Suitable mode of Public Transport – A Decision making model using Possibility Intuitionistic Fuzzy Soft Sets

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Abstract **– In this paper, the concept of Possibility Intuitionistic Fuzzy Soft Sets is used to develop a decision making model for choosing a suitable mode of transport.**

Keywords **– Soft Set, Fuzzy Set, Intuitionistic Fuzzy Soft Set, Possibility Intuitionistic Fuzzy Soft Set.**

I. Introduction

Application of mathematical concepts to real life problems facilitates the decision making authorities to take right decisions that resolve many social issues and help the society to lead a peaceful life.

At present, the urban population faces many hardships in discharging their routines right from the time they wake up till they go to bed. Of such problems, commuting through public transport, between residence and place of work and back is a problem of everyone, especially the women folk. The over-crowded buses, unreasonable tariff charged in autos and taxis, unsafe travel in share autos and like, are the problems that necessitate careful consideration in choosing the suitable mode of transport. So the authors intend to develop a decision making model using Possibility Intuitionistic Fuzzy Soft Sets that facilitates selection of right mode of public transport based on the criteria preferred by individuals.

Basic Definitions

Let U be a non-empty set. A **Fuzzy Set** in U is a function with domain U and values in the closed unit interval I = [0, 1].

An **Intuitionistic Fuzzy Set (IFS)** A in a nonempty set U (a universe of discourse) is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in U\}$, where the functions $\mu_A(x) : U \to [0,1]$, $\nu_A(x) : U \to [0,1]$, denotes the degree of membership and degree of non-membership of each element $x \in U$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all $x \in U$.

Let U be an initial universe set and E be the set of parameters. Let P(U) denotes the power set of U. A pair (F,E) is called a **Soft Set** over U where F is a mapping given by $F:E \to P(U)$.

Let U be an initial universe set and E be the set of parameters. Let IF^U denotes the collection of all intuitionistic fuzzy subsets of U. Let $A \subseteq E$. A pair (F, A) is called **Intuitionistic Fuzzy Soft Set** over U, where F is a mapping given by $F : A \to IF^U$.

Let $U = \{x_1, x_2, x_3, \ldots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, e_3, \ldots, e_m\}$ be the universal set of parameters. The pair (U,E) will be called a soft universe. Let $F : E \to (I \times I)^U \times I^U$ where $(I \times I)^U$ is the collection of all intuitionistic fuzzy subsets of U and I^U is the collection of all fuzzy subsets of U. Let p be a fuzzy subset of E, i.e., $p : E \to I^U$. Let $F_p : E \to (I \times I)^U \times I^U$ be a function defined as follows:

$F_p(e) = (F(e)(x), p(e)(x))$, where $F(e)(x) = (\mu(x), \nu(x))$, $p(e)(x) = \gamma(x), \forall x \in U$.

Then F_p is called a **Possibility Intuitionistic Fuzzy Soft Set (PIFSS in short)** over the soft universe (U, E). For each parameter e_i, $F_p(e_i) = (F(e_i)(x), p(e_i)(x))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $p(e_i)$. So one can write $F_p(e_i)$ as follows:

$$F_p(e_i) = \left\{ \left(\frac{x_1}{F(e_i)(x_1)}, p(e_i)(x_1) \right), \left(\frac{x_2}{F(e_i)(x_2)}, p(e_i)(x_2) \right), \ldots\ldots, \left(\frac{x_n}{F(e_i)(x_n)}, p(e_i)(x_n) \right) \right\}$$

Application of Possibility Intuitionistic Fuzzy Soft Sets

Based on the pilot study, the authors identified four modes of public transport which forms the universal set U = $\{x_1, x_2, x_3, x_4\}$ and five specific criteria which forms the parameter set E = $\{e_1, e_2, e_3\ e_4, e_5\}$.

Modes of transport (Universal Set U)

x_1 – Taxi

x_2 – Auto

x_3 – Mini Bus / Share Auto

x_4 – Government Bus

Criteria (Parameter Set E)

e_1 - Economical

It denotes the cost involved for the journey. The tariff charged varies from one mode of transport to the other.

e_2 - Flexibility

Flexibility in terms of timings, accessibility, departure and destination.

e_3 - Comfort

It refers to comfort during travel.

e_4 - Time saving

It refers to the time taken for the travel between the place of departure and destination.

e_5 - Safety

It refers to safety of self and belongings during the travel.

Construction of Possibility Intuitionistic Fuzzy Soft Sets

For the purpose of ascertaining membership function, non – membership function and degree of possibility of belongingness, the opinion of the women respondents about the five criteria for each mode of transport was collected by administering a questionnaire. A sample of 100 women respondents, working women and college students in Coimbatore city, was selected by adopting convenient sampling technique. Five point scaling technique (Strongly Agree, Agree, Neither Agree nor Disagree, Disagree, Strongly Disagree) was adopted to assess the degree of agreement and disagreement towards the five criteria for each mode of transport.

The number of respondents who marked either 'Strongly Agree' or 'Agree' was counted and its proportion to total respondents was used for framing membership function. The number of respondents who marked either 'Disagree' or 'Strongly Disagree' was used for framing non – membership functions as above. The counts of membership function of each criterion for all the four modes were added and the proportion of count of each mode to the total was ascertained to fix degree of possibility of belongingness.

Therefore, the Possibility Intuitionistic Fuzzy Soft Sets for this model are,

$$F_p(e_1)= \left\{\left(\frac{x_1}{0.20,0.73},0.13\right),\left(\frac{x_2}{0.26,0.74},0.17\right),\left(\frac{x_3}{0.50,0.43},0.32\right),\left(\frac{x_4}{0.60,0.37},0.38\right)\right\}$$

$$F_p(e_2)= \left\{\left(\frac{x_1}{0.57,0.40},0.3\right),\left(\frac{x_2}{0.61,0.27},0.32\right),\left(\frac{x_3}{0.30,0.64},0.16\right),\left(\frac{x_4}{0.42,0.53},0.22\right)\right\}$$

$$F_p(e_3)= \left\{\left(\frac{x_1}{0.73,0.24},0.41\right),\left(\frac{x_2}{0.53,0.40},0.30\right),\left(\frac{x_3}{0.39,0.60},0.22\right),\left(\frac{x_4}{0.12,0.78},0.07\right)\right\}$$

$$F_p(e_4)= \left\{\left(\frac{x_1}{0.59,0.39},0.40\right),\left(\frac{x_2}{0.52,0.46},0.35\right),\left(\frac{x_3}{0.23,0.71},0.16\right),\left(\frac{x_4}{0.14,0.70},0.09\right)\right\}$$

$$F_p(e_5)= \left\{\left(\frac{x_1}{0.52,0.46},0.24\right),\left(\frac{x_2}{0.51,0.41},0.23\right),\left(\frac{x_3}{0.44,0.50},0.20\right),\left(\frac{x_4}{0.72,0.21},0.33\right)\right\}$$

The authors developed a decision making model using these Possibility Intuitionistic Fuzzy Soft Sets by considering a set of parameters preferred by an individual to identify the mode of transport that suits best the requirement of the said individual.

Suppose the wishing parameters of Madam X is $A \subseteq E$ where A = $\{e_1, e_2, e_4\}$. Consider the following operations:

(i) For membership function: $\alpha(e_i) = \mu_i + \gamma_i - \mu_i\gamma_i$,
(ii) For non-membership function: $\beta(e_i) = \nu_i\,\gamma_i$, for i = 1,2,3,4

These two operations are taken to ascend the membership value and descend the non–membership value of $F(e_i)$ on the basis of the degree of preference of Madam X. Then the PIFSS $F_p(e_i)$ reduced to an intuitionistic fuzzy soft set $\psi(e_i)$ given as follows:

$$\psi(e_1)=\left\{\left(\frac{x_1}{0.30,0.09}\right),\left(\frac{x_2}{0.39,0.13}\right),\left(\frac{x_3}{0.66,0.14}\right),\left(\frac{x_4}{0.75,0.14}\right)\right\}$$

$$\psi(e_2)=\left\{\left(\frac{x_1}{0.70,0.12}\right),\left(\frac{x_2}{0.73,0.09}\right),\left(\frac{x_3}{0.41,0.10}\right),\left(\frac{x_4}{0.55,0.12}\right)\right\}$$

$$\psi(e_4)=\left\{\left(\frac{x_1}{0.75,0.16}\right),\left(\frac{x_2}{0.69,0.16}\right),\left(\frac{x_3}{0.35,0.11}\right),\left(\frac{x_4}{0.22,0.06}\right)\right\}$$

A comparison table is a square table in which number of rows and number of columns are equal and both are labeled by object name of the universe such as x_1, x_2, ….., x_n and the entries are c_{ij} where c_{ij} = the number of parameters for which the value of x_i exceeds or equal to the value of x_j.

Algorithm

(i) Input the set $A \subseteq E$ of choice of parameters of Madam X.
(ii) Consider the reduced intuitionistic fuzzy soft set.
(iii) Consider the tabular representation of membership function and non-membership function (see table 1 and table 4 respectively).
(iv) Compute the comparison table membership function and non-membership function (see table 2 and table 5 respectively).
(v) Compute the membership score and non-membership score (see table 3 and table 6 respectively).
(vi) Compute the final score by subtracting non-membership score from membership score(see table 7)
(vii) Find the maximum score, if it occurs in i^{th} row then Madam X will choose mode of transport x_i.

Table 1 : Tabular representation of membership function

U	e_1	e_2	e_4
x_1	0.30	0.70	0.75
x_2	0.39	0.73	0.69
x_3	0.66	0.41	0.35
x_4	0.75	0.55	0.22

Table 2 : Comparison table of Table 1

U	x_1	x_2	x_3	x_4
x_1	3	1	2	2
x_2	2	3	2	2
x_3	1	1	3	1
x_4	1	1	2	3

Table 3 : Membership score table

U	Row sum (a)	Column sum (b)	Membership score (a-b)
x_1	8	7	1
x_2	9	6	3
x_3	6	9	-3
x_4	7	8	-1

Table 4 : Tabular representation of non-membership function

U	e_1	e_2	e_4
x_1	0.09	0.12	0.16
x_2	0.13	0.09	0.16
x_3	0.14	0.10	0.11
x_4	0.14	0.12	0.06

Table 5 : Comparison table of Table 4

U	x_1	x_2	x_3	X_4
x_1	3	2	2	2
x_2	2	3	1	1
x_3	1	2	3	2
x_4	2	2	2	3

Table 6 : Non-membership score table

U	Row sum (c)	Column sum (d)	Membership score (c-d)
x_1	9	8	1
x_2	7	9	-2
x_3	8	8	0
x_4	9	8	1

Table 7 : Final score table

U	Membership score (m)	Non-membership score (n)	Final score (m-n)
x_1	1	1	0
x_2	3	-2	5
x_3	-3	0	-3
x_4	-1	1	-2

Therefore, the mode of transport which best satisfies the requirement of Madam X is the mode transport which has the maximum final score (5). Hence, Auto (x_2) is the mode of transport that suits the requirement of Madam X. Likewise, the model can be used for any set of criteria preferred by individuals to choose the suitable mode of transport that serves best their expectations.

Conclusion

The model can well be used for decision making in the fields of Marketing, Finance, Insurance, Banking, and other sectors offering products and services to consumers, whose taste, preference and attitude differ from one person to another.

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