Influence of Non-Selective Harvesting and Prey Reserve Capacity on Prey-Predator Dynamics

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***Abstract*-In the present paper, we have developed a simple two species prey-predator model in which the prey dispersal in a two patch environment, one is assumed to be a free hunting zone and the other is a reserved zone where hunting and other extractive activities are prohibited. We assume that the prey possesses heterogeneous intrinsic growth rate with uniform carrying capacity. We also see the effect of non selective harvesting on prey-predator system in a free hunting zone. Criteria for local stability of the non-negative equilibria are obtained. Using differential inequality, we obtain sufficient conditions that ensure the persistence of the system. Finally, numerical simulations are given to verify the analytical results with the help of different sets of parameters.**

***Keywords* – Prey-Predator, Reserve Area, Harvesting, Stability, Persistence.**

I. INTRODUCTION

It is well known that many species have already become extinct and many others are at the verge of extinction due to several natural or manmade reasons like over exploitation, indiscriminate harvesting, over predation, environmental pollution, loss of habitat and mismanagement of natural resources etc. To save the species from getting extinct we are taking measures like improving conditions of their natural habitat, reduce the interaction of the species with external agents which tend to decrease their numbers, impose restrictions on species harvesting, create natural reserves, establish protected areas etc. so that the species grow in these protected areas without any external disturbances and hence the protected population can improve their numbers. While creating protected areas for a species, several factors have to be taken into consideration like its capacity to house number of individuals of the species to be protected, dynamics of the ecosystem supporting these species, the number of the other important beings which depend on these protected species, economics associated with the maintenance of the protected area, bio-economics of the ecosystem and several others.

Over the past three decades, mathematics has made a considerable impact as a tool to model and understand biological phenomena. Dubey et al., 2002 analyzed a dynamic model for a single species fishery which depends partially on a logistically growing resource in a two patch environment. They showed that both the equilibrium density of the fish population as well as the maximum sustainability yield increases as the resource biomass density increases. Further, Kar et al., 2002 modified the model proposed by Dubey et al., 2002 in the presence of predator, which seems to be more realistic. They discussed the local and global stability. Kar and Matsuda (2006) investigated a prey-predator model with Holling type of predation and harvesting of mature predator species. From the study it is evident that using the harvesting effort as control, it is possible to break the cyclic behavior of the system and drive it to a required steady state.

Keeping all above things in the mind we see combined effort of harvesting and prey reserve on prey-predator system in this paper. We have formulated and analyzed two species prey-predator model in which the prey dispersal in a two patch environment. The organization of the paper is as follows: In Section 2, we introduce our mathematical model. In Section, 3 we find conditions of boundedness for the system and Section 4; we analyze our model with regard to equilibria and their stabilities. In Section 5, we derive the sufficient conditions for persistence and numerical simulation of variety of numerical solutions of system in Section 6. At last general discussion of the paper and biological implications of our model are presented in Section 7.

II. MATHEMATICAL MODEL

To introduce the model equations we make the following biological and technical assumptions:

(i) We suppose that the area under consideration is inhabited by two interacting stocks: a stock of prey (rabbit) and a stock of predators (natural enemies of rabbit such as fox, lion, cat etc).
(ii) The area under consideration is supposed to split into two subareas a reserve area where hunting/ predation (for prey species) is prohibited and an open area for predation.
(iii) We consider the heterogeneity in the intrinsic growth rates of prey species (Schnier, 2005a)
(iv) It is assumed that prey species grows logistically and predator functional response is of Holling type II.

Then the dynamics of the system may be governed by the following system of autonomous differential equations.

$$\begin{aligned}
\frac{dx_1}{dt} &= r_1 x_1\left(1-\frac{x_1}{sk}\right)-\frac{\sigma_1 x_1}{sk}+\frac{\sigma_2 x_2}{(1-s)k}-\frac{\alpha_1 x_1 y}{\beta_1+x_1}-q_1 E x_1,\\
\frac{dx_2}{dt} &= r_2 x_2\left(1-\frac{x_2}{(1-s)k}\right)+\frac{\sigma_1 x_1}{sk}-\frac{\sigma_2 x_2}{(1-s)k}, \qquad (2.1)\\
\frac{dy}{dt} &= -\gamma y^2+\frac{\alpha_2 x_1 y}{\beta_1+x_1}-dy-q_2 E y.
\end{aligned}$$

With initial conditions

$x_1(0)\geq 0,\ x_2(0)\geq 0,\ y(0)\geq 0.$

Here, x_1 and y are biomass densities of prey and predator species respectively inside the unreserved area which is an open access predating zone at time t. x_2 is the biomass density of prey species inside the reserved area where no predation is permitted at this time t. All the parameters are assumed to be positive. k is the carrying capacity of total prey species and s is the portion of land under protection. r_1 and r_2 are the intrinsic growth rates of prey species inside the unreserved and reserved area respectively. σ_1 and σ_2 are migration rates from the unreserved area to the reserved area and the reserved area to the unreserved area, respectively. α_1 is the capturing rate of predators and α_2 is the conversion rate of predators. β_1 the half saturating constant. d is the death rate of predator. $q_i(i=1,2)$ are the constant catchability coefficients for prey in unreserved area and predator respectively and E is the combined effort applied to harvest both prey in unreserved area and predator species.

III. BOUNDEDNESS OF SOLUTIONS

Theorem 3.1: Assume that $r_1+\delta>q_1E$ and $\alpha_2M>(q_2E+d)\delta\beta_1$. Then all solutions of system (2.1) are bounded within region Ω

$\Omega=\{(x_1,x_2,y): x_1(t)\le x_{\max}, x_2(t)\le x_{\max}, y(t)\le y_{\max}\}$,

where $x_{\max}=\dfrac{sk(r_1+\delta-q_1E)^2}{4r_1\delta}+\dfrac{(r_2+\delta)^2(1-s)k}{4r_2\delta}=\dfrac{M}{\delta}$, $y_{\max}=\dfrac{\alpha_2M-\delta\beta_1(q_2E+d)}{\delta\gamma\beta_1}$.

Proof:

We consider a function

$$x(t)=x_1(t)+x_2(t),$$

$$\frac{dx(t)}{dt}=\frac{dx_1(t)}{dt}+\frac{dx_2(t)}{dt},$$

$$\frac{dx(t)}{dt}+\delta\, x(t)\le\frac{sk(r_1+\delta-q_1E)^2}{4r_1}+\frac{(r_2+\delta)^2(1-s)k}{4r_2}.$$

Using comparison principle, we obtain

$$\limsup_{t\to\infty} x(t)\le\frac{sk(r_1+\delta-q_1E)^2}{4r_1\delta}+\frac{(r_2+\delta)^2(1-s)k}{4r_2\delta},$$

$$=\frac{M}{\delta},$$

$$=x_{\max}\ (\text{say}).$$

Where

$$M=\frac{sk(r_1+\delta-q_1E)^2}{4r_1}+\frac{(r_2+\delta)^2(1-s)k}{4r_2}.$$

Now, from the third equation of system (2.1), we obtain

$$\frac{dy(t)}{dt}\le\frac{\alpha_2My(t)}{\delta\beta_1}-q_2Ey-dy(t)-\gamma\, y^2(t).$$

Using comparison principle, we obtain

$$\limsup_{t\to\infty} y(t) \le \frac{\alpha_2 M - q_2 E\delta - d\delta}{\delta\,\beta_1\,\gamma} = y_{\max}\quad (\text{say}).$$

This completes Proof of Theorem 3.1.

IV. EQUILIBRIUM POINTS AND THEIR STABILITIES

Now, we analyze system (2.1) by finding its equilibria and studying their linear stability. Steady states of system satisfy the following system of equations:

$$r_1 x_1\left(1-\frac{x_1}{sk}\right)-\frac{\sigma_1 x_1}{sk}+\frac{\sigma_2 x_2}{(1-s)k}-\frac{\alpha_1 x_1 y}{\beta_1+x_1}-q_1 E x_1=0, \tag{4.1}$$

$$r_2 x_2\left(1-\frac{x_2}{(1-s)k}\right)+\frac{\sigma_1 x_1}{sk}-\frac{\sigma_2 x_2}{(1-s)k}=0, \tag{4.2}$$

$$-\gamma\, y+\frac{\alpha_2 x_1}{\beta_1+x_1}-d-q_2 E=0. \tag{4.3}$$

It is easy to check that system (2.1) may have following equilibria for certain parameter values

(i) $E_0(0,0,0)$ (ii) $E_1(\bar{x}_1,\bar{x}_2,0)$ (iii) $E_2(\hat{x}_1,\hat{x}_2,\hat{y})$.

Existence of $E_0(0,0,0)$ is obvious. We prove the existence of $E_1(\bar{x}_1,\bar{x}_2,0)$ as follows:

Existence of $E_1(\bar{x}_1,\bar{x}_2,0)$*:*

Here $\bar{x}_1$ and $\bar{x}_2$ are obtained by solving following equations:

$$r_1\bar{x}_1\left(1-\frac{\bar{x}_1}{sk}\right)-\frac{\sigma_1\bar{x}_1}{sk}+\frac{\sigma_2\bar{x}_2}{(1-s)k}-q_1\,E\,\bar{x}_1=0, \tag{4.4}$$

$$r_2\bar{x}_2\left(1-\frac{\bar{x}_2}{(1-s)k}\right)+\frac{\sigma_1\bar{x}_1}{sk}-\frac{\sigma_2\bar{x}_2}{(1-s)k}=0. \tag{4.5}$$

From equation (4.4), we get

$$\bar{x}_2=\frac{(1-s)k}{\sigma_2}\left[\frac{\sigma_1\bar{x}_1}{sk}+q_1 E\,\bar{x}_1-r_1\bar{x}_1\left(1-\frac{\bar{x}_1}{sk}\right)\right] \tag{4.6}$$

Substituting this value of $\bar{x}_2$ in equation (4.5), we have

$$c_1 x_1^3+c_2 x_1^2+c_3 x_1+c_4=0.$$

Where

$$c_1=\frac{r_1^2 r_2(1-s)}{s^2\sigma_2^2 k},\quad c_2=\frac{r_1 J_1}{sk\sigma_2}+\frac{J_2 r_1 r_2 k(1-s)}{\sigma_2 sk},\quad c_3=J_1 J_2-\frac{r_1 r_2 k(1-s)}{\sigma_2 sk}+\frac{r_1}{sk},\quad c_4=-J_1-r_1+q_1\,E,$$

$$J_1 = \frac{r_2\sigma_1(1-s)}{\sigma_2 s} + \frac{q_1 E r_2(1-s)k}{\sigma_2} - \frac{r_1 r_2(1-s)k}{\sigma_2}, \qquad J_2 = \frac{\sigma_1}{\sigma_2 sk} + \frac{q_1 E}{\sigma_2} - \frac{r_1}{\sigma_2}.$$

Let us consider a function

$$F(\bar{x}_1) = c_1\bar{x}_1^3 + c_2\bar{x}_1^2 + c_3\bar{x}_1 + c_4.$$

$$F(0) = c_4 < 0.$$

$$F(k) = c_1 k^3 + c_2 k^2 + c_3 k + c_4 > 0.$$

We note that $F(0) < 0$ and $F(k) > 0,$ showing the existence of $\bar{x}_1$ in the interval $0 < \bar{x}_1 < k.$ Now, the sufficient condition for $\bar{x}_1$ to be unique positive real is $F'(\bar{x}_1) > 0$, where

$$F'(\bar{x}_1) = 3c_1\bar{x}_1^2 + 2c_2\bar{x}_1 + c_3.$$

Existence of $E_2(\hat{x}_1, \hat{x}_2, \hat{y})$*:*

Here $\hat{x}_1, \hat{x}_2$ and $\hat{y}$ are the positive solutions of the system of algebraic equations given below:

$$r_1\hat{x}_1\left(1-\frac{\hat{x}_1}{sk}\right) - \frac{\sigma_1\hat{x}_1}{sk} + \frac{\sigma_2\hat{x}_2}{(1-s)k} - \frac{\alpha_1\hat{x}_1\hat{y}}{\beta_1+\hat{x}_1} - q_1E\hat{x}_1 = 0, \tag{4.7}$$

$$r_2\hat{x}_2\left(1-\frac{\hat{x}_2}{(1-s)k}\right) + \frac{\sigma_1\hat{x}_1}{sk} - \frac{\sigma_2\hat{x}_2}{(1-s)k} = 0, \tag{4.8}$$

$$-\gamma\hat{y} + \frac{\alpha_2\hat{x}_1}{\beta_1+\hat{x}_1} - d - q_2E = 0. \tag{4.9}$$

From equation (4.9) we will get

$$\hat{y} = \frac{1}{\gamma}\left(\frac{\alpha_2\hat{x}_1}{\beta_1+\hat{x}_1} - d - q_2E\right). \tag{4.10}$$

Putting the value of $\hat{y}$ in equation (4.7), we will get the value of $\hat{x}_2$ as:

$$\hat{x}_2 = \frac{(1-s)k}{\gamma(\beta_1+\hat{x}_1)^2\sigma_2}\left(A_1\hat{x}_1^4 + A_2\hat{x}_1^3 + A_3\hat{x}_1^2 + A_4\hat{x}_1\right),$$

where

$$A_1 = \frac{r_1\gamma}{sk},\ \ A_2 = Eq_1\gamma - r_1\gamma + \frac{2r_1\beta_1\gamma}{sk} + \frac{\gamma\sigma_1}{sk},\ \ A_3 = \alpha_1(-d-Eq_2) + \alpha_1\alpha_2 + 2Eq_1\beta_1\gamma + \frac{r_1\beta_1^2\gamma}{sk} + \frac{2\beta_1\gamma\sigma_1}{sk},$$

$$A_4 = (-d-Eq_2)\alpha_1\beta_1 + Eq_1\gamma\beta_1^2 - r_1\gamma\beta_1^2 + \frac{\beta_1^2\gamma\sigma_1}{sk}.$$

Substituting the value of $\hat{x}_2$ in equation (4.8), we get the following equation which give the value of $\hat{x}_1$,

$b_1\hat{x}_1^3 + b_2\hat{x}_1^2 + b_3\hat{x}_1 + b_4 = 0,$

where

$b_1 = A_1 sk\{r_2 k(1-s) - \sigma_2\},\quad b_2 = A_2 sk\{r_2 k(1-s) - \sigma_2\} + \gamma\sigma_1\sigma_2,\quad b_3 = A_3 sk\{r_2 k(1-s) - \sigma_2\} + 2\beta_1\gamma\sigma_1\sigma_2,$
$b_4 = A_4 sk\{r_2 k(1-s) - \sigma_2\} + \beta_1^2\gamma\sigma_1\sigma_2.$

Let us consider a function

$G(\hat{x}_1) = b_1\hat{x}_1^3 + b_2\hat{x}_1^2 + b_3\hat{x}_1 + b_4.$

$G(0) = b_4 > 0.$

$G(k) = b_1 k^3 + b_2 k^2 + b_3 k + b_4 < 0.$

We note that $G(0) > 0$ and $G(k) < 0$, showing the existence of $\hat{x}_1$ in the interval $0 < \hat{x}_1 < k$. Now, the sufficient condition for $\hat{x}_1$ to be unique positive real is $G'(\hat{x}_1) > 0$, where

$G'(\hat{x}_1) = 3b_1\hat{x}_1^2 + 2b_2\hat{x}_1 + b_3.$

General variational matrix of the system (2.1) is given by

$$V(E) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{pmatrix},$$

where

$$a_{11} = r_1 - \frac{2r_1x_1}{sk} - \frac{\sigma_1}{sk} - q_1E - \frac{\beta_1\alpha_1 y}{(\beta_1 + x_1)^2},\quad a_{12} = \frac{\sigma_2}{(1-s)k},\quad a_{13} = -\frac{\alpha_1 x_1}{\beta_1 + x_1},\quad a_{21} = \frac{\sigma_1}{sk},$$

$$a_{22} = r_2 - \frac{2r_2x_2}{(1-s)k} - \frac{\sigma_2}{(1-s)k},\quad a_{31} = \frac{\beta_1\alpha_2 y}{(\beta_1 + x_1)^2},\quad a_{33} = -2\gamma\, y - q_2E - d + \frac{\alpha_2 x_1}{\beta_1 + x_1}.$$

The variational matrix $V(E_0)$ at equilibrium point E_0 is given by

$$V(E_0) = \begin{pmatrix} r_1 - \frac{\sigma_1}{sk} - q_1E & \frac{\sigma_2}{(1-s)k} & 0 \\ \frac{\sigma_1}{sk} & r_2 - \frac{\sigma_2}{(1-s)k} & 0 \\ 0 & 0 & -q_2E - d \end{pmatrix}.$$

The eigenvalues of matrix $V(E_0)$ are $-(d+Eq_2)$ and $\dfrac{-n \pm \sqrt{n^2-4mp}}{2m}$.Where $n=(1-s)\{Ekq_1 s - kr_1 - kr_2 + \sigma_1\} + s\sigma_2$, $m = ks(1-s)$ and $p = -(1-s)\{kr_1r_2 + Ekq_1r_2 + r_2\sigma_1\} + s\sigma_2(Eq_1 - r_1)$. This implies that E_0 is stable in $y-$direction and unstable in $x_1 - x_2$ plane if $Ekq_1 s + \sigma_1 < k(r_1 + r_2)$. In this way equilibrium point E_0 is stable in $x_1 - x_2 - y$ plane if $Ekq_1 s + \sigma_1 > k(r_1 + r_2)$.

The variational matrix $V(E_1)$ at equilibrium point E_1 is given by

$$V(E_1) = \begin{pmatrix} r_1 - \dfrac{2r_1\bar{x}_1}{sk} - \dfrac{\sigma_1}{sk} - q_1 E & \dfrac{\sigma_2}{(1-s)k} & -\dfrac{\alpha_1\bar{x}_1}{\beta_1 + x_1} \\ \dfrac{\sigma_1}{sk} & r_2 - \dfrac{2r_2\bar{x}_2}{(1-s)k} - \dfrac{\sigma_2}{(1-s)k} & 0 \\ 0 & 0 & -q_2E - d + \dfrac{\alpha_2\bar{x}_1}{\beta_1 + \bar{x}_1} \end{pmatrix},$$

The eigenvalues of matrix $V(E_1)$ are $-(d+Eq_2) + \dfrac{\alpha_2\bar{x}_1}{\beta_1 + \bar{x}_1}$ and $\dfrac{-n_1 \pm \sqrt{n_1^{\,2} - 4m_1p_1}}{2m_1}$.Where $n_1 = (1-s)\{Ek^2q_1 s + k\sigma_1 + 2kr_1\bar{x}_1 - k^2r_1s - k^2r_2s\} + 2kr_2s\bar{x}_2 + ks\sigma_2$, $m_1 = k^2 s(1-s)$ and $p_1 = (1-s)\{-Ek^2q_1sr_2 + k^2r_1r_2s - 2kr_1r_2\bar{x}_1 - kr_2\sigma_1\} + 2Ekq_1r_2s\bar{x}_2 - 2kr_1r_2s\bar{x}_2 + 4r_1r_2\bar{x}_1\bar{x}_2 + 2r_2\bar{x}_2\sigma_1 + Ekq_1s\sigma_2 - kr_1s\sigma_2 + 2r_1\bar{x}_1\sigma_2$.This implies that E_1 is stable in $x_1 - x_2$ plane if $Ek^2q_1s + k\sigma_1 + 2kr_1\bar{x}_1 > k^2r_1s + k^2r_2s$ and unstable in $x_1 - x_2$ plane if $Ek^2q_1s + k\sigma_1 + 2kr_1\bar{x}_1 < k^2r_1s + k^2r_2s$. In this way equilibrium point E_1 is saddle point in $x_1 - x_2 - y$ plane.

The variational matrix $V(E_2)$ at equilibrium point E_2 is given by

$$V(E_2) = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & 0 \\ b_{31} & 0 & b_{33} \end{pmatrix},$$

where

$$b_{11} = r_1 - \frac{2r_1\hat{x}_1}{sk} - \frac{\sigma_1}{sk} - q_1E - \frac{\beta_1\alpha_1\hat{y}}{(\beta_1 + \hat{x}_1)^2},\quad b_{12} = \frac{\sigma_2}{(1-s)k},\quad b_{13} = -\frac{\alpha_1\hat{x}_1}{\beta_1 + \hat{x}_1},\quad b_{21} = \frac{\sigma_1}{sk},$$

$$b_{22} = r_2 - \frac{2r_2\hat{x}_2}{(1-s)k} - \frac{\sigma_2}{(1-s)k},\quad b_{31} = \frac{\beta_1\alpha_2\hat{y}}{(\beta_1 + \hat{x}_1)^2},\quad b_{33} = -2\gamma\,\hat{y} - q_2E - d + \frac{\alpha_2\hat{x}_1}{\beta_1 + \hat{x}_1}.$$

The characteristic equation corresponding to the variational matrix $V(E_2)$ is given by

$$\lambda^3 + G_1\lambda^2 + G_2\lambda + G_3 = 0, \tag{4.11}$$

where

$G_1 = -b_{11} - b_{22} - b_{33}$,

$G_2 = -b_{12}b_{21} + b_{11}b_{22} - b_{13}b_{31} - b_{23}b_{32} + b_{11}b_{33} + b_{22}b_{33}$,

$G_3 = b_{13}b_{22}b_{31} - b_{12}b_{23}b_{31} - b_{13}b_{21}b_{32} + b_{11}b_{23}b_{32} + b_{12}b_{21}b_{33} - b_{11}b_{22}b_{33}$.

Then by Routh-Hurwitz criterion, equilibrium E_2 is locally asymptotically stable if $G_1 > 0, G_3 > 0$, and $G_1G_2 > G_3$ and unstable if either of these conditions are not satisfied.

V. PERSISTENCE

Biologically, persistence means the survival of all populations in future time. Mathematically, persistence of a system means that strictly positive solutions do not have omega limit points on the boundary of a non-negative cone. A population $N(t)$ is said to be uniformly persistent if their exists a $\delta > 0$, independent of $N(0) > 0$ such that $\liminf_{t\to\infty} N(t) > \delta$. We say that a system persists uniformly whenever each component persists uniformly. Stability theory of ordinary differential equations is used to analyze the model.

Lemma If $p > 0, q > 0$ and $\frac{du}{dt} \le (\ge)\, u(t)(q - pu(t)),\ u(t_0) > 0,$ then we have $\limsup_{t\to\infty} u(t) \le \frac{q}{p}$ $\left(\liminf_{t\to\infty} u(t) \ge \frac{q}{p}\right)$.

Theorem (5.1): Let

$sk\beta_1(r_1 - q_1E) > \sigma_1\beta_1 + sk\alpha_1 y_{\max}$, $r_2 > \frac{\sigma_2}{k(1-s)}$ and $\delta\alpha_2 x_{1\min} > (\delta\beta_1 + M)(q_2E + d)$ hold, then the system (2.1) is uniformly persist.

Proof: From the first equation of the system (2.1), we have

$$\frac{dx_1}{dt} \ge \left(r_1 - \frac{\sigma_1}{sk} - \frac{\alpha_1 y_{\max}}{\beta_1} - q_1E\right)x_1 - \frac{r_1x_1^2}{sk}.$$

According to lemma and comparison principle, it follows that

$$\liminf_{t\to\infty} x_1(t) \ge \frac{sk\beta_1(r_1 - q_1E) - (\sigma_1\beta_1 + sk\alpha_1 y_{\max})}{r_1\beta_1},$$

$$x_{1\min}(t) = \frac{sk\beta_1(r_1 - q_1E) - (\sigma_1\beta_1 + sk\alpha_1 y_{\max})}{r_1\beta_1}.$$

From the second equation of the system (2.1), we have

$$\frac{dx_2}{dt} \ge \left(r_2 - \frac{\sigma_2}{(1-s)k}\right)x_2 - \frac{r_2x_2^2}{(1-s)k}.$$

According to lemma and comparison principle, it follows that

$$\lim_{t\to\infty}\inf\ x_2(t) \geq \frac{r_2(1-s)k-\sigma_2}{r_2},$$

$$x_{2\min}(t) = \frac{r_2(1-s)k-\sigma_2}{r_2}.$$

From the last equation of the system (2.1), we have

$$\frac{dy}{dt} \geq \left(\frac{\delta\alpha_2 x_{1\min} - (\delta\beta_1 + M)(q_2 E + d)}{\delta\beta_1 + M}\right) y - \gamma\, y^2.$$

According to lemma and comparison principle, it follows that

$$\lim_{t\to\infty}\inf\ y(t) \geq \frac{\delta\alpha_2 x_{1\min} - (\delta\beta_1 + M)(q_2 E + d)}{(\delta\beta_1 + M)\gamma},$$

$$y_{\min}(t) = \frac{\delta\alpha_2 x_{1\min} - (\delta\beta_1 + M)(q_2 E + d)}{(\delta\beta_1 + M)\gamma}.$$

This completes the proof of the theorem.

VI. NUMERICAL SIMULATION

In this section, we present numerical simulation to illustrate results obtained in previous sections. The system (2.1) is solved using fourth order Runge – Kutta Method with the help of MATLAB software package under the following set of parameters

$$r_1 = 6,\ r_2 = 4,\ \ k = 100,\ \beta = 0.01,\ \ \alpha_1 = 8,\ \alpha_2 = 6,\ \gamma = 0.01,\ s = 0.6,$$
$$\sigma_1 = 3,\ \sigma_2 = 2,\ \mathrm{d} = 0.01,\ \mathrm{q}_1 = 0.3,\ \mathrm{q}_2 = 0.3,\ \mathrm{E} = 1. \qquad (6.1)$$

We find that all the equilibrium points for the system (2.1) exist and given by,

$E_0(0,0,0)$, $E_1(56.8536, 40.207, 0)$, $E_2(0.000623492, 39.5, 4.21393)$.

The characteristic polynomial of the equilibrium point E_0 is

$$\lambda^3 - 9.29\,\lambda^2 + 19.339\,\lambda + 6.91765 = 0. \qquad (6.2)$$

The characteristic roots of equation (6.2) are

$-0.31, 5.65147, 3.94853$, this implies that equilibrium point E_0 is unstable equilibrium point of the system.

The characteristic polynomial of the equilibrium point E_1 is

$$\lambda^3 + 4.12318\,\lambda^2 - 32.4174\,\lambda - 133.14 = 0. \qquad (6.3)$$

The characteristic roots of equation (6.3) are

$-5.72225, 5.68894, -4.08987$, two roots are negative and one is positive this implies that equilibrium point E_1 is saddle point of the system.

At same time the characteristic polynomial of the equilibrium point E_2 is

$$\lambda^3 + 2985.39\,\lambda^2 + 12954.2\,\lambda + 4651.09 = 0. \tag{6.4}$$

The characteristic roots of equation (6.4) are

$-2981.05, -3.95, -0.394993$, this implies that equilibrium point E_2 is locally asymptotically stable equilibrium point.

The results of numerical simulation are displayed graphically. In Figures (1-3) prey (x_1), prey (x_2) and predator (y) population are plotted against time, from these figures it is noted that for a given initial value the population tend to their corresponding value of equilibrium point E_2 and hence coexist in the form of stable steady state, assuring the local stability of E_2. Simulation is performed for different initial starts I, II, III, IV in Figure (4) to graphically illustrate the global stability of the interior equilibrium point E_2, in the $x_1 - x_2$ plane, where
initial starts are
Initial start I: [0.02 1 0.2];
Initial start II: [0.02 2 0.2];
Initial start III: [0.02 3 0.2];
Initial start IV: [0.02 4 0.2].
It is depicted from the graph that the solutions of the system converge to equilibrium point E_2 for different value of initial starts, indicating that the system is globally asymptotically around this point. Now to depict the global stability of the interior equilibrium point E_2, in the $x_1 - y$ plane, we have performed simulations for different initial starts I,II, III, IV in Figure (5), where initial starts are
Initial start I: [0.02 1 0.2];
Initial start II: [0.02 1 0.4];
Initial start III: [0.02 1 0.6];
Initial start IV: [0.02 1 0.8].

Figure (6) is the plot of prey (x_1) against time for different values of E. Figures (7) is the plot of predator (y) against time for different values of E. From these figures, we can see the effect of prey reserve and harvesting on the system, it can be inferred that prey (x_1) population is increases as well as predator (y) population decreases as E increases and predator (y) population becomes extinct if $E \geq 6$ .Our model represents the nature of forest approximately. With the help of results of this model, we can save the wild life. We should make reserve area for preys in man-made parks.

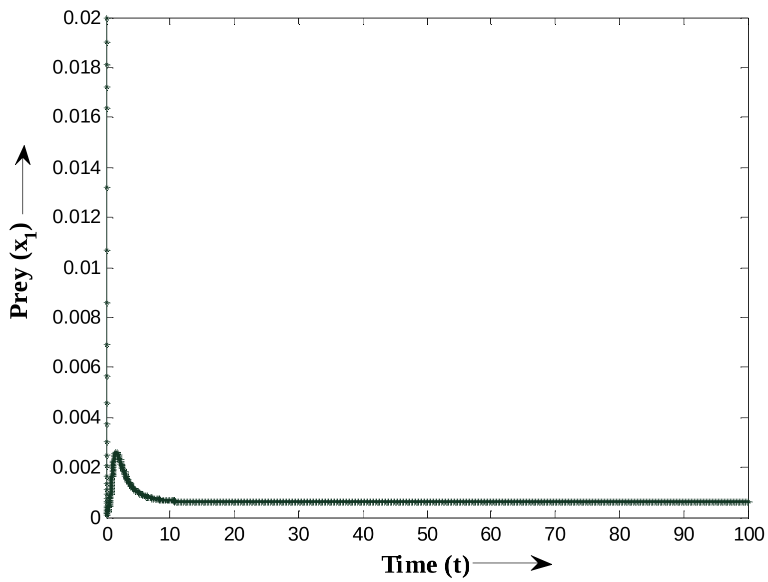

Figure (1), Stable behavior of x_1 with time and other parameter values are same as (6.1).

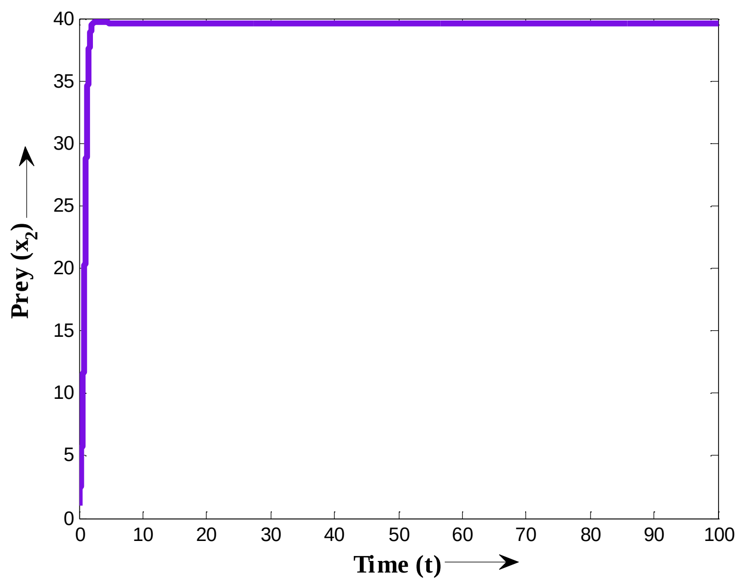

Figure (2), Stable behavior of x_2 with time and other parameter values are same as (6.1).

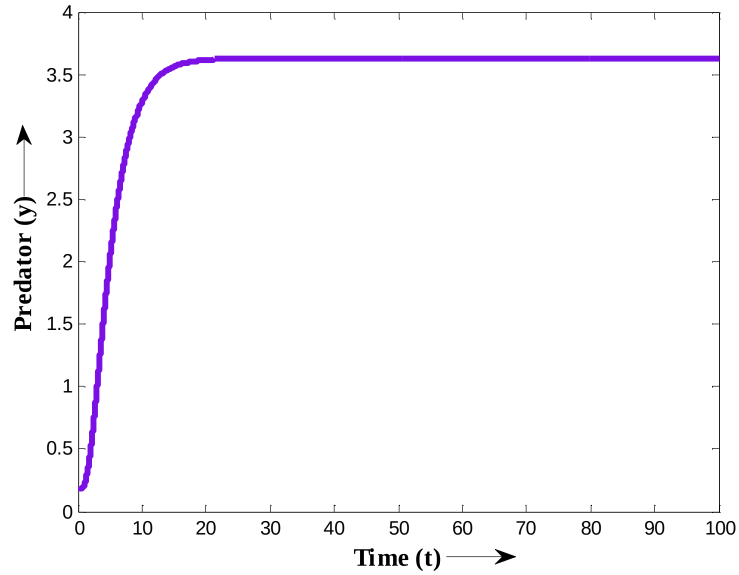

Figure (3), Stable behavior of y with time and other parameter values are same as (6.1).

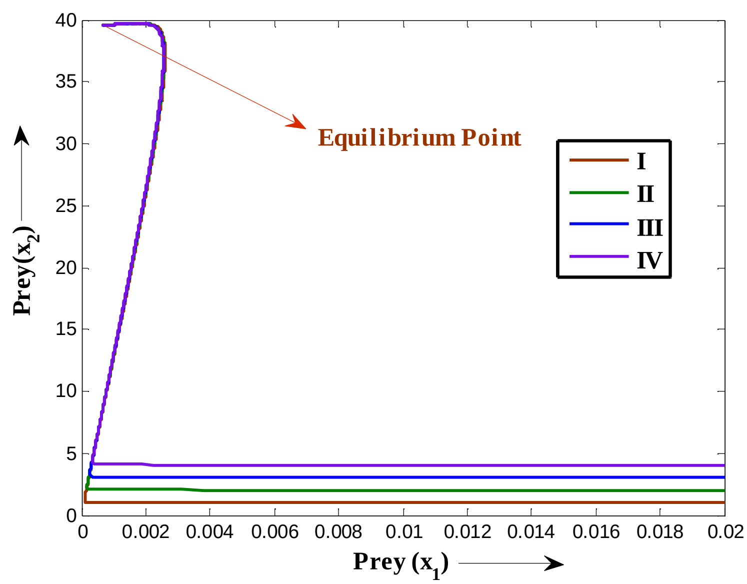

Figure (4), Graph of x_1 verses x_2 for different initial starts and other parameters are same as (6.1).

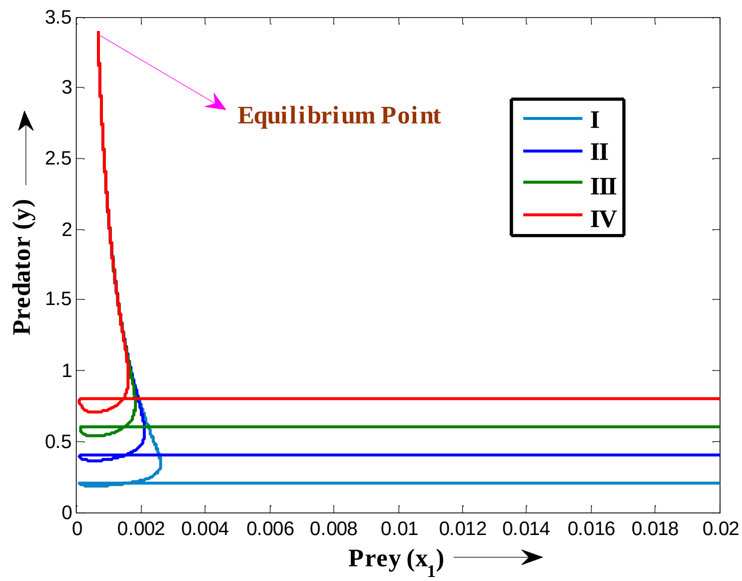

Figure (5), Graph of x_1 verses y for different initial starts and other parameters are same as (6.1).

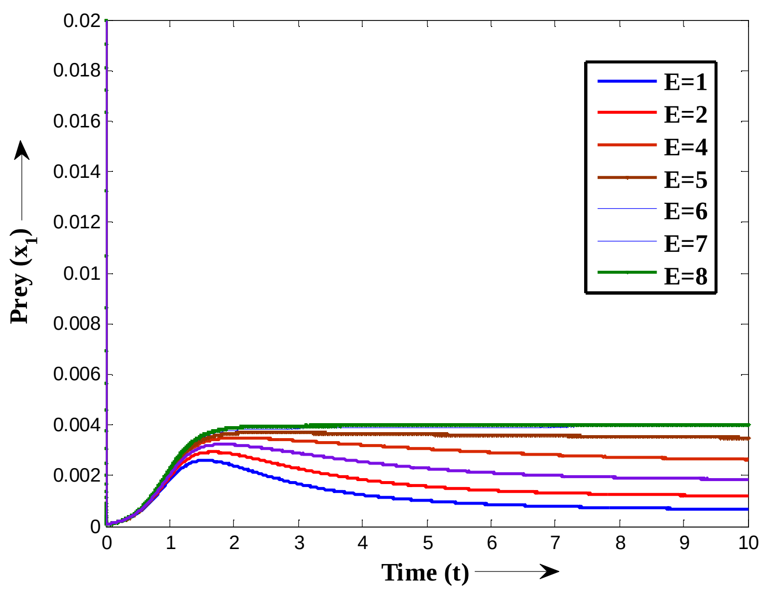

Figure (6), Graph of x_1 versus t for different value of E and other values of parameters are same as (6.1).

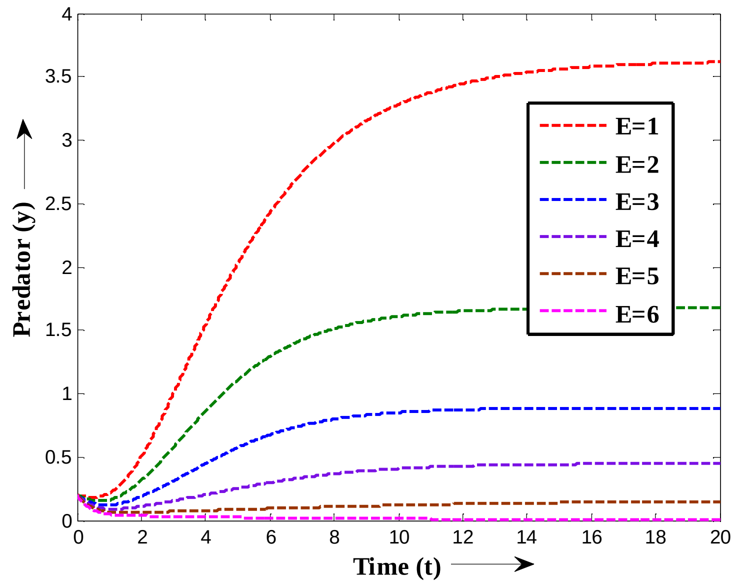

Figure (7), Graph of y versus t for different value of E and other values of parameters are same as (6.1).

VII. CONCLUSION

A nonlinear mathematical model is proposed and analyzed to see the effect of harvesting and prey reserve on prey-predator dynamics. Using stability theory of differential equations, we have obtained conditions for the existence of different equilibria and discussed their stabilities in local manner. Using, differential inequality, conditions have been obtained under which system persists. From our analysis, we have found preys reserve and harvesting are most important factors of ecology. Using these factors of ecology we can save the species from getting extinct. In this manuscript we have taken example from wild species for simplicity but our model is applicable for all species.

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