. By (iii) int(f^{-1}(f(A))) ⊆ f^{-1}(spint(f(A))). Now A = int(A) ⊆ int(f^{-1}(f(A))) ⊆ f^{-1}(spint(f(A))). Therefore f(A) ⊆ f(f^{-1}(spint(f(A)))) ⊆ spint(f(A)) ⊆ f(A). Therefore spint(f(A)) = f(A) is an IFSPOS in Y and hence an IFSPGOS in Y. Thus f is an IFSPGCM, by Theorem 3.23.

Theorem 3.28: A mapping f : (X, τ) → (Y, σ) is an IFSPGCM if f(spint(A)) ⊆ spint(f(A)) for every A ⊆ X.

Proof: Let A be an IFOS in X. Then int(A) = A. Now f(A) = f(int(A)) ⊆ f(spint(A)) ⊆ spint(f(A)), by hypothesis. But spint(f(A)) ⊆ f(A). Therefore f(A) is an IFSPOS in Y. Then f(A) is an IFSPGOS in Y. Hence f is an IFSPGCM, by Theorem 3.23.

Theorem 3.29: If f : (X, τ) → (Y, σ) is a mapping where X and Y are $IFSPT_{1/2}$ space, then the following statements are equivalent:

(i) f is an IFMSPGCM,
(ii) f(A) is an IFSPGOS in Y for every IFSPGOS A in X,
(iii) f(spint(B)) ⊆ spint(f(B)) for every IFS B in X,
(iv) spcl(f(B)) ⊆ f(spcl(B)) for every IFS B in X.

Proof: (i) ⇒ (ii) is obvious.
(ii) ⇒ (iii) Let B be any IFS in X. Since spint(B) is an IFSPOS, it is an IFSPGOS in X. Then by hypothesis, f(spint(B)) is an IFSPGOS in Y. Since Y is an $IFSPT_{1/2}$ space, f(spint(B)) is an IFSPOS in Y. Therefore f(spint(B)) = spint(f(spint(B))) ⊆ spint(f(B)).
(iii) ⇒ (iv) is obvious by taking complement in (iii).
(iv) ⇒ (i) Let A be an IFSPGCS in X. By Hypothesis, spcl(f(A)) ⊆ f(spcl(A)). Since X is an $IFSPT_{1/2}$ space, A is an IFSPCS in X. Therefore spcl(f(A)) ⊆ f(spcl(A)) = f(A) ⊆ f(spcl(A)). Hence f(A) is an IFSPCS in Y and hence an IFSPGCS in Y. Thus f is an IFMSPGCM.

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