Application of Intuitionistic Fuzzy Soft Matrices in Decision Making Problems

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Abstract

Fuzzy soft matrices and intuitionistic fuzzy soft matrices have recently been applied in many fields of real life scenario. Here we introduce different types of T-product of intuitionistic fuzzy soft matrices and study its properties. Then a new methodology has been developed involving T-product to solve intuitionistic fuzzy soft set based real life decision making problems which may involve more than one decision maker. Moreover, we have presented another example of decision making problem based on one of the products of intuitionistic fuzzy soft matrices.

Keywords: Soft sets, intuitionistic fuzzy soft sets, fuzzy soft matrices, intuitionistic fuzzy soft matrices.

I. INTRODUCTION

In the year 1999, Molodtsov [15] introduced soft set theory as a mathematical tool for dealing with the uncertainties which traditional mathematics failed to handle. Molodtsov has shown numerous applications of this theory in solving practical problems in engineering, medical sciences, economics, environment management and social sciences. In 2003 Maji et al. [13] studied the theory of soft sets initiated by Molodtsov [15] and developed as a mathematical discipline. In 2010, Ca $\breve{g}$ man et al. [6] introduced a new definition of soft set based decision making method which selects the optimum elements from the alternatives. They have also proposed a definition of soft matrices which are representation of soft sets and applied in decision making problems [7], [8]. This presentation has several advantages. During the last two years quite a good number of research papers have appeared on soft matrices, fuzzy soft matrices and intuitionistic fuzzy soft matrices [3], [4], [5]. Subsequently different methodologies have been developed for solving many of our day to day life problems.

In this paper we have defined T-product of intuitionistic fuzzy soft matrices in continuation of T-product of fuzzy soft matrices by Yong Yang at al.[16] and study some of its properties. Then we have presented two examples. In the first example, we use the product of intuitionistic fuzzy soft matrices(IFSM) based on the product of IFSM defined earlier by B. Chetia and P.K. Das [9] and the second example is based on T-product of intuitionistic fuzzy soft matrices. For this we have defined decision function and optimum fuzzy set.

II. PRELIMINARIES

Definition 2.1 [10]

Let U be an initial universe, P(U) be the power set of U, E be the set of all parameters and $A \subseteq E$. A pair (f_A, E) is called a soft set over U is defined as the set of ordered pairs $(f_A, E) = \{(e, f_A(e)) : e \in E, f_A(e) \in P(U)\}$, where f_A is a mapping given by $f_A : E \to P(U)$ such that $f_A(e) = \phi$ if $e \notin A$.

Here f_A is called approximate function of the soft set (f_A, E). The set $f_A(e)$ is called *e*- approximate value set which consists of related objects of the parameter $e \in A$. In other words, a soft set over *U* is a parameterized family of subsets of the universe *U*.

Definition 2.2 [14]
Let *U* be an initial universe set and *E* be a set of parameters (which are fuzzy words or sentences involving fuzzy words). Let $P(U)$ denotes the set of all fuzzy sets of *U* and $A \subseteq E$. A pair (f_A, E) is called a fuzzy soft set (FSS) over *U*, where f_A is a mapping given by $f_A : E \to P(U)$, such that $f_A(e) = \tilde{\phi}$ if $e \notin A$, where $\tilde{\phi}$ is a null fuzzy set.

Definition 2.3 [14]

Let *U* be an initial universe set and *E* be a set parameters. Let *P*(*U*) denotes the set of all intuitionistic fuzzy soft set over *U* and $A \subseteq E$. A pair (f_A, E) is called an intuitionistic fuzzy soft set (IFSS) over *U*, where f_A is a mapping given by, $f_A : E \to P(U)$ such that $f_A(e) = \tilde{\phi}$ if $e \notin A$, where $\tilde{\phi}$ is the null intuitionistic fuzzy set (i.e. the membership value of *x*, μ(*x*)=0; the non-membership value of $x, v(x) = 1$ are the indeterministic part of *x*, $\pi(x) = 0$ for all $x \in \tilde{\phi}$).

Definition 2.4 [7]
Let (f_A, E) be a fuzzy soft set over *U*. Then the subset R_A of $U \times E$ is uniquely defined by $R_A = \{(u,e) : e \in A, u \in f_A(e)\}$ is called a relation between elements of *A* and the elements of *U*. Then the characteristic function of R_A is denoted by

$$\mu_{R_A} : U \times E \to [0,1], \text{ where } \mu_{R_A}(u,e) \in [0,1] \text{ is the membership value of } u \in U \text{ for each } e \in E.$$

If $\mu_{ij} = \mu_{R_A}(u_i, e_j)$, we can define a matrix

$$[\mu_{ij}]_{m\times n} = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix}$$

which is called an $m \times n$ fuzzy soft matrix of the fuzzy soft set (f_A, E) over *U*.

Therefore, we can say that a fuzzy soft set (f_A, E) is uniquely characterized by the matrix $[\mu_{ij}]_{m\times n}$ and both concepts are interchangeable.
The set of all $m \times n$ fuzzy soft matrices over *U* will be denoted by $FSM_{m\times n}$.

Definition 2.5 [16]

Let $\tilde{A}_k = [a_{ij}^k] \in FSM_{m\times n}$, $1 \le k \le l$. Then the T-product of fuzzy soft matrices, denoted as $\prod_{k=1}^{l} \tilde{A}_k = \tilde{A}_1 \times \tilde{A}_2 \times \ldots \times \tilde{A}_l$, is defined by $\prod_{k=1}^{l} \tilde{A}_k = [c_i]_{m\times 1}$, where $c_i = \frac{1}{n} \sum_{j=1}^{n} \underset{k=1}{\overset{l}{T}} a_{ij}^k$ and $T = \wedge$ or $\vee$ according to the type of problems.

Theorem 2.6 [16]

Let $\tilde{A},\tilde{B},\tilde{C}\in FSM_{m\times n}$. Then

$(i)\tilde{A}\times\tilde{B}=\tilde{B}\times\tilde{A}.\quad (ii)(\tilde{A}\times\tilde{B})\times\tilde{C}=\tilde{A}\times(\tilde{B}\times\tilde{C}).$

Theorem 2.7 [16]

Let $\tilde{A},\tilde{B},\tilde{C}\in FSM_{m\times n}$ and $\tilde{B}\subseteq\tilde{C}$. Then $\tilde{A}\times\tilde{B}\subseteq\tilde{A}\times\tilde{C}.$

Corollary 2.7 [16]

Let $\tilde{A},\tilde{B},\tilde{C}\in FSM_{m\times n}$. Then

$(i)\tilde{A}\times(\tilde{B}\cup\tilde{C})\supseteq(\tilde{A}\times\tilde{B})\cup(\tilde{A}\times\tilde{C}).\qquad (ii)\tilde{A}\times(\tilde{B}\cap\tilde{C})\subseteq(\tilde{A}\times\tilde{B})\cap(\tilde{A}\times\tilde{C}).$

Definition 2.8 [9]

Let U be an initial universe, E be a set of parameters and $A\subseteq E$. Let (f_A,E) be an intuitionistic fuzzy soft set over U. Then a subset of $U\times E$ is uniquely defined by

$$R_A=\{(u,e):e\in A,u\in f_A(e)\}$$

is called a relation form of (f_A,E). The membership and non-membership functions are denoted by

$$\mu_{R_A}:U\times E\to[0,1] \text{ and } \nu_{R_A}:U\times E\to[0,1],$$

where $\mu_{R_A}(u,e)\in[0,1]$ and $\nu_{R_A}(u,e)\in[0,1]$ are the membership and non-membership value respectively of $u\in U$ for each $e\in E$.

If $[(\mu_{ij},\nu_{ij})]_{m\times n}=(\mu_{R_A}(u_i,e_j),\nu_{R_A}(u_i,e_j))$, then we can define a matrix

$$[(\mu_{ij},\nu_{ij})]_{m\times n}=\begin{bmatrix}(\mu_{11},\nu_{11}) & (\mu_{12},\nu_{12}) & \cdots & (\mu_{1n},\nu_{1n})\\ (\mu_{21},\nu_{21}) & (\mu_{22},\nu_{22}) & \cdots & (\mu_{2n},\nu_{2n})\\ \vdots & \vdots & \vdots & \vdots\\ (\mu_{m1},\nu_{m1}) & (\mu_{m2},\nu_{m2}) & \cdots & (\mu_{mn},\nu_{mn})\end{bmatrix}$$

which is called an $m\times n$ IFSM of IFSS (f_A,E) over U.

Therefore, we can say that an IFSS (f_A,E) is uniquely characterized by the matrix $[(\mu_{ij},\nu_{ij})]_{m\times n}$ and both concepts are interchangeable. The set of all $m\times n$ IFSM over U will be denoted by $IFSM_{m\times n}$.

Example 2.9

Assume that $U=\{u_1,u_2,u_3,u_4\}$ be a universal set and $E=\{e_1,e_2,e_3,e_4\}$ be the set of all parameters. If $A\subseteq E$ and $A=\{e_1,e_2,e_3\}$, then $R_A=\{(u,e):e\in A,u\in f_A(e)\}$ and

$f_A(e_1)=\{u_1/(0.3,0.6),u_2/(0.4,0.2),u_3/(0.3,0.2),u_4/(0.4,0.3)\}.$

$f_A(e_2)=\{u_1/(0.5,0.4),u_2/(0.3,0.6),u_3/(0.5,0.2),u_4/(0.3,0.5)\}.$

$f_A(e_3)=\{u_1/(0.7,0.2), u_2/(0.5,0.3), u_3/(0.4,0.4), u_4/(0.5,0.2)\}$.

Then the IFSM $[(\mu_{44},\nu_{44})]$ is written as

$$[(\mu_{44},\nu_{44})]=\begin{pmatrix} (0.3,0.6) & (0.5,0.4) & (0.7,0.2) & 0 \\ (0.4,0.2) & (0.3,0.6) & (0.5,0.3) & 0 \\ (0.3,0.2) & (0.5,0.2) & (0.4,0.4) & 0 \\ (0.4,0.3) & (0.3,0.5) & (0.5,0.2) & 0 \end{pmatrix}$$

PRODUCT OF INTUITIONISTIC FUZZY SOFT MATRICES:

In [9] B. Chetia et.al. has defined five different types of product of intuitionistic fuzzy soft matrices.

Definition 2.10

Let $\tilde{A}=[(\mu_{ij},\nu_{ij})], \tilde{B}=[(\mu^{/}_{ik},\nu^{/}_{ik})]\in IFSM_{m\times n}$. Then '$\times_1$' product of $\tilde{A}$ and $\tilde{B}$ is defined by

$\times_1 : IFSM_{m\times n}\times IFSM_{m\times n}\to IFSM_{m\times n^2}$ such that $\tilde{A}\times_1\tilde{B}=[(\mu_{ij},\nu_{ij})]\times_1[(\mu^{/}_{ik},\nu^{/}_{ik})]=[(\mu^{//}_{ip},\nu^{//}_{ip})]$,

where $\mu^{//}_{ip}=\mu_{ij}.\mu^{/}_{ik}$ and $\nu^{//}_{ip}=\nu_{ij}.\nu^{/}_{ik}$ such that $p=n(j-i)+k$.

Definition 2.11

Let $\tilde{A}=[(\mu_{ij},\nu_{ij})], \tilde{B}=[(\mu^{/}_{ik},\nu^{/}_{ik})]\in IFSM_{m\times n}$. Then '$\times_2$' product of $\tilde{A}$ and $\tilde{B}$ is defined by

$\times_2 : IFSM_{m\times n}\times IFSM_{m\times n}\to IFSM_{m\times n^2}$ such that $\tilde{A}\times_2\tilde{B}=[(\mu_{ij},\nu_{ij})]\times_2[(\mu^{/}_{ik},\nu^{/}_{ik})]=[(\mu^{//}_{ip},\nu^{//}_{ip})]$,

where $\mu^{//}_{ip}=\mu_{ij}+\mu^{/}_{ik}-\mu_{ij}.\mu^{/}_{ik}$ and $\nu^{//}_{ip}=\nu_{ij}.\nu^{/}_{ik}$ such that $p=n(j-i)+k$.

Definition 2.12

Let $\tilde{A}=[(\mu_{ij},\nu_{ij})], \tilde{B}=[(\mu^{/}_{ik},\nu^{/}_{ik})]\in IFSM_{m\times n}$. Then '$\times_3$' product of $\tilde{A}$ and $\tilde{B}$ is defined by

$\times_3 : IFSM_{m\times n}\times IFSM_{m\times n}\to IFSM_{m\times n^2}$ such that $\tilde{A}\times_3\tilde{B}=[(\mu_{ij},\nu_{ij})]\times_3[(\mu^{/}_{ik},\nu^{/}_{ik})]=[(\mu^{//}_{ip},\nu^{//}_{ip})]$,

where $\mu^{//}_{ip}=\mu_{ij}.\mu^{/}_{ik}$ and $\nu^{//}_{ip}=\nu_{ij}+\nu^{/}_{ik}-\nu_{ij}.\nu^{/}_{ik}$ such that $p=n(j-i)+k$.

Definition 2.13

Let $\tilde{A}=[(\mu_{ij},\nu_{ij})], \tilde{B}=[(\mu^{/}_{ik},\nu^{/}_{ik})]\in IFSM_{m\times n}$. Then '$\times_4$' product of $\tilde{A}$ and $\tilde{B}$ is defined by

$\times_4 : IFSM_{m\times n}\times IFSM_{m\times n}\to IFSM_{m\times n^2}$ such that $\tilde{A}\times_4\tilde{B}=[(\mu_{ij},\nu_{ij})]\times_4[(\mu^{/}_{ik},\nu^{/}_{ik})]=[(\mu^{//}_{ip},\nu^{//}_{ip})]$,

where $\mu^{//}_{ip}=\min\{\mu_{ij},\mu^{/}_{ik}\}$ and $\nu^{//}_{ip}=\max\{\nu_{ij},\nu^{/}_{ik}\}$ such that $p=n(j-i)+k$.

Definition 2.14

Let $\tilde{A}=[(\mu_{ij},\nu_{ij})], \tilde{B}=[(\mu^{/}_{ik},\nu^{/}_{ik})]\in IFSM_{m\times n}$. Then '$\times_5$' product of $\tilde{A}$ and $\tilde{B}$ is defined by

$\times_5 : IFSM_{m\times n}\times IFSM_{m\times n}\to IFSM_{m\times n^2}$ such that $\tilde{A}\times_5\tilde{B}=[(\mu_{ij},\nu_{ij})]\times_5[(\mu^{/}_{ik},\nu^{/}_{ik})]=[(\mu^{//}_{ip},\nu^{//}_{ip})]$, where $\mu_{ip}^{//} = \max\{\mu_{ij}, \mu_{ik}^{/}\}$ and $\nu_{ip}^{//} = \min\{\nu_{ij}, \nu_{ik}^{/}\}$ such that $p = n(j-i)+k.$

III. T-PRODUCT OF INTUITIONISTIC FUZZY SOFT MATRICES

T-product of fuzzy soft matrices defined by Yong Yang and Chenli Ji[16]. In this section, we extend that work and define two types of T-product of intuitionistic fuzzy soft matrices and study their properties.

Definition 3.1

Let $\tilde{A}_k = [(\mu_{ij}^k, \nu_{ij}^k)] \in IFSM_{m\times n}$, for $1 \le k \le l$. Then T−product X_1 of $\tilde{A}_k$'s for all $1 \le k \le l$,

denoted by $\prod_{k=1}^{l} \tilde{A}_k$, is defined as

$$\prod_{k=1}^{l} \tilde{A}_k = [(\mu_{ij}^1, \nu_{ij}^1)\,\mathrm{X}_1 (\mu_{ij}^2, \nu_{ij}^2)\,\mathrm{X}_1 \ldots \mathrm{X}_1 (\mu_{ij}^l, \nu_{ij}^l)] = [(\mu_{i1}^{//}, \nu_{i1}^{//})]_{m\times 1},$$

where $\mu_{i1}^{//} = \frac{1}{n}\sum_{j=1}^{n}[\max\{\mu_{i1}^1, \mu_{i1}^2, \ldots, \mu_{i1}^l\}]$ and $\nu_{i1}^{//} = \frac{1}{n}\sum_{j=1}^{n}\min[\nu_{i1}^1, \nu_{i1}^2, \ldots, \nu_{i1}^l];\ \ i = 1, 2, \ldots, m.$

Definition 3.2

Let $\tilde{A}_k = [(\mu_{ij}^k, \nu_{ij}^k)] \in IFSM_{m\times n}$, for $1 \le k \le l$. Then T−product X_2 of $\tilde{A}_k$'s for all $1 \le k \le l$,

denoted by $\prod_{k=1}^{l} \tilde{A}_k$, is defined as

$$\prod_{k=1}^{l} \tilde{A}_k = [(\mu_{ij}^1, \nu_{ij}^1)\,\mathrm{X}_2 (\mu_{ij}^2, \nu_{ij}^2)\,\mathrm{X}_2 \ldots \mathrm{X}_2 (\mu_{ij}^l, \nu_{ij}^l)] = [(\mu_{i1}^{//}, \nu_{i1}^{//})]_{m\times 1},$$

where $\mu_{i1}^{//} = \frac{1}{n}\sum_{k=1}^{l}[\min\{\mu_{ij}^1, \mu_{ij}^2, \ldots, \mu_{ij}^l\}]$ and $\nu_{i1}^{//} = \frac{1}{n}\sum_{j=1}^{l}[\max\{\nu_{ij}^1, \nu_{ij}^2, \ldots, \nu_{ij}^n\}];\ i = 1, 2, \ldots, m.$

To illustrate these products, let us the following example.

Example 3.3

Assume that $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3 \in IFSM_{3\times 2}$ are given as follows:

$$\tilde{A}_1 = \begin{pmatrix} (0.5,0.3) & (0.5,0.2) \\ (0.4,0.3) & (0.6,0.3) \\ (0.6,0.2) & (0.3,0.1) \end{pmatrix}, \tilde{A}_2 = \begin{pmatrix} (0.4,0.2) & (0.5,0.3) \\ (0.5,0.2) & (0.5,0.4) \\ (0.5,0.3) & (0.6,0.2) \end{pmatrix}, \tilde{A}_3 = \begin{pmatrix} (0.4,0.3) & (0.5,0.2) \\ (0.5,0.2) & (0.4,0.3) \\ (0.4,0.1) & (0.3,0.2) \end{pmatrix}$$

Then the T-product X_1 of intuitionistic fuzzy soft matrices $\tilde{A}_1, \tilde{A}_2$ and $\tilde{A}_3$ is given by

$$\prod_{k=1}^{3}\tilde{A}_k = \tilde{A}_1 \mathrm{X}_1 \tilde{A}_2 \mathrm{X}_1 \tilde{A}_3 = \frac{1}{2}\begin{pmatrix} (\max\{0.5,0.4,0.4\}+\max\{0.5,0.5,0.5\}, \min\{0.3,0.2,0.3\}+\min\{0.2,0.3,0.2\}) \\ (\max\{0.4,0.5,0.5\}+\max\{0.6,0.5,0.4\}, \min\{0.3,0.2,0.2\}+\min\{0.3,0.4,0.3\}) \\ (\max\{0.6,0.5,0.4\}+\max\{0.3,0.6,0.3\}, \min\{0.2,0.3,0.1\}+\min\{0.1,0.2,0.2\}) \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} (0.5+0.5, 0.2+0.2) \\ (0.5+0.6, 0.2+0.3) \\ (0.6+0.6, 0.1+0.1) \end{pmatrix} = \frac{1}{2}\begin{pmatrix} (1.0, 0.4) \\ (1.1, 0.5) \\ (1.2, 0.2) \end{pmatrix} = \begin{pmatrix} (0.50, 0.20) \\ (0.55, 0.25) \\ (0.60, 0.10) \end{pmatrix}$$

Similarly T-product X_2 of intuitionistic fuzzy soft matrices $\tilde{A}_1, \tilde{A}_2$ and $\tilde{A}_3$ is given by

$$\prod_{k=1}^{3} A_k = \tilde{A}_1 \mathrm{X}_2 \tilde{A}_2 \mathrm{X}_2 \tilde{A}_3$$

$$= \frac{1}{2}\begin{pmatrix} (\min\{0.5,0.4,0.4\}+\min\{0.5,0.5,0.5\}, \max\{0.3,0.2,0.3\}+\max\{0.2,0.3,0.2\}) \\ (\min\{0.4,0.5,0.5\}+\min\{0.6,0.5,0.4\}, \max\{0.3,0.2,0.2\}+\max\{0.3,0.4,0.3\}) \\ (\min\{0.6,0.5,0.4\}+\min\{0.3,0.6,0.3\}, \max\{0.2,0.3,0.1\}+\max\{0.1,0.2,0.2\}) \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} (0.4+0.5, 0.3+0.3) \\ (0.4+0.4, 0.2+0.3) \\ (0.4+0.3, 0.3+0.2) \end{pmatrix} = \frac{1}{2}\begin{pmatrix} (0.9, 0.6) \\ (0.8, 0.5) \\ (0.7, 0.5) \end{pmatrix} = \begin{pmatrix} (0.45, 0.30) \\ (0.40, 0.25) \\ (0.35, 0.25) \end{pmatrix}$$

Proportion 3.4

Let $\tilde{A}, \tilde{B}, \tilde{C} \in IFSM_{m\times n}$. Then

(i) $\tilde{A} \otimes \tilde{B} = \tilde{B} \otimes \tilde{A}$. (ii) $(\tilde{A} \otimes \tilde{B}) \otimes \tilde{C} = \tilde{A} \otimes (\tilde{B} \otimes \tilde{C})$, where $\otimes$ denotes the T- product X_1 or X_2 of IFSM.

Proof: (i)

Let $\tilde{A} = [(\mu_{ij}, \nu_{ij})]$ and $\tilde{B} = [(\mu'_{ik}, \nu'_{ik})]$.

Then $\tilde{A} \otimes \tilde{B} = \sum_{i=1, j=1}^{m,n} [(\mu_{ij}, \nu_{ij})] \otimes [(\mu'_{ik}, \nu'_{ik})] = \sum_{i=1, j=1}^{m,n} [(\mu'_{ik}, \nu'_{ik})] \otimes [(\mu_{ij}, \nu_{ij})] = \tilde{B} \otimes \tilde{A}$.

(ii) This prove is similar to (i).

Proportion 3.5

Let $\tilde{A}, \tilde{B}, \tilde{C} \in IFSM_{m\times n}$. Then

(i) $\tilde{A} \cup (\tilde{B} \otimes \tilde{C}) = (\tilde{A} \cup \tilde{B}) \otimes (\tilde{A} \cup \tilde{C})$. (ii) $\tilde{A} \otimes (\tilde{B} \cup \tilde{C}) = (\tilde{A} \otimes \tilde{B}) \cup (\tilde{A} \otimes \tilde{C})$.

where $\otimes$ denotes the T- product X_1 or X_2 of IFSM.

IV. APPLICATION OF INTUITIONISTIC FUZZY SOFT MAATRICES IN DECISION MAKING PROBLEM

In this section we are presenting two different decision making problems. First problem based on product $\times_5$ of intuitionisticfuzzy soft matrices and second problem is based on the T-product X_1 of intuitionistic fuzzy soft matrices.

4.1 DECISION MAKING PROBLEM USING THE PRODUCT $\times_5$ OF INTUITIONISTIC FUZZY SOFT MATRICES

Here we construct an IFS-max-min decision making method by using IFS-max-min decision function. We define these max-min decision function and optimum fuzzy set based on the definitions by N. Ca$\breve{g}$man and S. Engino$\breve{g}$lu [7] for the decision making problem by using product $\times_5$ of intuitionistic fuzzy soft matrices. The method selects optimum alternatives from the set of the alternatives.

Definition 4. 1.1

Let $[(\mu_{ip},\nu_{ip})] \in IFSM_{m\times n^2}$, where $I_k=\{p:(\mu_{ip},\nu_{ip})\neq 0 \textit{ for some } 1\le i\le m; (k-1)n<p\le kn\}$ for all $k\in\{1,2,\ldots,n\}$. The IFS-max-min decision function denoted by D_m^M is defined as

$$D_m^M : IFSM_{m\times n^2} \to IFSM_{m\times 1}$$

such that $D_m^M(\mu_{ip},\nu_{ip}) = [d_{i1}] = [(\mu_{i1}^{//},\nu_{i1}^{//})] = [(\max_{k=1}^{n}\{\mu_{ip_k}^{/}\}, \min_{k=1}^{n}\{\nu_{ip_k}^{/}\})]$,

where $D_m^M = D_{\min}^{\max}$,

$$\mu_{ip_k}^{/} = \begin{cases} \min_{p_k\in I_k}\{\mu_{ip_k}\} & if\ I_k\neq\phi \\ 0 & if\ I_k=\phi \end{cases}$$

and $$\nu_{ip_k}^{/} = \begin{cases} \max_{p_k\in I_k}\{\nu_{ip_k}\} & if\ I_k\neq\phi \\ 0 & if\ I_k=\phi \end{cases}$$

Then the one column IFSM $D_m^M(\mu_{ip},\nu_{ip})$ is called max-min decision function of IFSM.

Definition 4.1.2

Let $U=\{u_1,u_2,\ldots,u_m\}$ be the initial universe and $D_m^M(\mu_{ip},\nu_{ip})=[d_{i1}]$. Then the set defined by

$\underset{i=1\ [d_{i1}]}{\overset{m}{opt}}(U)=\{u_i/d_i : u_i\in U, d_i = \max\{\mu_{i1}^{//}-\nu_{i1}^{//}\}, d_{i1}\neq 0\}$ which is called an optimum fuzzy set on *U*.

ALGORITHM:

The algorithm for the solution is given below.

Step 1: choose feasible subsets of the set of parameters.
Step 2: construct the IFS-matrices for each parameter.
Step 3: choose a product of the IFS-matrices, say $\times_5$.

Step 4*:* find the method max-min decision IFS matrices.
Step 5: find an optimum fuzzy set on *U*.

REMARK: We can also define IFSM-min-max or IFSM-min-min or IFSM-max-max decision making methods. One of them may be more useful than the others according to the type of the problems.

CASE STUDY:

Assume that, a car dealer stores four different types of cars $U=\{c_1,c_2,c_3,c_4\}$ which may be characterized by the set of parameters $E=\{e_1,e_2,e_3\}$, where e_1 stands for costly, e_2 stands for getup and e_3 stands for fuel efficiency. Then we consider the following example.

Suppose a couple, Mr. X and Mrs. X come to the dealer to buy a car before Durga Puja. If each partner has to consider his/her own set of parameters, then we select the car on the basis of partner's parameters by using IFSM-max–min decision making as follow.

Step 1: First, Mr. X and Mrs. X have to choose the sets of their parameters $A=\{e_2,e_3\}$ and $B=\{e_1,e_2\}$ respectively.

Step 2: Then we construct the IFS-matrices $\tilde{P}$ and $\tilde{Q}$ according to their set of parameters A and B respectively as follow:

$$\tilde{P}=\begin{bmatrix} 0 & (0.6,0.3) & (0.7,0.2) \\ 0 & (0.3,0.6) & (0.5,0.3) \\ 0 & (0.5,0.2) & (0.6,0.2) \\ 0 & (0.4,0.5) & (0.3,0.5) \end{bmatrix} \quad \text{and } \tilde{Q}=\begin{bmatrix} (0.5,0.4) & (0.8,0.2) & 0 \\ (0.8,0.1) & (0.5,0.3) & 0 \\ (0.5,0.3) & (0.3,0.6) & 0 \\ (0.4,0.3) & (0.5,0.2) & 0 \end{bmatrix}$$

Step 3: Now, we can find a product of the IFS-matrices $\tilde{P}$ and $\tilde{Q}$ by using max-min ($\times_5$) product as follow:

$$\tilde{P}\times_5\tilde{Q}=\begin{bmatrix} 0 & (0.6,0.3) & (0.7,0.2) \\ 0 & (0.3,0.6) & (0.5,0.3) \\ 0 & (0.5,0.2) & (0.6,0.2) \\ 0 & (0.4,0.5) & (0.3,0.5) \end{bmatrix}\times_5\begin{bmatrix} (0.5,0.4) & (0.8,0.2) & 0 \\ (0.8,0.1) & (0.5,0.3) & 0 \\ (0.5,0.3) & (0.3,0.6) & 0 \\ (0.4,0.3) & (0.5,0.2) & 0 \end{bmatrix}$$

$$=\begin{bmatrix} (0.5,0) & (0.8,0) & 0 & (0.6,0.3) & (0.8,0.2) & 0 & (0.7,0.2) & (0.8,0.2) & (0.7,0) \\ (0.8,0) & (0.5,0) & 0 & (0.8,0.1) & (0.5,0.3) & 0 & (0.8,0.1) & (0.5,0.3) & (0.5,0) \\ (0.5,0) & (0.3,0) & 0 & (0.5,0.2) & (0.5,0.2) & 0 & (0.6,0.2) & (0.6,0.2) & (0.6,0) \\ (0.4,0) & (0.5,0) & 0 & (0.4,0.3) & (0.5,0.2) & 0 & (0.4,0.3) & (0.5,0.2) & (0.3,0) \end{bmatrix}$$

Step 3: Now, to calculate

$$[d_{i1}] = \begin{bmatrix} (\mu_{11}^{//}, \nu_{11}^{//}) \\ (\mu_{21}^{//}, \nu_{21}^{//}) \\ (\mu_{31}^{//}, \nu_{31}^{//}) \\ (\mu_{41}^{//}, \nu_{41}^{//}) \end{bmatrix} \quad for\ all\ i \in \{1,2,3,4\}.$$

To demonstrate, let us find d_{21} for $i = 2$.

Since $i = 2$ and $k \in \{1,2,3\}$ so $d_{21} = (\mu_{21}^{//}, \nu_{21}^{//})$.

Let $t_{2k} = \{t_{21}, t_{22}, t_{23}\}$, where $t_{2k} = (\mu_{2p}^{//}, \nu_{2p}^{//})$.

Here, we have to find t_{2k} for all $k \in \{1,2,3\}$.

First to find t_{21}, $I_1 = \{\, p : 0 < p \le 3\}$ for k=1 and n=3.

We have $t_{21} = (\min\{\mu_{2p}\}, \max\{\nu_{2p}\})$, here $p = 1,2,3.$

$= (\min\{\mu_{21}, \mu_{22}, \mu_{23}\}, \max\{\nu_{21}, \nu_{22}, \nu_{23}\}) = (\min\{0.8, 0.5, 0\}, \max\{0,0,0\}) = (0,0)$

Then $t_{22} = (\min\{\mu_{2p}\}, \max\{\nu_{2p}\}),\ \ p = 4,5,6.$

$$= (\min\{\mu_{24}, \mu_{25}, \mu_{26}\}, \max\{\nu_{24}, \nu_{25}, \nu_{26}\}) = (\min\{0.8, 0.5, 0\}, \max\{0.1, 0.3, 0\}) = (0, 0.3).$$

$and\ \ t_{23} = (\min\{\mu_{2p}\}, \max\{\nu_{2p}\}),\ p = 7,8,9.$

$$= (\min\{\mu_{27}, \mu_{28}, \mu_{29}\}, \max\{\nu_{27}, \nu_{28}, \nu_{29}\}) = (\min\{0.8, 0.5, 0.5\}, \max\{0.1, 0.3, 0\}) = (0.5, 0.3).$$

Hence $d_{21} = (\max\{0,0,0.5\}, \min\{0,0,0.3\}) = (0.5, 0.3).$

Similarly, we can find out d_{11}, d_{31} and d_{41} as $d_{11} = (0.7, 0.2)$, $d_{31} = (0.6, 0.2)$ and $d_{41} = (0.3, 0.3)$.

Then, we can obtain the IFS-max-min column matrix $[d_{i1}]$ as

$$[d_{i1}] = \begin{bmatrix} (0.7, 0.2) \\ (0.5, 0.3) \\ (0.6, 0.2) \\ (0.3, 0.3) \end{bmatrix}$$

Step 5: Finally, we can find an optimum fuzzy set on *U* as

$$\underset{i=1\ [d_{i1}]}{\overset{4}{Opt}} (U) = \{c_1 / (\mu_{11}^{//} - \nu_{11}^{//}), c_2 / (\mu_{21}^{//} - \nu_{21}^{//}), c_3 / (\mu_{31}^{//} - \nu_{31}^{//}), c_4 / (\mu_{41}^{//} - \nu_{41}^{//})\}$$

$$= \{c_1 / 0.5, c_2 / 0.2, c_3 / 0.4, c_4 / 0\}.$$

Thus c_1 has the maximum value. Therefore the couple may decide to buy the car c_1.

4.2 DECISION MAKING PROBLEM OF T- PRODUCT OF INTUITIONISTIC FUZZY SOFT MATRICES

Let N number of decision makers want to select an object jointly from the m number of objects which have n number of features i.e., parameters (E). Suppose that each decision maker has autonomy to take decision of inserting and evaluation of parameters associated with the selected object, i.e., each decision maker has his own choice parameters belonging to the parameter set E and his own view of evaluation. In this decision making problem, it is assumed that the parameter evaluation of the objects by the decision makers must be an intuitionistic fuzzy soft set and may be presented in linguistic form or intuitionistic fuzzy soft set format, and then it is converted in the form of intuitionistic fuzzy soft matrix. Now the problem is to find out the object out of these *m* objects.

Here we apply T-product of IFSM in decision making problem. For this, we first define maximum score set and optimum decision set.

Definition 4.2.1

$Let\ \tilde{A}_k \in IFSM_{m\times n}, k=1,2,...,l$. The T- product (say X_1) is $\prod_{k=1}^{l} \tilde{A}_k = [(\mu_{i1}, \nu_{i1})]_{m\times 1}$.

Then the set O_s= { j : c_j =max { $\mu_{i1} - \nu_{i1}$ }, i =1,2,...,m } is called the maximum score set and

$$O_d = \{u_j : u_j \in U \text{ and } j \in O_s\}$$

is called the optimum decision set of *U*.

SOLUTION PROCEDURE :

The algorithm for the solution is given below.

ALGORITHM:

Input : Intuitionistic fuzzy soft sets with m objects, each of which has n parameters.
Output: An optimum set.

Step 1: choose the set of parameters.
Step 2: construct the intuitionistic fuzzy soft matrices for each of the parameters.
Step 3: compute T-product of the intuitionistic fuzzy soft matrices.
Step 4: find the maximum score set.
Step 5: find the optimum decision set.

CASE STUDY:

Let $U = \{u_1, u_2, u_3, u_4\}$ be the set of four persons related with a political party of a particular district.
The party high command has to select only one person for nominating a candidate in the upcoming Lokshabha Election.

Step 1: The party high command consider $E = \{e_1, e_2, e_3, e_4\}$ as the set of parameters for selecting the person, where

$e_1 =$ commitment to the society,

$e_2 =$ experience in the field of politics,

$e_3 =$ relation with the peoples,

and $e_4 =$ clean image.

Step 2: Let $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ be the intuitionistic fuzzy soft matrices on the basic of the choice of the President, General Secretary and Chief Minister of the state.

$$\tilde{A}=\begin{pmatrix}(0.7,0.2) & (0.6,0.3) & (0.5,0.2) & (0.6,0.3)\\ (0.6,0.3) & (0.7,0.2) & (0.6,0.3) & (0.5,0.4)\\ (0.5,0.3) & (0.5,0.4) & (0.5,0.2) & (0.6,0.3)\\ (0.4,0.2) & (0.5,0.3) & (0.4,0.5) & (0.4,0.2)\end{pmatrix}$$

$$\tilde{B}=\begin{pmatrix}(0.5,0.2) & (0.7,0.3) & (0.6,0.2) & (0.4,0.3)\\ (0.6,0.4) & (0.5,0.2) & (0.6,0.3) & (0.5,0.4)\\ (0.5,0.3) & (0.6,0.4) & (0.5,0.4) & (0.6,0.3)\\ (0.3,0.2) & (0.5,0.2) & (0.4,0.2) & (0.6,0.2)\end{pmatrix}$$

$$\tilde{C}=\begin{pmatrix}(0.5,0.2) & (0.7,0.3) & (0.6,0.2) & (0.4,0.3)\\ (0.6,0.4) & (0.5,0.2) & (0.6,0.3) & (0.5,0.4)\\ (0.5,0.3) & (0.6,0.4) & (0.5,0.4) & (0.6,0.3)\\ (0.3,0.2) & (0.5,0.2) & (0.4,0.2) & (0.6,0.2)\end{pmatrix}$$

Step 3: Now we compute T-product X_1 between the three matrices i.e.

$$\tilde{A}\,\mathrm{X}_1\,\tilde{B}\,\mathrm{X}_1\,\tilde{C}=\tfrac{1}{4}\begin{pmatrix}(0.7+0.7+0.6+0.6,0.2+0.3+0.2+0.3)\\ (0.6+0.7+0.6+0.5,0.3+0.2+0.3+0.4)\\ (0.5+0.6+0.6+0.6,0.3+0.4+0.2+0.3)\\ (0.4+0.6+0.5+0.6,0.2+0.2+0.2+0.2)\end{pmatrix}=\tfrac{1}{4}\begin{pmatrix}(2.5,1.0)\\ (2.4,1.2)\\ (2.3,1.2)\\ (1.9,0.8)\end{pmatrix}=\begin{pmatrix}(0.62,0.25)\\ (0.60,0.30)\\ (0.57,0.30)\\ (0.47,0.20)\end{pmatrix}$$

Step 4: Then maximum score set

$$O_s=\{c_1=0.62-0.25,c_2=0.60-0.30,c_3=0.57-0.30,c_4=0.47-0.20\}=\{c_1=0.37,c_2=0.30,c_3=0.27,c_4=0.27\}.$$

Step 5: The maximum score $c_1=0.37$, scored by u_1 and the decision is in favor of nominating the candidate u_1.

V. Conclusion

The theory of intuitionistic fuzzy soft matrices is being applied in many fields of theoretical as well as practical problems. In this paper we applied intuitionistic fuzzy soft matrices in two decision making problems. Firstly, we applied product of intuitionistic fuzzy soft matrices in decision making problems based on product of intuitionistic fuzzy soft matrices defined by B.Chetia [9]. Then we defined two different types T-product of intuitionistic fuzzy soft matrices and used one of them through a new solution procedure to solve a real life decision problem which may involve more than one decision makers.

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