RW-CONTINUOUS MAPS AND RW-IRRESOLUTE MAPS IN TOPOLOGICAL SPACES

***M.Karpagadevi* [1], *A.Pushpalatha* [2]**
[1] *Assistant Professor, Karpagam College of Engineering, Coimbatore, India*
[2] *Assistant Professor, Government Arts College, Udumalpet, India*

Abstract

In this paper we introduce and study the concept of regular weakly continuity (briefly rw-continuity) and regular weakly irresolute (briefly rw-irresolute) in topological spaces and discuss some of their properties in topological spaces.

Mathematics Subject Classification: 54C05

KEYWORDS: rw-closed set, rw-continuous, rw-irresolute

1. Introduction

Topologist studied weaker and stronger forms of continuous functions in topology using the sets stronger and weaker than open and closed sets. Balachandran et.al [4], Levine [14], Mashhour et.al [16], Gnanmbal et.al [11] have introduced g- continuity, Semi - continuity, pre- continuity, gpr - continuity respectively.

In 1972, Crossley and Hiledebrand [6] introduced the notion of irresoluteness.In 1981, Munshi and Bassan [17] introduced the notion of generalized continuous (briefly g - continuous) functions which are called in [4] as g – irresolute functions. Furthermore, the notion of gs-irresolute [7] (resp.gp-irresolute [2] , αg-irresolute[8] , gb – irresolute[3],gsp-irresolute[21]) functions is introduced.
S.S. Benchalli and R.S Wali [5] introduced new class of sets called regular weakly - closed (briefly rw - closed) sets in topological spaces which lies between the class of all w - closed sets and the class of all regular g - closed sets.
The aim of this paper is to introduce and study the concepts of new class of maps namely rw-continuous maps and rw-irresolute maps.

Throughout this paper (X, τ) and (Y,σ) (or simply X and Y) represents the non-empty topological spaces on which no separation axiom are assumed, unless otherwise mentioned. For a subset A of X, cl(A) and int(A) represents the closure of A and interior of A respectively.

2. Preliminaries

In this section we recollect the following basic definitions which are used in this paper.

Definition 2.1 [5]: A subset A of a topological space (X, τ) is called rw-closed (briefly rw-closed) if cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is regular semiopen in X.

Definition 2.2 [18]: A subset A of a topological space (X, τ) is called regular generalized closed (briefly rg-closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X.

Definition 2.3 [19]: A subset A of a topological space (X, τ) is called weakly closed (briefly w-closed) if cl(A)) $\subseteq$ U whenever A $\subseteq$ U and U is semi open in X.

Definition 2.4 [18] : A map f: (X, τ) $\rightarrow$ (Y,σ) from a topological space X into a topological space Y is called rg continuous if the inverse image of every closed set inY is rg-closed in X.

Definition 2.5 [19] : A map f: (X, τ) $\rightarrow$ (Y,σ) from a topological space X into a topological space Y is called w-continuous if the inverse image of every closed set in Y is w-closed in X.

Definition 2.6 [6] : A map f: (X, τ) $\rightarrow$ (Y,σ) from a topological space X into a topological space Y is called irresolute if the inverse image of every semi-closed set in Y is semi-closed in X.

3. RW - continuous mappings

In this chapter we introduce and study rw-continuous mappings in topological spaces.

Definition 3.1: Let f: X $\rightarrow$ Y from a topological space X into a topological space Y is called rw-continuous if the inverse image of every closed set inY is rw closed in X.

Theorem 3.2: If a map f: X $\rightarrow$ Y from a topological space X into a topological space Y is continuous, then it is rw continuous but not conversely.

Proof: Let f: X $\rightarrow$ Y be continuous and F be any closed set in Y. Then the inverse image $f^{-1}(F)$ is closed in X. Since every closed set is rw-closed, $f^{-1}(F)$ is rw-closed in X. Therefore f is rw-continuous.

Remark 3.3: The converse of the above theorem need not be true as seen from the following example

Example 3.4: Let X = Y = {a,b,c} with toplogies τ = {X,ϕ, {a},{a,b},{b}},σ= {Y,ϕ,{c}}. Let f: X $\rightarrow$ Y be a map defined by f(a) = a, f(b) = b, f(c) = c. Here f is rw continuous but not continuous since for the closed set F = {a,b} in Y $f^{-1}(F)$ = {a,b} is not closed in X.

Theorem 3.5: If a map f: X $\rightarrow$ Y from a topological space X into a topological space Y is rw-continuous, then it is rg continuous but not conversely.

Proof: Let f: X $\rightarrow$ Y be rw - continuous and F be any closed set in Y. Then the inverse image $f^{-1}(F)$ is rw - closed in X. Since every rw-closed set is rg-closed, $f^{-1}(F)$ is rg-closed in X. Therefore f is rg - continuous.

Remark 3.6: The converse of the above theorem need not be true as seen from the following example

Example 3.7: Let X = Y = {a,b,c,d} with toplogies τ = {X,ϕ, {a},{a,b},{b},{a,b,c}}, σ= {Y,ϕ,{b},{a,b,d}}. Let f: X $\rightarrow$ Y be a map defined by f (a) = a, f (b) = b, f(c) = c. Here f is rg continuous but not rw-continuous since for the closed set F = {c} in Y $f^{-1}(F)$ = {c} is not rw-closed in X.

Theorem 3.8: If a map f: X $\rightarrow$ Y from a topological space X into a topological space Y is w - continuous, then it is rw continuous but not conversely.

Proof: Let f: X $\rightarrow$ Y be w - continuous and F be any closed set in Y. Then the inverse image $f^{-1}(F)$ is w - closed in X. Since every w-closed set is rw-closed, $f^{-1}(F)$ is rw-closed in X.Therefore f is rw - continuous.

Remark 3.9: The converse of the above theorem need not be true as seen from the following example

Example 3.10: Let X = Y = {a,b,c} with toplogies τ = {X,ϕ, {a},{a,b},{b}},σ= {Y,ϕ,{c}}. Let f: X $\rightarrow$ Y be a map defined by f(a) = a,f(b) = b, f(c) = c. Here f is rw continuous but not w-continuous since for the closed set F = {a,b} in Y $f^{-1}(F)$ = {a,b} is not w-closed in X.

Theorem 3.11: A function f: (X, τ) $\rightarrow$ (Y,σ) is rw-continuous if and only if $f^{-1}(U)$ is rw-open in (X, τ) for every open set U in (Y,σ).

Proof: Let f: (X, τ) $\to$ (Y,σ) be rw-continuous and U an open set in (Y,σ). Then $f^{-1}(U^c)$ is rw-closed in (X, τ). But $f^{-1}(U^c) = (f^{-1}(U))^c$ and so $f^{-1}(U)$ is rw-open in (X, τ).

Theorem 3.12: If f: X $\to$ Y and g: Y $\to$ Z be any two functions, then g ^{o} f : X $\to$ Z is rw-continuous if g is continuous and f is rw-continuous.

Proof: Let F be any closed set in Z. Since g is continuous, $g^{-1}(F)$ is closed in Y and since f is rw-continuous, $f^{-1}(g^{-1}(F))$ is rw-closed in X. Hence $(g\,^o\, f)^{-1}$ is rw-closed in X. Thus g ^{o} f is rw-continuous.

Remark 3.13: The Composition of two rw-continuous maps need not be rw-continuous. Let us prove the remark by the following example.

Example 3.14: Let X = Y = Z ={a,b,c} with toplogies τ = {X,ϕ, {a},{a,b},{b}},σ= {Y,ϕ,{b},{a,b}}, η = {Z,ϕ, {a},{a,c},{c}}. Let g: (X, τ) $\to$ (Y,σ) be a map defined by g(a) = a, g(b) = b, g(c) = c. Let f: (Z, η) $\to$ (X, τ) be a map defined by f(a) = b, f(b) = a, f(c) = c. Both f and g are rw-continuous. Define g ^{o} f: (Z, η) $\to$ (Y,σ). Here {c} is closed set of (Y,σ).Therefore $(g\,^o\, f)^{-1}(c)$ = {c} is not a rw-closed set of (Z, η). Hence g ^{o} f is not rw-continuous.

Theorem 3.15: Let f: X $\to$ Y be a rw-continuous map from a topological space X into a topological space Y and let H be a closed subset of X. Then the restriction f / H: H $\to$ Y is rw – continuous where H is endowed with the relative topology.

Proof: Let F be any closed subset in Y. Since f is rw-continuous, $f^{-1}(F)$ is rw-closed in X. If $f^{-1}(F) \cap H = H_1$ then H1 is a rw-closed set in X, since the intersection of two rw-closed set is rw-closed set. Since $(f / H)^{-1}(F) = H_1$, it is sufficient to show that H1 is rw-closed set in H. Let G_1 be any open set of H such that G1 contains H_1. Let $G_1 = G \cap H$ where G is open in X. Now $H_1 \subset G \cap H \subset G$ since H_1 is rw-closed in X. $\bar{H}_1 \subset G$. Now $cl_H(H_1) = \bar{H}_1 \cap H \subset G \cap H = G_1$ where $cl_H(A)$ is the closure of a subset A of the subspace H of X. Therefore f / H is rw-continuous.

Theorem 3.16: Let f: X $\to$ Y be a map from a topological space X into a topological space Y

i) The following statements are equivalent

a) f is rw-continuous

b) The inverse image of each open set in Y is rw-open in X.

ii) If f: X $\to$ Y is rw-continuous then f(rw cl(A)) $\subset$ cl(f(A))) for every subset A of X.

iii) The following statements are equivalent

a) For each point x in X and each open set V in Y with f(x)$\in$ V, there is a rw-open set U in X such that x$\in$ U, f(U) $\subset$ V.

b) For every subset A of X, f (rw cl(A)) $\subset$ cl(f(A)) holds.

c) For each subset B of Y, rw cl(f^{-1}(B)) $\subset$ f^{-1}(cl(B)).

Proof: i) Assume that f: X $\to$ Y be rw-continuous. Let G be open in Y. Then G^c is closed in Y. Since f is rw-continuous, $f^{-1}(G^c)$ is rw-closed in X. But $f^{-1}(G^c)$ = X - f^{-1}(G). Thus X - f^{-1}(G) is rw-closed in X and so f^{-1}(G) is rw-open in X. Therefore (a) implies (b).

Conversely assume that the inverse image of each open set in Y is rw-open in X. Let F be any closed set in Y. Then F^c is open in Y. By assumption, $f^{-1}(F^c)$ is rw-open in X. But

$f^{-1}(F^c) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is rw-open in X and so $f^{-1}(F)$ is rw-closed in X. Therefore f is rw-continuous. Hence (b) implies (a). Thus (a) and (b) are equivalent.

ii) Since A $\subset$ $f^{-1}f(A)$, we have A $\subset$ $f^{-1}(cl(f(A)))$. Now cl (f (A)) is a closed set in Y and hence $f^{-1}(cl(f(A)))$ is a rw-closed set containing A. Consequently rw cl(A) $\subset$ $f^{-1}(cl(f(A)))$. Therefore f (rw cl(A)) $\subset$ f $f^{-1}(cl(f(A))) \subset$ cl(f(A))

iii) (a) $\Leftrightarrow$ (b)

Suppose that a) holds and let y $\in$ f (rw cl(A) and let V be any open neighbourhood of y. Then there exists a point x $\in$ X and rw-open set U such that f(x) = y, x $\in$ U, x $\in$ rw cl(A) and f(U) $\subset$ V. Since x $\in$ rw cl(A), U $\cap$ A $\neq \varphi$ holds and hence f(A) $\cap$ V $\neq \varphi$. Therefore we have y = f(x) $\in$ cl(f(A)).

Conversely if b) holds and let x $\in$ X and let V be any open set containing f(x).

Let A = $f^{-1}(V^c)$. Since f (rw cl(A)) $\subset$ cl(f(A)) $\subset$ V^c, it is shown that rw cl(A) = A. Then since x $\notin$ A, there exists an rw-open set U containing x such that U $\cap$ A = φ and hence f(U) $\subset$ f (A^c) $\subset$ V. (b) $\Leftrightarrow$ (c)

Suppose that (b) holds and let B be any subset of Y. Replacing A by $f^{-1}(B)$ we get from (c) f (rw cl($f^{-1}(B)$)) $\subset$ cl(f $f^{-1}(B)$) $\subset$ (B). Hence rw cl($f^{-1}(B)$) $\subset$ $f^{-1}(cl(B))$.

Conversely suppose that(c) holds, let B = f(A) where A is a subset of X. Then rw cl(A) $\subset$ rw cl($f^{-1}(B)$) $\subset$ $f^{-1}(cl(f(A)))$. Therefore f (rw cl(A)) $\subset$ cl(f(A)). This completes the proof.

4. RW - irresolute mappings

Definition 4.1: Let f: X $\rightarrow$ Y from a topological space X into a topological space Y is called rw-irresolute if the inverse image of every rw-closed set in Y is rw-closed in X.

Theorem 4.2: A map f: X $\rightarrow$ Y is rw-irresolute if and only if the inverse image of every rw-open set in Y is rw-open in X.

Proof: Assume that f is rw-irresolute. Let A be any rw-open set in Y. Then A^c is rw-closed set in Y. Since f is rw-irresolute, $f^{-1}(A^c)$ is rw-closed in X. But $f^{-1}(A^c) = X - f^{-1}(A)$ and so $f^{-1}(A)$ is rw-open in X. Hence the inverse image of every rw-open set in Y is rw-open in X.

Conversely assume that the inverse image of every rw-open set in Y is rw-open in X. Let A be any rw-closed set in Y. Then A^c is rw-open in Y. By assumption, $f^{-1}(A^c)$ is rw-open in X.But $f^{-1}(A^c) = X - f^{-1}(A)$ and so $f^{-1}(A)$ is rw-closed in X. Therefore f is rw-irresolute.

Theorem 4.3: If a map f: X $\rightarrow$ Y is rw-irresolute, then it is rw-continuous but not conversely.

Proof: Assume that f is rw-irresolute. Let F be any closed set in Y. Since every closed set is rw-closed, F is rw-closed in Y.Since f is rw-irresolute, $f^{-1}(F)$ is rw-closed in X. Therefore f is rw-continuous.

Remark 4.4: The converse of the above theorem need not be true as seen from the following example.

Example 4.5: Let X = Y = {a,b,c} with toplogies τ = {X,ϕ, {a},{a,c},{c}} σ= {Y,ϕ,{b}, {b,c},{c}}. Let f: X $\rightarrow$ Y be a map defined by f (a) = a, f(b) = b, f(c) = c. Here f is rw continuous. However {a} is rw-closed in Y but $f^{-1}(a)$ = {a} is not rw-closed in X. Therefore f is not rw-irresolute.

Theorem 4.6: Let X, Y and Z be any topological spaces.For any rw-irresolute map f: X $\rightarrow$ Y and any rw-continuous map g: Y $\rightarrow$ Z, the composition $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is rw-continuous.

Proof: Let F be any closed set in Z. Since g is rw-continuous, $g^{-1}(F)$ is rw-closed in Y. Since f is rw-irresolute, $f^{-1}(g^{-1}(F))$ is rw-closed in X. But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}$. Therefore $g \circ f$: X $\rightarrow$ Z is rw-continuous.

Remark 4.7: The irresolute maps and rw-irresolute maps are independent of each other. Let us prove the remark by the following examples.

Example 4.8: Let X = Y = {a,b,c} with toplogies $\tau = \{X,\phi, \{a\},\{a,c\},\{c\}\}$, $\sigma = \{Y,\phi,\{b\}, \{b,c\},\{c\}\}$. Let f: X $\rightarrow$ Y be a map defined by f(a) = a, f(b) = b, f(c) = c. Then f is irresolute but it is not rw-irresolute since F = {a} is rw -closed in (Y,σ) but $f^{-1}(F)$ = {a} is not rw -closed in (X, τ).

Example 4.9: Let X = Y = {a,b,c} with toplogies $\tau = \{X,\phi, \{a\},\{a,c\},\{c\}\}$, $\sigma = \{Y,\phi,\{b\}, \{b,c\},\{c\}\}$. Let f: X $\rightarrow$ Y be a map defined by f(a) = a, f(b) = b, f(c) = c. Then f is rw- irresolute but it is not irresolute since F = {a,c} is semi -closed in (Y,σ) where $f^{-1}(F)$ = {a,c} is not semi -closed in (X, τ).

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