Contra $g^{\#}$p-Continuous Functions

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Abstract

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called $g^{\#}$p -continuous[2] if f^{-1} (V) is $g^{\#}$p -closed in(X,τ) for every closed set V in (Y,σ). The notion of contra continuity was introduced and investigated by Dontchev[6]. In this paper we introduce and investigate a new generalization of contra continuity called contra $g^{\#}$p -continuity.

Key Words

Conta $g^{\#}$p-continuous funtions, $g^{\#}$p-closed sets,contra pre continuous functions

MSC 1991: 54 C 08, 54 C 10, 54 C 05, 54 H 05

I. INTRODUCTION

Dontchev[6] introduced the notion of contracontinuity. Dontchev and Noiri [7] introduced and investigated contra semi-continuous functions and RC continuous functions between topological spaces.Veerakumar also introduced contra pre-semicontinuous functions. In this paper we introduce and investigate contra $g^{\#}$p-continuous functions. This new class properly contains the class of contra continuous, contra α-continuous, contra pre-continuous, contra g^{*}p-continuous functions and contra gα-continuous functions and is properly contained in the class of contra gsp-continuous functions and contra gpr-continuous functions.

II. PRELIMINARIES

Throughout this paper (X,τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, *cl*A and *int*A denote the closure of A and the interior of A respectively. X \ A denotes the complement of A in X. We recall the following definitions and results.

Definition 2.1

A subset A of a topological space X is called

(i) regular-open if A = *int cl*A and regular-closed if A = *clint*A, [18]

(ii) semi-open if A ⊆ □*cl int*A and semi-closed if *int cl*A⊆ A,[11]

(iii) α-open if A ⊆ *int cl int*A and α-closed if *cl int clA* ⊆ A,[14]

(iv) pre-open if A ⊆ □*int cl*A and pre-closed if *cl int*A⊆ A,[13]

(v) semi-pre-open [3] or β-open [1] if A⊆*cl int cl*A and semi-pre-closed [3] or β-closed [1] if *int cl int*A ⊆ □A,

The semi-pre-closure of a subset A of X is the intersection of all semi-pre-closed sets containing A and is denoted by *spcl*A and the semi-closure of A is the intersection of all semi-closed sets containing A and is denoted by *scl*A. Andrijevic [3] established the relationships among the above operators.

Definition 2.2

A subset A of a space X is called

(i) rg-closed if *cl*A ⊆ U whenever A ⊆ U and U is regular open,[16]

(ii) g-closed if *cl*A ⊆ U whenever A ⊆ U and U is open,[12]

(iii) sg-closed if $sclA \subseteq U$ whenever $A \subseteq U$ and U is semiopen.[4]

(iv) $g^{\#}$-closed set if $clA \subseteq U$ whenever $A \subseteq U$ and U is αg-open,[22]

The complement of an rg-closed set is rg-open. The g-open, sg-open and $g^{\#}$p-open sets can be analogously defined.

Definition 2.3

A subset A of a space X is called

(i) pre-semiclosed if $spclA \subseteq U$ whenever $A \subseteq U$ and U is g-open,[20]

(ii) g^{*}p-closed if $pclA \subseteq U$ whenever $A \subseteq U$ and U is g -open.[23]

(iii) $g^{\#}$p-closed if $pclA \subseteq U$ whenever $A \subseteq U$ and U is $g^{\#}$ -open.[17]

The complement of an pre-semiclosed set is pre-semi-open ,g^{*}p-open and $g^{\#}$p-open sets can be analogously defined.

Definition 2.4:[17]

A topological space (X, τ) is said to be

(i).a $T_p^{\#}$space if every $g^{\#}$p- closed set is closed.

(ii).a ${}^{\#}T_p$ space if every gp- closed set is $g^{\#}$p- closed.

(iii).a $T_p^{\#\#}$ space if every $g^{\#}$p- closed set is gα- closed.

(iv).a ${}_{\alpha}T_p^{\#}$ space if every $g^{\#}$p- closed set is preclosed.

(v).a ${}_{\alpha}T_p^{\#\#}$ space if every $g^{\#}$p- closed set is α-closed.

(vi).a ${}^{\#}{}_{s}T_p$ space if every gsp-closed set is $g^{\#}$p- closed.

Lemma2.5 [17]

Let (X, τ) be a topological space .Then

(i)..Every pre-closed set is a $g^{\#}$p-closed .

(ii).Every closed set **,** α-closed set and gα-closed set is $g^{\#}$p-closed.

(iii).Every g^{*}p-closed set is $g^{\#}$p-closed.

(iv).Every $g^{\#}$p-closed set is gsp-closed

(v). Every $g^{\#}$p-closed set is gpr-closed set.

Definition 2.6[10]

A space X is locally indiscrete if every open subset of X is closed.

Definition 2.7

A function f: (X,τ) → (Y,σ) is called

(i) contra continuous if $f^{-1}(V)$ is closed in (X,τ) for every open set V in (Y,σ), [6]

(ii) contra semi-continuous if $f^{-1}(V)$ is semi-closed in (X,τ) for every open set V in (Y,σ), [7]

(iii) RC continuous if $f^{-1}(V)$ is regular-closed in (X,τ) for every open set V in (Y,σ), [7]

(iv) contra gp-continuous if $f^{-1}(V)$ is gp-closed in (X,τ) for every open set V in (Y,σ),

(v) contra α-continuous if $f^{-1}(V)$ is α-closed in (X,τ) for every open set V in (Y,σ), [8]

(vi) contra pre-continuous if $f^{-1}(V)$ is pre-closed in (X,τ) for every open set V in (Y,σ), [9]

(vii) contra gsp-continuous if $f^{-1}(V)$ is gsp-closed in (X,τ) for every open set V in (Y,σ),

(viii) contra gα-continuous if $f^{-1}(V)$ is gα-closed in (X,τ) for every open set V in (Y,σ),

(ix) contra g^{*}p –continuous if $f^{-1}(V)$ is g^{*}p -closed in (X,τ) for every open set V in (Y,σ),

(x) contra gpr-continuous if $f^{-1}(V)$ is gpr-closed in (X,τ) for every open set V in (Y,σ),

(xi) $g^{\#}p$ -continuous if $f^{-1}(V)$ is $g^{\#}p$ -closed in (X,τ) for every closed set V in (Y,σ), [2]

Lemma 2.8 [2]

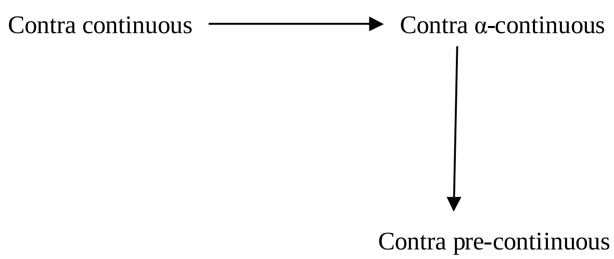
Let f: (X,τ) → (Y,σ) be a function. Then the following are equivalent.

(i) f is $g^{\#}p$-continuous.

(ii) The inverse image of each closed set in Y is $g^{\#}p$-closed in X.

(iii) The inverse image of each open set in Y is $g^{\#}p$--open in X.

Diagram 2.9



Examples can be constructed to show that the reverse implications are not true.

III. CONTRA $g^{\#}p$ -CONTINUOUS FUNCTIONS

In this section we introduce contra $g^{\#}p$ -continuous functions.

Definition 3.1

A function f: (X,τ) → (Y,σ) is called contra $g^{\#}p$ -continuous if $f^{-1}(V)$ is $g^{\#}p$ -closed in (X,τ) for each open set V in (Y,σ).

Theorem 3.2

Every contra pre-continuous function is contra $g^{\#}p$-continuous.

Proof

Suppose f: (X,τ) → (Y,σ) is contra pre-continuous function. Let V be an open set in Y. Since f is contra pre-continuous, using Definition 2.7 (vi), f^{-1} (V) is pre-closed in X. Again usingLemma 2.5 (i), f^{-1} (V) is $g^{\#}p$ -closed in X. Therefore by using Definition 3.1, f is contra $g^{\#}p$ -continuous.

The converse of Theorem 3.2 need not be true as seen from the following example.

Example 3.3

Let X = Y = {a,b,c,} with topologies τ ={Ø,{a}, X } and σ = {Ø,{a},{a,c},Y}. Define f: (X,τ) → (Y,σ) by f(a)=c,f(b)=b,f(c)=a. Then f is contra $g^{\#}p$ -continuous ,since every open set V in Y, f^{-1} (V) is $g^{\#}p$ –closed in (X,τ). But f is not contra pre –continuous, since {a,c} is an open set in Y,f^{-1}{a,c}={c,a} is not preclosed in (X,τ).

Corollary 3.4

(i) Every contra continuous function is contra $g^{\#}p$ -continuous.

(ii) Every contra α-continuous function is contra $g^{\#}p$ -continuous.

Proof

Follows from Diagram 2.9 and Theorem 3.2.

The converses of Corollary 3.4 need not be true as seen from the following example.

Example 3.5

Let X ,Y , τ , σ and f be as in the example 3.3.

Then f is contra $g^{\#}p$ -continuous but not contra continuous and not contra α-continuous.

Theorem 3.6

Every contra $g^{*}p$- continuous function is contra $g^{\#}p$-continuous.

Proof

Suppose f: (X,τ) → (Y,σ) is contra $g^{*}p$ -continuous function. Let V be an open set in Y. Since f is contra $g^{*}p$ -continuous, using Definition 2.7(ix), f^{-1} (V) is $g^{*}p$ -closed in X. Again using Lemma 2.5 (iii), f^{-1} (V) is $g^{\#}p$ -closed in X. Therefore by using Definition 3.1, f is contra $g^{\#}p$ -continuous.

The converse of Theorem 3.6 need not be true as seen from the following example.

Example 3.7

Let X ,Y , τ, σ and f be as in the example 3.3. Then f is contra $g^{\#}p$ -continuous ,since every open set V in Y, f^{-1} (V) is $g^{\#}p$ –closed in (X,τ). But f is not contra $g^{*}p$ –continuous, since {a,c} is an open set in Y, f^{-1}{a,c}={c,a} is not $g^{*}p$ -closed in (X,τ).

Theorem 3.8

Every contra $g^{\#}p$ -continuous function is contra gsp-continuous.

Proof

Suppose f: (X,τ) → (Y,σ) is a contra $g^{\#}p$ -continuous function.Let V be an open set in Y. Since f is contra $g^{\#}p$ -continuous,using Definition 3.1, f^{-1} (V) is $g^{\#}p$ -closed in X. Again using Lemma 2.5(iv), f^{-1} (V) is gsp-closed in X. Therefore by using Definition 2.7(vii), f is contra gsp -continuous.This proves the theorem.

The converse of Theorem 3.8 need not be true as seen from thefollowing example.

Example 3.9

Let X = Y = {a,b,c,} with τ = {Ø,{a},{b},{a,b},X } and σ = {Ø,{a},{a,c},Y}. Let f: (X,τ) → (Y,σ) be the identity function. Then f is contra gsp-continuous but not contra $g^{\#}p$ –continuous , since {a} is an open set in (Y,σ) but f^{-1}{a}={a} is not $g^{\#}p$ –closed in (X,τ).

Theorem 3.10

Every contra $g^{\#}p$ -continuous function is contra gpr-continuous.

Proof

Suppose f: (X,τ) → (Y,σ) is a contra $g^{\#}p$ -continuous function.Let V be an open set in Y. Since f is contra $g^{\#}p$ -continuous,using Definition 3.1, f^{-1} (V) is $g^{\#}p$ -closed in X. Again using Lemma 2.5(v), f^{-1}(V) is gpr-closed in X. Therefore by using Definition 2.7(x), f is contra gpr -continuous.This proves the theorem.

The converse of Theorem 3.10 need not be true as seen from thefollowing example.

Example 3.11

Let X ,Y, τ , and σ be as in the example 3.3 .Let f: (X,τ) → (Y,σ) be the identity function. Then f is contra gpr-continuous but f is not contra $g^{\#}p$ –continuous , since {a} is an open set in (Y,σ) but f^{-1}{a}={a} is not $g^{\#}p$ –closed in (X,τ).

Definition 3.12

A space (X,τ) is called $g^{\#}p$ -locally indiscrete if every $g^{\#}p$ -open set is closed.

Theorem 3.13 Let f: (X,τ) → (Y,σ) be a function.

(i) If f is $g^{\#}p$ -continuous and if X is $g^{\#}p$ -locally indiscrete then f is contra continuous.

(ii) If f is contra $g^{\#}p$ -continuous and if X is $\alpha T_P^{\#\#}$ space, then f is contra α-continuous.

(iii) If f is contra $g^{\#}p$ –continuous and if X is $T_P^{\#}$ space, then f is contra continuous.

(iv) If f is $g^{\#}p$ -continuous and if Y is locally indiscrete, then f is contra - $g^{\#}p$ continuous.

(v) If f is contra $g^{\#}p$ -continuous and if X is $\alpha T_P^{\#}$ space, then f is contra pre-continuous.

(vi) If f is contra $g^{\#}p$ -continuous and if X is $T_P^{\#\#}$ space, then f is contra gα-continuous.

(vii) If f is contra gp-continuous and if X is $^{\#}T_P$ space, then f is contra $g^{\#}p$ -continuous.

(viii) If f is contra gsp-continuous and if X is $^{\#}{}_{s}T_P$ space, then f is contra $g^{\#}p$ -continuous.

Proof

(i)Suppose f is $g^{\#}p$ -continuous. Let X be $g^{\#}p$ -locally indiscrete. Let V be open in Y. Since f is $g^{\#}p$ -continuous, by using Definition 2.7(xi), f^{-1} (V) is $g^{\#}p$ -open in X. Since X is $g^{\#}p$ -locally indiscrete, using Definition 3.12, f^{-1} (V) is closed in X. Therefore by using Definition 2.7(i), f is contra continuous. This proves (i).

(ii) Suppose f is contra $g^{\#}p$ -continuous. Let X be an $\alpha T_P^{\#\#}$ space. Let V be open in Y. Since f is contra $g^{\#}p$ -continuous, by using Definition 3.1, f^{-1} (V) is $g^{\#}p$ -closed in X. Since X is an $\alpha T_P^{\#\#}$ space, by using Definition 2.4(v), f^{-1} (V) is α-closed in X. Therefore by using Definition 2.7(v), f is contra α-continuous. This proves (ii).

(iii) Suppose f is contra $g^{\#}p$ -continuous. Let X be a $T_P^{\#}$ space. Let V be open in Y. Since f is contra $g^{\#}p$ -continuous, by using Definition 3.1, f^{-1} (V) is $g^{\#}p$ -closed in X. Since X is a $T_P^{\#}$ space, using Definition 2.4(i), f^{-1} (V) is closed in X. Therefore by using Definition 2.7(i), f is contra continuous. This proves (iii).

(iv) Suppose f is $g^{\#}p$- continuous. Let Y be locally indiscrete. Let V be an open subset of Y. Since Y is locally indiscrete, by using Definition 3.12, V is closed. Since f is $g^{\#}p$ -continuous, by using Definition 2.7(xi), f^{-1} (V) is $g^{\#}p$ -closed in X. Therefore by using Definition 3.1, f is contra $g^{\#}p$ -continuous. This proves (iv).

(v) Suppose f is contra $g^{\#}p$ -continuous. Let X be an $\alpha T_P^{\#}$ space. Let V be open in Y. Since f is contra $g^{\#}p$ -continuous, by using Definition 3.1, f^{-1} (V) is $g^{\#}p$ -closed in X. Since X is an $\alpha T_P^{\#}$ space, by using Definition 2.4(iv), f^{-1} (V) is pre-closed in X. Therefore by using Definition 2.7(vi), f is contra pre-continuous. This proves (v).

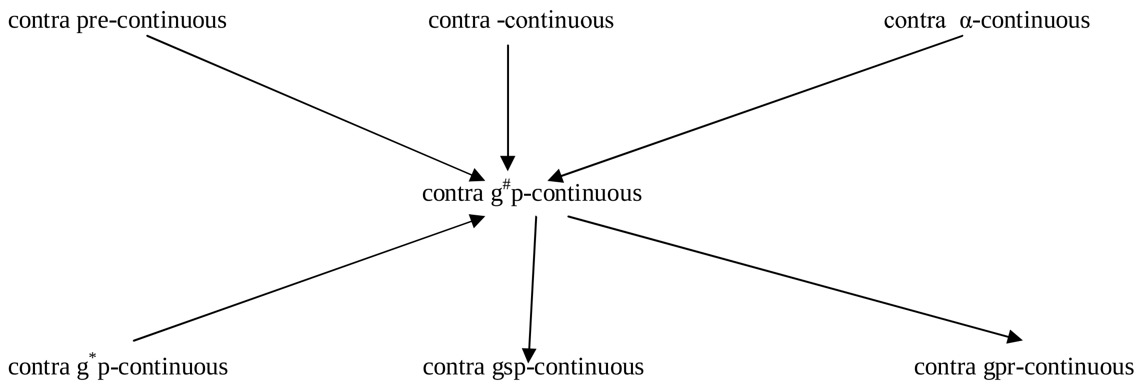
(vi) Suppose f is contra $g^{\#}p$ -continuous. Let X be a $T_P^{\#\#}$ space. Let V be open in Y. Since f is contra $g^{\#}p$ -continuous, by using Definition 3.1, f^{-1} (V) is $g^{\#}p$ -closed in X. Since X is a $T_P^{\#\#}$ space, by using Definition 2.4(iii), f^{-1} (V) is gα-closed in X. Therefore by using Definition 2.7(viii), f is contra gα-continuous. This proves (vi).

(vii) Suppose f is contra gp-continuous. Let X be a $^{\#}T_P$ space. Let V be open in Y. Since f is contra gp-continuous, by using Definition 2.7(iv), f^{-1} (V) is gp-closed in X. Since X is a $^{\#}T_P$ space, using Definition 2.4(ii), f^{-1} (V) is $g^{\#}p$ -closed in X. Therefore by using Definition 3.1, f is contra $g^{\#}p$ -continuous. This proves (vii).

(viii) Suppose f is contra gsp-continuous. Let X be a $^{\#}{}_{s}T_P$ space. Let V be open in Y. Since f is contra gsp-continuous, by using Definition 2.7(vii), f^{-1} (V) is gsp-closed in X. Since X is a $^{\#}{}_{s}T_P$ space, using Definition 2.4(vi), f^{-1} (V) is $g^{\#}p$ -closed in X. Therefore by using Definition 3.1, f is contra $g^{\#}p$ -continuous. This proves (viii).

Remark 3.14:

The following diagram shows that the relationships between contra $g^{\#}$p-continuous function and some other contra continuous functions.



Where A→B represents A implies B and B need not imply A.

REFERENCES

[1] M.E. Abd El-Monsef, S.N. El- Deeb and R.A. Mahmoud, β-open sets and β-continuous mappings, *Bull. Fac. Sci. Assiut Univ.*, 12(1983), 77-90.

[2] K.Alli.,S.Pious Missier and A.Subramanian.,on $g^{\#}$p-continuous maps in topological spaces, *International Journal of Modern Engineering Research,Vol.3,Issue 1,Jan-Feb*.(2013) 360- 364.

[3] D. Andrijevic, Semi-pre-open sets. *Mat.Vesnik.,* 38(1986), 24-32.

[4] P. Bhattacharya and B.K. Lahiri, Semi-generalized closed sets in topology, *Indian. J.Math*., 29(3)(1987), 375-382.

[5] M. Caldas and S. Jafari, Some properties of contra β-continuous functions, *mem.Fac.Sci. Kochi Univ. Ser. A Math.*, 22(2001), 19-28.

[6] J.Dontchev, Contra continuous functions and strongly S-closed spaces, *Int.Math. Math.Sci*.,19(2)(1996), 303-310.

[7] J. Dontchev and T. Noiri, Contra semi-continuous functions, *Math. Pannon*., 10(2)(1999), 159-168.

[8] S. Jafari and T. Noiri, Contra α-continuous functions between topological spaces, *Iran. Int. J. Sci*., 2(2)(2001), 153-167.

[9] S. Jafari and T. Noiri, On contra pre-continuous functions, *Bull. Malays. Math Sci. Soc.*, 25(2)(2002), 115- 128.

[10] D.S. Jankovic, On locally irreducible space, *Anna. De. La. Soc. Sci. de bruxelles,* 97(2)(1983), 59-72.

[11] N. Levine, Semi-open sets and semi continuity in topological spaces, A*mer. Math. Monthly*, 70(1963), 36-41.

[12] N. Levine, Generalized closed sets in topology, *Rend. Circ. Mat. Palermo*, 19(2)(1970), 89-96.

[13] A.S. Mashhour, M.E Abd El-Monsef and S.N El-Deeb, On pre-continuous and weak pre-continuous functions, *Proc. Math.*

Phys. Soc. Egypt, 53(1982), 47-53.

[14] O. Njastad, On some classes of nearly open sets, *Pacific J. Math*., 15(1965), 961-970.

[15] T. Noiri, Super continuity and some strong forms of continuity, *Indian J. pure. appl. Math*., 15(3)(1984), 241-250.

[16] N. Palaniappan and K.C. Rao, Regular generalized closed sets, *Kyungpook Math. J*., 33(2)(1993), 211-219.

[17] S.Pious Missier, K. Alli and A.Subramanian , $g^{\#}$p-closed sets in a topological spaces, *International Journal of Mathematical Archieve (IJMA)* - 4(1),(2013),176-181.

[18] M. Stone, Applications of the theory of Boolean rings to the general topology, *Trans. Amer. Math. Soc.*, 41(1937), 375-481.

[19] M.K.R.S. Veerakumar, Between semi-closed sets and semi-preclosed sets, *Rend. Istit. Mat.Univ. Trieste*, 32(1-2)(2000), 25-41.

[20] M.K.R.S. Veerakumar, Pre-semiclosed sets, *Indian J.Math*., 44(2)(2002), 165-181.

[21] M.K.R.S. Veerakumar, Contra pre-semicontinuous functions, *Bull. Malays. Math. Sci. Soc.*, (2)28(1)(2005),67-71.

[22] M.K.R.S.Veera kumar,$g^{\#}$-closed sets in topological spaces, *Mem.Fac.Sci.Kochi Univ.Ser.A.Math 24(2003),1-13.*

[23] M.K.R.S. Veera kumar, g^{*}-preclosed sets *Acta ciencia Indica (Maths) Meerut XXVIII(M)(1) 2002.*