Soft gsg-closed Set in Soft Topological Spaces

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Abstract: **In this paper we consider a new class of soft set called soft generalized semi generalized closed sets in soft topological spaces. Also, we discuss some basic properties of soft gsg-closed sets.**

Key words and phrase**: Soft gsg-closed set, Soft gsg-closure, Soft Tgsg space.**

I. INTRODUCTION

In 1999, Molodtsov[5] introduced the concept of soft set theory and started to develop the basics of the corresponding theory as a new approach for modeling uncertainties. Maji et al [4] proposed several operations on soft sets and some basic properties. Shabir and Naz[6] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Many researchers extended the results of generalization of various soft closed sets in many directions. The concept generalized semi generalized closed sets was introduced and studied by Lellis et al [3] in classical topology. In this present study, we discuss the soft generalized semi generalized closed sets in soft topological and some of their properties.

II. PRELIMINARIES

Definition 2.1 [5]*:*
Let U be an initial universe and E be a set of parameters. Let $P\,(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F: A \rightarrow P\,(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.2 [5]*:*
The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C, H(e) = F(e)$ if $e \in A - B, H(e) = G(e)$ if $e \in B - A$ and $H\,(e) = F\,(e) \cup G\,(e)$ if $e \in A \cap B$. We write $(F, A) \tilde{\cup} (G, B) = (H, C)$.

Definition 2.3 [6]*:*
The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted $(F, A) \tilde{\cap} (G, B)$, is defined as $C = A \cap B$, and $H(e) = F\,(e) \cap G(e)$ for all $e \in C$.

Definition 2.4: Let τ be the collection of soft sets over X, then it is said to be a soft topology on X if

(1) Φ, X belong to τ ,
(2) the union of any number of soft sets in τ belongs to τ,
(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of , τ are said to be soft open sets in X.

Definition 2.5 [6]*:*
Let (A, E) be a soft set over X. Then, the soft closure of (A, E), denoted by $Cl\,(A, E)$ is the intersection of all soft closed supersets of (A, E). (A, E) is the smallest soft closed set over X which contains (A, E). The soft interior of (A, E), denoted by $Int\,(A, E)$ is the union of all soft open subsets of (A, E). Clearly (A, E) is the smallest soft closed set over X which contains (A, E) and the largest soft open set over X which is contained in (A, E).

Definition 2.5 [3] Let (X, τ) be a topological space. Then a set A of (X, τ) is called a generalized semi generalized closed set (in short, gsg-closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi generalized open in X

Definition 2.6 [1] :
A subset A of a topological space (X, τ, E) is called a soft semi open set if $(A, E) \tilde{\subseteq} Cl(Int(A, E))$.

Definition 2.7 [2]:
A soft set (A, E) is called soft generalized closed set (soft g-closed) in a soft topological space (X, τ, E) if $Cl(A, E) \tilde{\subseteq} (U, E)$ whenever $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft open in X.

Definition 2.8:
A soft set (A, E) is called a soft semi generalized closed set (soft sg-closed) in a soft topological spaces (X, τ, E) if $ssCl(A, E) \tilde{\subseteq} (U, E)$ whenever $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft semi open in X.

Definition 2.9:
A soft topological space (X, τ, E) is a soft $T_{1/2}$ space if every soft g-closed set is soft closed in (X, τ, E).

Definition 2.10:
A soft topological space (X, τ, E) is a soft T_ω space if every soft ω-closed set is soft closed in (X, τ, E).

Theorem 2.1:
Every soft closed set is soft sg-closed.

Proof: Let (A, E) be soft closed. Let (U, E) be any soft semi open in X such that $(A, E) \tilde{\subseteq} (U, E)$. Then $ssCl(A, E) \tilde{\subseteq} Cl(A, E) = (A, E) \tilde{\subseteq} (U, E)$. Therefore (A, E) be soft sg-closed.

Theorem 2.2:
Every soft semi closed set is soft sg-closed.

Proof: Let (A, E) be soft semi closed and (U, E) be any soft semi open set such that $(A, E) \tilde{\subseteq} (U, E)$. Now $ssCl(A, E) = (A, E) \tilde{\subseteq} (U, E)$. Hence (A, E) soft sg-closed.

III. SOFT GENERALIZED SG-CLOSED SETS AND SOFT GENERALIZED SG-OPEN SETS

Definition 3.1:
A soft set (A, E) is called a soft generalized semi generalized closed set (soft gsg-closed) in a soft topological spaces (X, τ, E) if $Cl(A, E) \tilde{\subseteq} (U, E)$ whenever $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft sg-open in X.

Proposition 3.1:
Every soft closed is soft gsg -colsed.

Proof: Let (A, E) be soft closed in (X, τ, E) and (U, E) be soft sg-open such that $(A, E) \tilde{\subseteq} (U, E)$. Consider$Cl(A, E) = (A, E) \tilde{\subseteq} (U, E)$. Therefore (A, E) is soft gsg -closed.

Proposition 3.2:
Every soft gsg-closed is soft g-closed.

Proof: Let (A, E) be soft gsg-closed. Let (U, E) be any soft open in X such that $(A, E) \tilde{\subseteq} (U, E)$. Since every soft open set is soft sg-open set, we have $Cl(A, E) \tilde{\subseteq} (U, E)$ such that $(A, E) \tilde{\subseteq} (U, E)$. Therefore (A, E) is soft g-closed.

Proposition 3.3:
Every soft gsg-closed set is soft ω-closed.

Proof: Let (A, E) be soft gsg-closed and (U, E) be any soft semi open set such that $(A, E) \tilde{\subseteq} (U, E)$. Since every soft semi open set is soft sg-open, (U, E) is soft sg-open. Therefore (A, E) is soft gsg-closed and $(A, E) \tilde{\subseteq} (U, E)$. Hence (A, E) soft ω-closed.

Definition 3.2:
The complement of soft gsg-closed set soft gsg-open set.

Proposition 3.4:
- Every soft open is soft gsg-open
- Every soft gsg- open is soft g- open.
- Every soft gsg- open set is soft ω- open.

IV. CHARACTERIZATION OF SOFT GSG-CLOSED SET AND SOFT GSG-OPEN SET

Theorem 4.1:
If (A, E) and (B, E) are soft gsg-closed sets in X then $(A, E) \tilde{\cup} (B, E)$ is soft gsg-closed set.

Proof: Suppose (A, E) and (B, E) are soft gsg-closed set in X. Then $Cl(A, E) \tilde{\subseteq} (U, E)$ and $Cl(B, E) \tilde{\subseteq} (U, E)$ where $(A, E) \tilde{\subseteq} (U, E)$, $(B, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft sg-open in X. Consider $Cl\big((A, E) \tilde{\cup} (B, E)\big) = Cl(A, E) \tilde{\cup} Cl(B, E) \tilde{\subseteq} (U, E)$. Hence $(A, E) \tilde{\cup} (B, E)$ is soft gsg-closed set.

Theorem 4.2:
If a soft set (A, E) is soft gsg-closed then $Cl(A, E) - (A, E)$ containts only null soft closed set.

Proof: Let (F, E) be a soft closed set and $(F, E) \tilde{\subseteq} Cl(A, E) - (A, E)$. Then $(F, E) \tilde{\subseteq} Cl(A, E)$ and $(F, E) \tilde{\subseteq} (A, E)^C$. This implies $(A, E) \tilde{\subseteq} (F, E)^C$. Then $Cl(A, E) \tilde{\subseteq} (F, E)^C$, as $(F, E)^C$ as soft sg-open set. This implies $(F, E) \tilde{\subseteq} (Cl(A, E))^C$. Therefore $(F, E) \tilde{\subseteq} Cl(A, E) \tilde{\cap} (Cl(A, E))^C$. Hence (F, E) is null soft closed set.

Theorem 4.3:
A soft (A, E) is soft gsg-closed if and only if $Cl(A, E) - (A, E)$ containts only null soft sg-closed set.

Proof:
Necessity Part: Let (A, E) be a soft gsg-closed. Let (F, E) be soft sg-closed set such that $(F, E) \tilde{\subseteq} Cl(A, E) - (A, E)$. Then $(F, E) \tilde{\subseteq} Cl(A, E)$ and $(F, E) \tilde{\subseteq} (A, E)^C$. This implies $(A, E) \tilde{\subseteq} (F, E)^C$. Then $Cl(A, E) \tilde{\subseteq} (F, E)^C$. This implies $(F, E) \tilde{\subseteq} (Cl(A, E))^C$. Therefore $(F, E) \tilde{\subseteq} Cl(A, E) \tilde{\cap} (Cl(A, E))^C$. Hence (F, E) is null soft closed set.

Suffiency Part: Assume $Cl(A, E) - (A, E)$ containts only null soft sg-closed set. Let (U, E) be soft sg-open such that $(A, E) \tilde{\subseteq} (U, E)$. Suppose $(A, E) \tilde{\nsubseteq} (U, E)$, then $Cl(A, E) \tilde{\cap} (U, E)^C$ is non null set. As $Cl(A, E)$ and $(U, E)^C$ is soft sg-closed, $Cl(A, E) \tilde{\cap} (U, E)^C$ is non null sg-closed which is contained in $Cl(A, E) - (A, E)$. This is a contradiction. Therefore $Cl(A, E) \tilde{\subseteq} (U, E)$ and hence (A, E) is a soft gsg-closed.

Theorem 4.4:
If (A, E) is soft gsg-closed in X and $(A, E) \tilde{\subseteq} (B, E) \tilde{\subseteq} Cl(A, E)$ then (B, E) is soft gsg-closed.

Proof: Let (B,E) be soft sg-open such that $(B,E) \tilde{\subseteq} (U,E)$. Since (A,E) is soft gsg-closed in X, we have $Cl(A,E) - (A,E)$ contains no non-null soft sg-closed set. Now, $(B,E) \tilde{\subseteq} Cl(A,E)$ implies $Cl(B,E) \tilde{\subseteq} Cl(A,E)$. We have $Cl(B,E) - (B,E) \tilde{\subseteq} Cl(A,E) - (A,E)$. This implies $Cl(B,E) - (B,E)$ does not contain any non-null soft sg-closed set. By the above theorem (4.3), (B,E) is soft gsg-closed.

Theorem 4.5:
If (A,E) is soft sg-open and soft gsg-closed then (A,E) soft closed.
Proof: Easy to prove.

Theorem 4.6
A soft set (A,E) soft gsg-open if and only if $(F,E) \tilde{\subseteq} Int(A,E)$ where (F,E) is soft sg-closed and $(F,E) \tilde{\subseteq} (A,E)$.

Proof: Assume $(F,E) \tilde{\subseteq} Int(A,E)$ where (F,E) is soft sg-closed and $(F,E) \tilde{\subseteq} (A,E)$. Then $(F,E)^c$ is soft sg-open and $(A,E)^c \tilde{\subseteq} (F,E)^c$.
Now, $(F,E) \tilde{\subseteq} Int(A,E) \Longrightarrow (Int(A,E))^c \tilde{\subseteq} (F,E)^c \Longrightarrow Cl(A,E)^c \tilde{\subseteq} (F,E)^c$. This gives $(A,E)^c$ is gsg-closed. Hence (A,E) is gsg-open.

Conversely,
(A,E) soft gsg-open and (F,E) is soft sg-closed with $(F,E) \tilde{\subseteq} (A,E)$. This implies $(A,E)^c$ is soft gsg-closed and $(F,E)^c$ is soft sg-open with $(A,E)^c \tilde{\subseteq} (F,E)^c$. Then $Cl(A,E)^c \tilde{\subseteq} (F,E)^c \Longrightarrow (Int(A,E))^c \tilde{\subseteq} (F,E)^c \Longrightarrow (F,E) \tilde{\subseteq} Int(A,E)$. Hence proved.

Theorem 4.7:
If (A,E) and (B,E) are soft gsg-open sets then $(A,E) \tilde{\cap} (B,E)$ is soft gsg-open.
Proof: Easy to prove.

Theorem 4.8:
If $Int(B,E) \tilde{\subseteq} (B,E) \tilde{\subseteq} (A,E)$ and if (A,E) is soft gsg-open then (B,E) is soft gsg-open in X.
Proof: Easy to prove.

Theorem 4.9:
If a soft set (A,E) is soft gsg-closed in (X,τ,E) then $Cl(A,E) - (A,E)$ is soft gsg-open in (X,τ,E).

Proof: Suppose that (A,E) is soft gsg-closed in (X,τ,E). Let $(F,E) \tilde{\subseteq} Cl(A,E) - (A,E)$ and (F,E) is soft sg-closed. Then (F,E) is null sg-closed set. Now, $(F,E) \tilde{\subseteq} Int[Cl(A,E) - (A,E)]$ and hence by theorem (4.6), $Cl(A,E) - (A,E)$ is soft gsg-open.

V. SOFT T_{GSG} SPACE

Definition 5.1:
A soft topological space (X,τ,E) is called a soft T_{gsg} Space if every soft gsg-closed set in it is soft closed.

Theorem 5.2:
Every soft $T_{1/2}$ space is soft T_{gsg} space.

Proof: Let (X,τ,E) be a soft $T_{1/2}$ space and let (A,E) be soft gsg-closed set in (X,τ,E).Then (A,E) is soft g-closed, by proposition (3.2). Since (X,τ,E) is soft $T_{1/2}$ space, (A,E) is soft closed in (X,τ,E). Hence (X,τ,E) is soft T_{gsg} space.

Theorem 5.3:
Every soft T_ω space is a soft T_{gsg} space.

Proof: Let (X, τ, E) be a soft T_ω space and let (A, E) be soft gsg-closed set in (X, τ, E). Then (A, E) is soft ω-closed, by proposition (3.3). Since (X, τ, E) is soft T_ω space, (A, E) is soft closed in (X, τ, E). Hence (X, τ, E) is soft T_{gsg} space.

VI. Conclusion

In this paper, soft gsg-closed sets were introduced and studied with already existing soft sets in soft topological spaces. A new space soft T_{gsg} space is also been introduced. A very few basic ideas are studied and hence it is the initiative to the study of soft gsg-closed sets. The scope for further research can be focused on the applications of soft topological spaces in real life problems.

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