Projective Synchronization by SMC for a class of Uncertain Chaotic System with Time Varying Delays

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***Abstract*—This paper is further to investigate the projective synchronization of a class of uncertain chaotic system with time varying delays. Based on the Lyapunov stability theorem, by using the LMI technique and designing a sliding mode control (SMC) approach, a less conservative yet sufficient condition is derived to guarantee the global stability of the error system. The feasible SMC law is designed such that the trajectory of the error system is globally driven onto the specified sliding surface. Numerical simulation result is represented to show the effectiveness of the proposed SMC laws. The projective synchronization is obtained at last.**

***Keywords*—Chaotic system, Delay, Projective synchronization, sliding mode control.**

I. INTRODUCTION

Since Lorenz found that there exists chaos in determined systems, people have realized that chaos is a ubiquitous phenomena in nature. From then on, the study on the theory and application of chaos systems has always been the focus on nonlinear systems. As the typical kinds of dynamics, control and synchronization in chaotic systems have become of significant interest in recent years. Sliding mode control for uncertain new chaotic dynamical system is investigated. The chattering phenomenon, which was inherent in conventional switching-type sliding controllers, due to the control law in Ref. [1] was very small and was damped quickly. Then modified projective synchronization of chaotic systems with disturbances via active sliding mode control was studied in Ref. [2]. Synchronization of chaotic systems with known and unknown parameters using a modified active sliding mode control was derived in Ref. [3]. The Ref. [4] designed a sliding mode controller for a class of fractional-order chaotic systems. The chaos control is implemented in the fractional-order Chen system, the fractional-order Lorenz system, and the same to the fractional-order financial system. In order to overcome the chattering, two new sliding mode controllers were proposed to ensure robust synchronization for classes of chaotic systems with input nonlinearities and external uncertainty in Ref. [5].

The sliding mode control (SMC) is a popular robust nonlinear control strategy. The main advantages of the SMC theory are as follows: fast response, good transient performance and robustness to variations of system parameters. One may know that delay is an important cause of instability and deterioration of system performance, research of time-delay systems has received wide attention from many scholars in the past decades. In Ref. [6], the switching surface was designed technically to realize fast convergence. The controller derived from such switching surface was non-singular and it could stabilize the chaotic systems in a finite time. The Ref. [7] proposed an adaptive fuzzy sliding mode control to synchronize two different uncertain fractional-order time-delay chaotic systems, which were infinite dimensional in nature, and time delay was a source of instability. The projective synchronization of the Lorenz system and the Chen system with fully unknown parameters was proposed in Ref. [8].

The Ref. [9] introduced a new sort of dynamical sliding mode surface with both integral and differential operators. Stability analysis was performed and a theorem serving as designing the chatter free sliding mode control input was also proposed. The Ref. [9] did not consider the condition about time varying delays. This paper will try to add time varying delays to the uncertain chaotic system, and use a modified SMC method to study the projective synchronization of the systems.

This paper is organized as follows: In section 2, system description and preliminaries are stated and a lemma and a corollary are listed. The SMC approach will be developed to address the projective synchronization problem of the chaotic system in section 3. We first properly construct an integral sliding surface. Then projective synchronization of the system with time varying delays is derived, where an integral sliding mode controller is designed to guarantee the reach ability of the specified sliding surface. In order to show the advantage of eliminating chatter in control input of our method, we give the simulation results in section 4. We give the conclusions of this study in section 5

II. PAGE LAYOUT

In paper [9], the author proposed a novel chatter free SMC strategy for chaos control and synchronization to the nonlinear uncertain chaotic system. The chaotic system described by the following equation, $\dot{x} = Ax + F(t,x) + d(t) + u(t)$. This paper will consider add delays to the above system, and then study the stability of the system. The drive system is

$$\dot{x}(t) = Ax(t) + Df(x(t)) + \sum_{j=1}^{n} H_j f(x_j(t-\tau)) + d(t) \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \dots x_n(t))^{\mathsf{T}} \in R^n$ denotes the state vector of the system, $d(t) \in R^n$ is an uncertain term presenting the external disturbance, un-modelled dynamics or structural variation, $f(x(t)): R^+ \times R^n \to R^n$ indicates a continues nonlinear vector field, τ is the time varying delays with $1 - \dot{\tau} \le 1$, $A, D \in R^{n \times n}$ represent two coefficient matrices, H_j is time varying delays connection matrix.

It is assumed that the measured output of system (1) is dependent on both the state and the delays state with the following form:

$$Z(t) = Bx(t) - C\sum_{j=1}^{n} x_j(t-\tau) \tag{2}$$

where $Z(t) \in R^m, B, C \in R^{m \times n}$ are constant matrices.

In order to study the synchronization of system (1), the response system is represented by

$$\dot{y}(t) = \hat{A}y(t) + \hat{D}\hat{f}(y(t)) + \sum_{j=1}^{n} \hat{H}_j \hat{f}(y_j(t-\tau)) + \hat{d}(t) + u(t) \tag{3}$$

where $y(t) \in R^n$ denotes the state vector of the response system, $u(t)$ is the sliding mode controller to be designed. The purpose of this study is to propose a method to designing a suitable controller such that: $t \to \infty$, $\|e(t)\| = \|x(t) - \lambda y(t)\| \to 0.$

Define the error signal $e(t) = x(t) - \lambda y(t),\ \lambda \ne 1,$ then the error system of (1) and (3) can be obtained as follow

$$\begin{aligned}
\dot{e}(t) &= \dot{x}(t) - \lambda\dot{y}(t) \\
&= [Ax(t) - \lambda\hat{A}y(t)] + [Df(x(t)) - \lambda\hat{D}\hat{f}(y(t))] + \sum_{j=1}^{n}[H_j f(x_j(t-\tau)) - \lambda\hat{H}_j\hat{f}(y_j(t-\tau))] + d(t) - \lambda\hat{d}(t) - \lambda u(t) \\
&= Ae(t) + Dg(e(t)) + \sum_{j=1}^{n} H_j g(e_j(t-\tau)) + d(t) - \lambda\hat{d}(t) - \lambda u(t) - \lambda u(t) + \lambda[(A-\hat{A})y(t) + Df(y(t)) - \hat{D}\hat{f}(y(t)) \\
&\quad + \sum_{j=1}^{n} H_j f(y_j(t-\tau)) - \hat{H}_j\hat{f}(y_j(t-\tau))]
\end{aligned}$$

where $g(e(t)) = f(x(t)) - \lambda f(y(t))$. Let

$$R(t) = d(t) - \lambda\hat{d}(t) + \lambda[(A-\hat{A})y(t) + Df(y(t)) - \hat{D}\hat{f}(y(t)) + \sum_{j=1}^{n}(H_j f(y_j(t-\tau)) - \hat{H}_j\hat{f}(y_j(t-\tau)))],$$

then we can get

$$\dot{e}(t) = Ae(t) + Dg(e(t)) + \sum_{j=1}^{n} H_j g(e_j(t-\tau)) + R(t) - \lambda u(t) \tag{4}$$

We can get from (4) that

$$e(t) = e(0) + \int_0^t [Ae(s) + Dg(e(s)) + \sum_{j=1}^{n} H_j g(e_j(s-\tau)) + R(s) - \lambda u(s)]ds \tag{5}$$

In order to meet the proof of the theorem, we use the following lemma.

Lemma 1. [10] $\forall\ x(t) = (x_1(t), x_2(t), \dots x_n(t))^{\mathsf{T}}$, $y(t) = (y_1(t), y_2(t), \dots y_n(t))^{\mathsf{T}} \in R^n$, there exists a positive definite matrix $P \in R^{n \times n}$, the following matrix inequality holds

$$2x^{\mathsf{T}}y \le x^{\mathsf{T}}Px + y^{\mathsf{T}}P^{-1}y.$$

Corolary 1. $\forall\ x(t) = (x_1(t), x_2(t), \dots x_n(t))^{\mathsf{T}}$, $y(t) = (y_1(t), y_2(t), \dots y_n(t))^{\mathsf{T}} \in R^n$, $\forall\ Q$ is a constant matrix; we get the next matrix inequality

$$2x^{\mathsf{T}}Qy \le x^{\mathsf{T}}QQ^{\mathsf{T}}x + y^{\mathsf{T}}y.$$

Proof. $\forall\ x(t) = (x_1(t), x_2(t), \dots x_n(t))^{\mathsf{T}}$, $y(t) = (y_1(t), y_2(t), \dots y_n(t))^{\mathsf{T}} \in R^n$, from the Lemma 1, we have

$$2x^{\mathsf{T}}Qy \le 2(Q^{\mathsf{T}}x)^{\mathsf{T}}y \le (Q^{\mathsf{T}}x)^{\mathsf{T}}P(Q^{\mathsf{T}}x) + y^{\mathsf{T}}P^{-1}y = x^{\mathsf{T}}QPQ^{\mathsf{T}}x + y^{\mathsf{T}}P^{-1}y,$$

then let *P=I*, we get $2x^{\mathsf{T}}Qy \le x^{\mathsf{T}}QQ^{\mathsf{T}}x + y^{\mathsf{T}}y.$

III. PAGE STYLE

For the drive system in Eq. (1), a sliding mode controller is to be chosen so that: (i) the sliding motion is globally asymptotically stable; (ii) the state trajectory in Eq. (1) is globally driven onto the specified sliding surface and maintained there for all subsequent time.

To utilize the information of the measured output $Z(t)$ sufficiently, a suitable sliding surface is constructed as

$$S(t)=e(t)-\int_0^t[Ae(s)+Dg(e(s))+\sum_{j=1}^{n}H_j g(e_j(s-\tau))-K(Z(s)-\lambda By(s)+\lambda C\sum_{j=1}^{n}y_j(s-\tau))]ds$$

where the gain matrix $K\in R^{m\times n}$ is to be designed. Substitute Eq. (5) to above Eq., one can obtain

$$S(t)=e(0)-\int_0^t[KBe(s)-KC\sum_{j=1}^{n}e_j(s-\tau)+R(s)-\lambda u(s)]ds \tag{6}$$

where $e(0)$ is an initial condition of the error system (4).

According to the SMC theory, after the error system driving onto the specified sliding surface, the sliding mode controller is a strong nonlinear input, so it is difficult to analyse. But if we use a continuous equivalent input instead of the switching input, then the analysis can be simplified. When the state trajectories of the error system (4) enter into the sliding surface, we have: $S(t)=0$, and $\dot{S}(t)=0$, then we get the following equivalent control law

$$u(t)=\frac{1}{\lambda}[KBe(t)-KC\sum_{j=1}^{n}e_j(t-\tau)+R(t)] \tag{7}$$

Substitute Eq. (7) to Eq. (4), the error sliding mode dynamics can be obtained and represented by

$$\dot{e}(t)=(A-KB)e(t)+Dg(e(t))+\sum_{j=1}^{n}H_j g(e_j(t-\tau))+KC\sum_{j=1}^{n}e_j(t-\tau) \tag{8}$$

The next work is to study the stability of the system (8). The next theorem is obtained.

Theorem 1. For given scalar $\tau>0$, the error sliding mode dynamics (8) is globally asymptotically stable, if there exist real matrices satisfy $P=P^{\mathrm{T}}>0, H_{1j}=H_{1j}^{\mathrm{T}}>0, H_{2j}=H_{2j}^{\mathrm{T}}>0.$ Such that

$$\Xi_j=\begin{pmatrix}\Phi_1+\Phi_2+\Phi_3+\Phi_4+I & H_{1j} & 0 & 0\\ * & \Psi_1 & 0 & 0\\ * & * & \Psi_2 & 0\\ * & * & * & \Psi_3\end{pmatrix}<0 \tag{9}$$

where $*$ always denotes the symmetric block in a symmetric matrix, and

$\Phi_1=P(A-KB)P^{\mathrm{T}}(A-KB)^{\mathrm{T}}, \Phi_2=PKC(PKC)^{\mathrm{T}}, \Phi_3=PD(PD)^{\mathrm{T}},$

$\Phi_4=P\sum_{j=1}^{n}H_j(P\sum_{j=1}^{n}H_j)^{\mathrm{T}}, \Psi_1=I-H_{1j}, \Psi_2=I+H_{2j}, \Psi_3=I-H_{3j}.$

Proof. Design the Lyapunov function as $V(t)=V_1(t)+V_2(t)+V_3(t)$, where

$$V_1(t)=e^{\mathrm{T}}(t)Pe(t),\ V_2(t)=\sum_{j=1}^{n}\int_{t-\tau}^{t}e_j^{\mathrm{T}}(s)H_{1j}e_j(s)ds,$$
$$V_3(t)=\sum_{j=1}^{n}\int_{t-\tau}^{t}g^{\mathrm{T}}(e_j(s))H_{2j}g(e_j(s))ds. \tag{10}$$

Calculating the derivative of (10) along the trajectories of (8), by the Corollary 1, one get

$$\begin{aligned}\dot{V}_1(t)&=2e^{\mathrm{T}}(t)P\dot{e}(t)\\&=2e^{\mathrm{T}}(t)[P(A-KB)e(t)+PDg(e(t))+P\sum_{j=1}^{n}H_j g(e_j(t-\tau))+PKC\sum_{j=1}^{n}e_j(t-\tau)]\\&\le e^{\mathrm{T}}(t)[P(A-KB)P^{\mathrm{T}}(A-KB)^{\mathrm{T}}+I+PKC(PKC)^{\mathrm{T}}+PD(PD)^{\mathrm{T}}+P\sum_{j=1}^{n}H_j(P\sum_{j=1}^{n}H_j)^{\mathrm{T}}]e(t)\\&\quad+[\sum_{j=1}^{n}e_j(t-\tau)]^{\mathrm{T}}\sum_{j=1}^{n}e_j(t-\tau)+g^{\mathrm{T}}(e(t)g(e(t)+[\sum_{j=1}^{n}g(e_j(t-\tau))]^{\mathrm{T}}\sum_{j=1}^{n}g(e_j(t-\tau))\end{aligned}$$

$$\dot{V}_2(t)=\sum_{j=1}^{n}[e_j^{\mathrm{T}}(t)H_{1j}e_j(t)-(1-\dot{\tau})e_j^{\mathrm{T}}(t-\tau)H_{1j}e_j(t-\tau)]\le\sum_{j=1}^{n}[e_j^{\mathrm{T}}(t)H_{1j}e_j(t)-e_j^{\mathrm{T}}(t-\tau)H_{1j}e_j(t-\tau)]$$

$$\dot{V}_3(t) \le \sum_{j=1}^{n}[g^{\mathrm{T}}(e_j(t))H_{2j}g(e_j(t)) - g^{\mathrm{T}}(e_j(t-\tau))H_{2j}g(e_j(t-\tau))]$$

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t)$$

$$\le e^{\mathrm{T}}(t)[\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + H_{1j} + I]e(t) + \sum_{j=1}^{n}[e_j^{\mathrm{T}}(t-\tau)\Psi_1 e_j(t-\tau)] + g^{\mathrm{T}}(e(t))\Psi_2 g(e(t))$$

$$+ \sum_{j=1}^{n}[g^{\mathrm{T}}(e_j(t-\tau))\Psi_3 g(e_j(t-\tau))] = \sum_{j=1}^{n}\mathrm{E}_j^{\mathrm{T}}(t)\Xi_j \mathrm{E}_j(t)$$

where $\mathrm{E}_j(t) = [e^{\mathrm{T}}(t), e_j^{\mathrm{T}}(t-\tau), g^{\mathrm{T}}(e(t)), g^{\mathrm{T}}(e_j(t-\tau))]$ From Eq. (9) we have

$$\dot{V}(t) \le \sum_{j=1}^{n}\mathrm{E}_j^{\mathrm{T}}(t)\Xi_j \mathrm{E}_j(t) < 0 \tag{11}$$

Therefore, based on the Lyapunov stability theorem, $\|e(t)\| = \|x(t) - \lambda y(t)\| \to 0$, when $t \to \infty$, then the error sliding mode dynamics (8) is globally asymptotically stable. Thus the proof is completed. The theorem guaranteed the synchronization of the system with time varying delays. Compare with the result of [9], our method solved the time varying delays condition, and the method could apply to other uncertain chaotic systems.

We have finished the condition (i), in order to guarantee the reach ability of the specific switching surface, we choose a feasible SMC law as follows.

Theorem 2. Consider the error system (4). Assume that the sliding function is given by (6) with *P*, *K* and *r*, where *P*, *K* and *r* are the suitable solution to (9). Let $\theta > 0$ be a constant scalar, if the SMC law is designed as follows:

$$u(t) = \frac{1}{\lambda}[KBe(t) - KC\sum_{j=1}^{n} e_j(t-\tau) - \omega(t)\,\mathrm{sgn}(S(t))) \tag{12}$$

with the switching gain $\omega(t)$ being taken as

$$\omega(t) = -\theta - [\|I - \lambda I\| + \|\lambda(A - \hat{A})\| + \|D\|\|f\| - \|\hat{D}\|\|\hat{f}\| + \sum_{j=1}^{n}(\|H_j\|\|f\| - \|\hat{H}_j\|\|\hat{f}\|)\|y_j(t-\tau)\|]. \tag{13}$$

Then the trajectories of the error system can be globally driven onto the sliding surface $S(t)=0$.

Proof. We are in a position to design a feasible SMC law to guarantee the reach ability of the sliding surface. It is follows from (4) and (12) that

$$\dot{S}(t) = R(t) + \omega(t)\,\mathrm{sgn}(S(t)) \tag{14}$$

By designing the following Lyapunov function:

$$V_4(t) = \frac{1}{2}S^{\mathrm{T}}(t)S(t) \tag{15}$$

Calculating the derivative of (15) along the trajectories of (14), by the Eq. (13), one get

$$\dot{V}_4(t) = S^{\mathrm{T}}(t)[R(t) + \omega(t)\,\mathrm{sgn}(S(t))]$$

$$\le \|S(t)\|[\|I - \lambda I\| + \|\lambda(A - \hat{A})\| + \|D\|\|f\| - \|\hat{D}\|\|\hat{f}\| + \sum_{j=1}^{n}(\|H_j\|\|f\| - \|\hat{H}_j\|\|\hat{f}\|)\|y_j(t-\tau)\| + \omega(t)\|S(t)\|]$$

$$= -\theta\|S(t)\|$$

It means that $\dot{V}_4(t) < 0$, for any $S(t) \ne 0$, so the trajectories of the error system can be globally driven onto the sliding surface. The SMC law guaranteed the reach ability of the sliding surface. The above two theorems not only obtain the globally asymptotically stable but also drive the SMC reach ability.

IV. NUMERICAL SIMULATION

In this section, without loss of generality, we consider the following neural networks with multi-delays,

$$\dot{x}(t) = Ax(t) + Df(x(t)) + \sum_{j=1}^{n}H_j f(x_j(t-\tau)) + d(t).$$

Following the conditions (9) of the theorem 1 and by employing the LMI toolbox in matlab 2012b, we design the argument as follows,

$f(x)=\tanh(x), a_{11}(x)=-6x+\sin(x),\ a_{22}(x)=-8x+\cos(x),\ \tau(t)=0.2|\cos(t)|, d_1(t)=-36\sin(\pi t),$
$d_2(t)=36\cos(\pi t),\ \ P=B=I_2, K=diag\{-9.836,-6.386\}, D=diag\{-0.5,-2.0\},$
$H_{1j}=diag\{1.2530, 1.5427\}>0,\ H_{2j}=diag\{-1.0537,-1.0539\}<0, H_{3j}=diag\{1.8530, 1.6437\}>0.$

Furthermore, the initial condition of the $x(t)$=(-10,9), it is easy to verify the condition of the assumptions, then the state trajectories of the neurons $x_1(t)$ and $x_2(t)$ is shown in the figure 1.

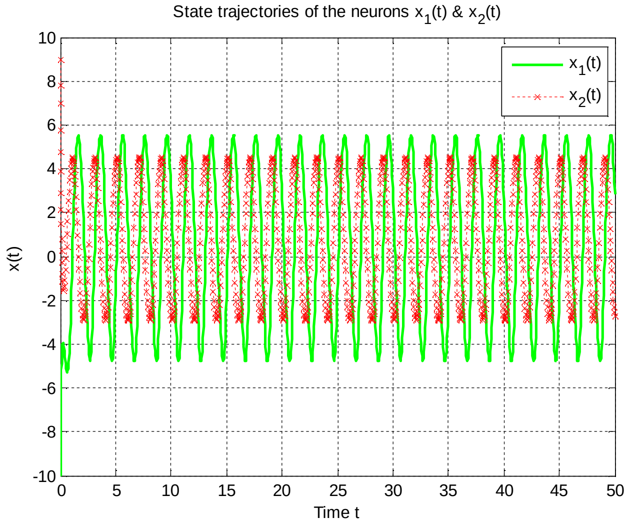

Figure 1. State trajectories of the neurons $x_1(t)$ &$x_2(t)$.

Because the exponential convergence rate and specific kerner function are not known in advance, so we can prove the conclusion's correctness in mathematics.

V. Conclusions

In this paper, we investigate the projective synchronization of a class of uncertain chaotic system with time varying delays. Based on the Lyapunov stability theorem, by using the LMI technique and designing a sliding mode control (SMC) approach, we confirm the projective synchronization between drive system and response system. The feasible SMC law is designed such that the trajectory of the error system is globally driven onto the specified sliding surface. The SMC law guarantees the reach ability of the specific switching surface. Numerical simulation result is represented to show the effectiveness of the proposed SMC laws. The projective synchronization is obtained at last. Compared with Ref. [9], we solve the condition with time varying delays, and our method could apply in other networks.

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