An Inventory Model for Time Varying Holding Cost and Weibull Distribution for Deterioration with Fully Backlogged Shortages

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Abstract **- To find optimum order quantity deterioration plays crucial role in inventory management therefore handling of deterioration is in highlight and attracted of many researchers. In this model Economic Production Problem under time varying holding cost, demand is constant, price discount is offered for deteriorated items, deterioration rate is considered as Weibull distribution and shortages are permitted and kept fully backlogging. The model is supported by numerical example.**

Keywords **- EPQ model, Deteriorating product, Shortage, Fully backlogging, Time varying holding cost, Weibull distribution.**

I. Introduction

In inventory management finding of total cost for optimal order quantity with deterioration is a challenging work due to limited useful life of items. The items which are not used before the time would outdate and would be an additional cost for such outdated items. Many researchers suggested different models for deterioration under various situations. Deterioration is defined as change, damage, decay spoilage, obsolescence, evaporation, pilferage and loss of utility or loss of original value in a commodity from the original one. In such situation price discount can be another option to cover somewhat loss on these deteriorated items. Price discount is an attraction to the customers, therefore price discount are common practices by supplier it encourages the customer to purchase a lot size or defective items other than regular purchase.

P.N. Ghare and G.P. Schrader [1963] were the pioneer in deterioration they have considered constant deterioration rate in their inventory model. N.H. Shah and K.T. Shukla [2009] have developed deterministic inventory model for deterioration with shortages. C.K. Tripathy and U. Mishra [2010] have used Weibull distribution for deteriorating items and price dependent demand with time varying holding cost. S.V. Kawale and P.B. Bansode [2012] continued this work and developed EPQ model for deteriorating items with Weibull deterioration under time-varying holding cost. H.M. Wee [1999] considered deteriorating inventory model for quantity discount, pricing and partial backordering. S.K. Goyal and B.C.Giri [2001] gave recent trend of modeling in deteriorating item inventory. They classified inventory models on the basis of demand variations and various other conditions. T. Chakrabarty, et.al. [1998] developed inventory model using three parameters Weibull distribution in deterioration.

S. Sarkar and T. Chakrabarti [2013] developed EPQ model using Weibull distribution for electronic items in which holding cost is time varying and same work continued with exponential demand for permissible delay in payment. C. Sugapriya and K. Jeyaraman [2008] considered the EPQ model for non-instantaneous deteriorating item with holding cost varies.

V.K. Mishra, et.al. [2013] developed deterministic inventory model for time-dependent demand and time-varying holding cost under partial backlogging. N. Ghasemi and B.A. Nadjafi [2013] used holding cost is an increasing function of period length to develop inventory model for deteriorated items.

B.C. Giri, et.al. [1996] generalized an EOQ model for deteriorating items over a finite time horizon with demand and costs are varying with time.

In present paper we develop a model using three parameters Weibull distribution for deterioration with time-varying holding cost and shortages are permitted with fully backlogging. Price discount is offered to deteriorated items. Numerical example is given to support developed model and analyzed the same.

II. Assumptions and Notations

Assumptions:

a. The demand rate for the product is known and finite.
b. Shortage is allowed and fully backlogged.
c. An infinite planning horizon is assumed.
d. Once a unit of the product is produced, it is available to meet the demand.
e. Once the production is terminated the product starts deterioration and the price discount is considered.
f. The deterioration follows the three-parameter Weibull distribution.
g. There is no replacement or repair for a deteriorated item.

Notations:

- $I_1(t)$: inventory level for the product during the production and demand i.e. $0 \le t \le T_1$.
- $I_2(t)$: inventory level for the product during the period when there is no production only demand with deterioration of the rate Weibull probability distribution i.e. $T_1 \le t \le T_2$.
- $I_3(t)$: Inventory level for the product during the period there is no stock in hand and demand is continued and fully backlogged.
- $I_4(t)$: Inventory level for the product during the period there is production starts and fulfil demand which was backlogged.
- Q : maximum inventory level of the product.
- S : maximum shortage of the product.
- p : production rate per unit time.
- d : actual demand of the product per unit time.
- A : Set up cost.
- $h= a+bt$: inventory carrying cost per unit time, where a and b are positive constants.
- K : production cost per unit.
- r : price discount per unit cost.
- T : optimal cycle time.
- T_1 : production period with demand.
- T_2 : time during which there is no production of the product only demand and deterioration.
- T_3 : time during which there is only shortage period.
- T_4 : time during the production period and backlog fulfilment.
- TVC (T) : total cost/unit time.

III. Mathematical formulation and solution

Here we assume production starts at time $t = 0$ and supply is also same time. The production stop at $t = T_1$ where the maximum inventory level Q is reached. In the interval $[0, T_1]$ the inventory built up at a rate p - d. There is no deterioration in this interval. The production stop at time T_1 and deterioration starts and supply is also with discount rate. The inventory is finitely decreasing up to until inventory reaches zero in the time interval $[T_1, T_2]$. Now shortages occur at a time $t = T_2$ where the maximum shortage level

(S) built in the interval [T_2, T_3].The production starts again at time t = T_3 and the backlog is fulfilled in interval [T_3, T_4]. When the stock is again zero at time T_4.The cycle repeats itself at time T_4.

The model is represented by the following diagram

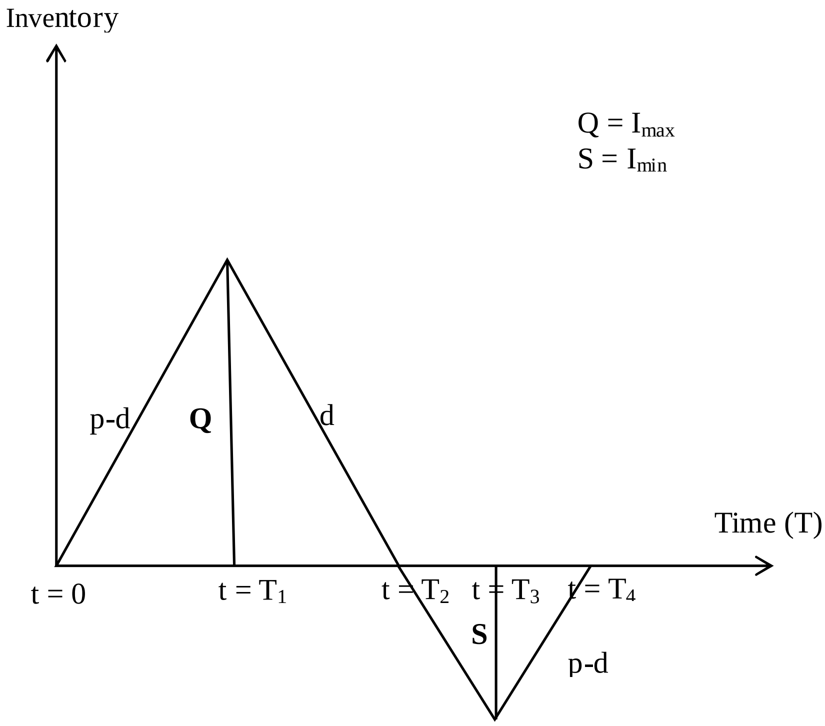

Figure-I: Graphical representation of inventory situation

Let I (t) be the inventory level of the product at time t over period [0, T] can be described by the following equation.

$$\frac{dI_1(t)}{dt} = (p - d), \qquad 0 \le t \le T_1 \qquad (1)$$

$$\frac{dI_2(t)}{dt} + \alpha\beta(\mu - \gamma)^{\beta-1} I_2(t) = -d, \qquad T_1 \le t \le T_2 \qquad (2)$$

$$\frac{dI_3(t)}{dt} = -d, \qquad T_2 \le t \le T_3 \qquad (3)$$

$$\frac{dI_4(t)}{dt} = -(p - d), \qquad T_3 \le t \le T_4 \qquad (4)$$

With the boundary condition: $I_1(t_0) = 0$, $I_1(t_1) = Q$, $I_2(t_1) = Q$, $I_2(t_2) = 0$, $I_3(t_2) = S$, $I_4(t_3) = S$ and $I_4(t_4) = 0$.
Now solving the above differential equation, we get

$$I_1(t) = (p - d)t, \qquad 0 \le t \le T_1 \qquad (5)$$

$$I_2(t) = \frac{d\left\{\exp\left[\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)t\right] - 1\right\}}{\exp\left[\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)t\right]}, \qquad T_1 \le t \le T_2 \qquad (6)$$

$$I_3(t) = -dt, \qquad T_2 \le t \le T_3 \qquad (7)$$

$$I_4(t) = -(p-d)t, \qquad T_3 \le t \le T_4 \tag{8}$$

Production Cost: The production cost per unit time is given by

$$PC = pk\left(\frac{T_1}{T} + \frac{T_4}{T}\right) \tag{9}$$

Setup Cost: The setup cost per unit time is given by

$$SC = \frac{A}{T} \tag{10}$$

Holding Cost: The holding cost per unit time is

$$HC = \frac{1}{T}\left[\int_0^{T_1} (a+bt)I_1(t)dt + \int_{T_1}^{T_2} (a+bt)I_2(t)dt\right]$$

$$= \frac{1}{T}\left\{\int_0^{T_1} (a+bt)(p-d)tdt + \int_{T_1}^{T_2} (a+bt)\left[\frac{d\left(\exp(\alpha\beta(\mu-\gamma)^{\beta-1})t-1\right)}{\exp\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)t}\right]dt\right\} \tag{11}$$

Using Tailor's expansion and assuming $\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)T<1$ and neglecting the terms of power greater than or equal to 2, resulting equation (11) is as follows.

$$HC = a(p-d)\frac{T_1^2}{2T} + b(p-d)\frac{T_1^3}{3T} + \frac{adT_2^2}{2T} \tag{12}$$

Deterioration cost: The deterioration cost is in period [T_1, T_2].

$$DC = \frac{k}{T}\left[I_2(0) - \int_{T_1}^{T_2} d.dt\right]$$

$$= \frac{kd\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)T_2^2}{2T} \tag{13}$$

Shortage cost: The shortage cost is in period [T_2, T_3].

$$C_1 = Sd\left(\frac{T_3}{T} - \frac{T_4}{T}\right) \tag{14}$$

Price discount: Price discount is offered as a fraction of production cost for the period [T_1, T_2].

$$PD = \frac{kr}{T}\int_{T_1}^{T_2} d.dt$$

$$= \frac{krdT_2}{T} \tag{15}$$

Therefore the average total cost per unit time is given by

Total average cost $(TVC(T)) = PC + SC + HC + DC + C_1 + PD$

$$TVC(T) = pk\left(\frac{T_1}{T} + \frac{T_4}{T}\right) + \frac{A}{T} + a(p-d)\frac{T_1^2}{2T} + b(p-d)\frac{T_1^3}{3T} + \frac{adT_2^2}{2T} + \frac{kd\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)T_2^2}{2T} + Sd\left(\frac{T_3}{T} - \frac{T_4}{T}\right) + \frac{krdT_2}{T} \tag{16}$$

To solve this equation let us express T_1, T_2, T_3 and T_4 in terms of T. So T_1, T_2, T_3 and T_4 is as follows

$$T_1 = \frac{d}{p}T$$

$$T_2 = \frac{(p-d)}{p}T$$

$$T_3 = \frac{d}{p}T$$

and

$$T_4 = \frac{d^2T}{p(p-d)}T$$

$$T = T_1 + T_2 + T_3 + T_4 = \frac{d}{p}T + \frac{(p-d)}{p}T + \frac{d}{p}T + \frac{d^2T}{p(p-d)}T$$

Therefore equation (16) becomes

$$TVC(T) = kd + \frac{kd^2}{(p-d)} + \frac{A}{T} + a(p-d)\frac{d^2T}{2p^2} + b(p-d)\frac{d^3T^2}{3p^3} + \frac{ad(p-d)^2T}{2p^2} + \frac{kd\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)(p-d)^2T}{2p^2} + \frac{Sd^2}{p} - \frac{Sd(p-d)}{p} + \frac{krd(p-d)}{p} \quad (17)$$

Our objective is to minimize the total cost per unit time TVC (T). Therefore differentiate TVC (T) with respect to T and set the result equal to zero. We get,

$$\frac{dTVC(T)}{dT} = -\frac{A}{T^2} + a(p-d)\frac{d^2}{2p^2} + 2b(p-d)\frac{d^3T}{3p^3} + \frac{ad(p-d)^2}{2p^2} + \frac{kd\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)(p-d)^2}{2p^2} = 0 \quad (18)$$

$$\frac{d^2TVC(T)}{dT^2} = \frac{2A}{T^3} + 2b(p-d)\frac{d^3}{3P^3} > 0$$

i.e. the second derivative is found to be positive.

IV. EXAMPLE

1) A = \$1000/set up, p = 100unit/unit time, d = 30units/unit time, a = 4, b = 0.4, k = \$20/unit time, α = 0.02, β = 1, γ = 0.5, μ = 3, r = 0.01/unit, S = 25unit/unit time, T = 4.70unit time, TVC (T) = \$986.8337 and HC = \$202.9667.
2) A = \$1000/set up, p = 100unit/unit time, d = 30units/unit time, a = 4, b = 0.4, k = \$20/unit time, α = 0.02, β = 1, γ = 0.5, μ = 3, r = 0.02/unit, S = 25unit/unit time, T = 4.70unit time, TVC (T) = \$986.9737 and HC = \$202.9667.
3) A = \$1000/set up, p = 100unit/unit time, d = 30units/unit time, a = 4, b = 0.4, k = \$20/unit time, α = 0.02, β = 1, γ = 0.5, μ = 3, r = 0.03/unit, S = 25unit/unit time, T = 4.70unit time, TVC (T) = \$987.1137 and HC = \$202.9667.

Computed Total cost (TVC) and Holding cost (HC) for different sets of deterioration rates and price discount. After that we compared the results. The following tables show the total cost and holding cost with reference to deterioration rate and price discount.

SENSITIVITY ANALYSIS

r	a	b	α	B	γ	Holding Cost (HC)	Total Cost TVC(T)
0.01	**4**	**0.4**	**0.02**	**1**	**0.5**	**202.9667**	**986.8337***
	3	0.3	0.04	2	1.0	152.225	1032.818
	2	0.2	0.06	3	1.5	101.4833	1151.3468
0.02	**4**	**0.4**	**0.02**	**1**	**0.5**	**202.9667**	**986.9737***
	3	0.3	0.04	2	1.0	152.225	1032.958
	2	0.2	0.06	3	1.5	101.4833	1151.4868
0.03	**4**	**0.4**	**0.02**	**1**	**0.5**	**202.9667**	**987.1137***
	3	0.3	0.04	2	1.0	152.225	1033.90
	2	0.2	0.06	3	1.5	101.4833	1151.6268

V. CONCLUSION

In this paper, Weibull distribution is considered for deterioration with time varying holding cost, shortage are permitted and kept backlogged. Above sensitivity analysis shows that total cost increases as a, b decreases and deterioration parameter increases, whereas holding cost decreases for decreasing values of a, b the same trend can see for different value of price discount. In this situation deterioration parameter is much effective for total cost. Whereas the relation between values of a, b is directly proportional to holding cost. There for the value marked by (*) is optimal for that situation.

VI. REFERENCES

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