Hematocrit effects of the axisymmetric blood flow through an artery with stenosis arteries

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***Abstract*— When the nature of blood flow changes from its usual state to a disturbed flow condition due to the presence of a stenosis in an artery. Then it has been suggested that the deposits of cholesterol on the arterial wall and proliferation of connective tissue may be responsible for the abnormal growth in the lumen of an artery. Therefore a study about the blood flow in an arterial segment having a stenosis is important because of its unusual state disturbe the flow behaviour. An axisymmetric flow of blood through a circular tube with an axially symmetric stenosis is considered. The unsteady nonlinear Navier-Stokes equations in cylindrical coordinate system governing flow assuming axial symmetry under laminar flow condition is then solved numerically so that the problem effectively becomes two-dimensional. In this investigation, we observe the effects of red cell concentration (hematocrit) on blood flow characteristics in the presence of stenosis, and it is found that the flow resistance and the wall shear stress increases with hematocrits.**

Keywords*— *Stenosis, blood flow, hematocrit.

I. Introduction

Blood is made up of a suspension of particles in a solution of proteins and electrolytes called plasma. Erythrocytes, leukocytes and platelets are the main constituents of blood. The erythrocytes or Red Blood Cells (RBCs) are more than a thousand times more numerous than the leukocytes or White Blood Cells (WBCs) and much larger than platelets. For this reason, the flow properties of blood mainly involve the RBCs. The hematocrit (percentage of the blood volume that is made up of red blood cells) is the major determinant of blood viscosity. Initially red blood cells have round shape. But in people who have sickle-cell anemia, the blood cells break down. They lose their round shape and take the form of a sickle, a farmer's implement. A lot of work available in this direction but theoretical observations of Haynes [1]. Young [2] observed the effects of time dependent stenosis of flow through a tube, and experimental investigation of Cokelet [3] indicate that the blood can no longer be treated as a single-phase homogenous viscous fluid in small size vessels (of diameter <1000μm). It is surprising to note that the individuality of the red cells (of diameter 8μm) is important even in such large vessels (with diameter up to 100cells diameter). The effects of peripheral layer on blood flow through artery with mild stenosis obtained Shukla et al. [5] while Srivastava et al. [5] discussed on two phase model of pulsatile blood flow and observed the entrance effects. Srivastava [7] also worked on the flow of a couple-stress fluid through stenotic blood vessels while Chaturani and Samy [8] discussed about the pulsatile flow of casson fluid through stenosed arteries with application to blood flow. Chakravarty and Choudhary [9] made a report for the response of blood flow through an artery under stenotic conditions. Mann et al. [10] discussed about the flow of non-Newtonian blood analog fluids in rigid curved and straight artery models, while Sud [11] worked on the simulation of steady cardiovascular flow in the presence of stenosis using a finite element method. As for as the approximate solution Taylor et al. [12] discussed also about the finite element modeling of three-dimensional pulsatile flow in the abdominal aorta: Relevance to atherosclerosis, while Waters et al. [13] discussed about the oscillatory flow in a tube of time dependent curvature part-1 perturbation to flow in a stationary curved tube. Zhang et al. [14] studied on the blood constitutive parameters in different blood constitutive equations, although Secomb et al. [1] discussed about the blood flow and red blood cell deformation in non uniform capillaries effects of the endothelial surface layer. Steinman et al. [16] studied the flow imaging and computing the large artery hemodynamics**.** Kumar et al. [17] numerically worked on the study of the axisymmetric blood flow in a constricted rigid tube, while Bali et al. [18] observed that the effect of a magnetic field on the resistance of blood flow through stenotic artery. Verma and Parihar [19] studied effects of magneto-hydrodynamic and hematocrit on blood flow in an artery with multiple mild stenosis. Verma and Parihar [20] worked on a mathematical model of blood flow through a tapered artery with mild stenosis and hematocrit while Misra et al. [21] worked on a numerical model for the magnetohydrodynamic (MHD) flow of blood in a porous channel. Eldesoky [22] observe that the influence of slip condition on peristaltic transport of a compressible maxwell fluid through porous medium in a tube. Recently Kumar and Diwakar [23] worked on a biomagentic fluid dynamic model for the MHD couette flow between two infinite horizontal parallel porous plates and give that the main flow component decreases with the increase of Hartmann number and the velocity decreases with the increases of the injection suction parameter. Further in the same direction Kumar and Diwakar [24] discussed about a mathematical model for Newtonian blood flow in the presence of applied magnetic field and found that results concerning the velocity and temperature field, and rate of heat transfer indicates that the presence of magnetic field appreciable influence the flow field, while the flow is appreciably influenced by the application of the magnetic field and in particularly by the strength and the magnetic field gradient.

Our main motto is to give a general solution for the problem of flow through symmetric stenosis, and as we know the major constituents of blood that characterise several blood properties and responsible for many diseases, are the red blood cells. They play a vital an important role for blood flow through small vessels (of diameter 0.2-0.4 cm). Thus the model, which we are considering, is a particle-fluid suspension model, and the main aim of this analysis is to made an observation about the effects of stenosis on blood flow characteristics.

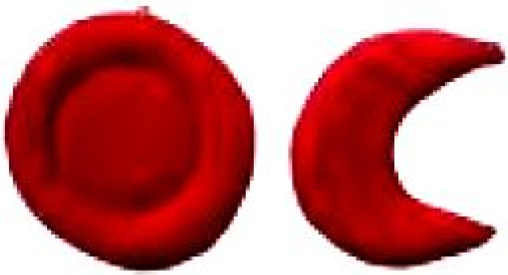

Fig.1 Round red blood cell and obustructed red blood cell

II. Mathematical formulation

Consider a circularly cylindrical arterial segment having axisymmetric flow of blood with an axially symmetric stenosis. If a is the constant radius of artery, l is the length of stenosis, δ is the maximum height of the stenosis and d is the location of stenosis. Here l is being chosen as the finite length of the arterial segment under consideration and also $\delta << a$.

Then the geometry of the stenosis (Fig-2), which is assumed to be revealed in arterial segment may be described by:

$$\frac{R(z)}{a} = \begin{cases} 1 - \frac{\delta}{2a}\left(1 + \cos\frac{2\pi}{l}(z - d - l/2)\right) & d \le z \le d + l \\ 1 & otherwise \end{cases} \quad \ldots(i)$$

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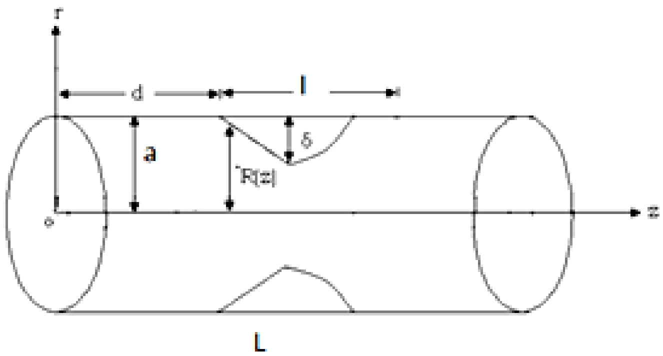

Fig. 2 Geometry of stenosis in an artery

Now considering a laminar, steady, one dimensional axisymmetric flow of blood through an arterial segment for $\delta << a$, with the idea of the Young [2] work. The general constitutive equation for this two phase macroscopic model is simplified to:

$$(1-c)\,\frac{dp}{dz}=(1-c)\,\frac{\mu_c}{r}\,\frac{\partial}{\partial r}\,(r\frac{\partial}{\partial r})u_1+cD\left(u_2-u_1\right) \quad \text{… (ii)}$$

$$-\frac{dp}{dz}=D\left(u_1-u_2\right) \quad \text{…(iii)}$$

where D and μ_c represents the drug coefficient and viscosity of suspension respectively, and they are given by following the equations.

$$D=\frac{9}{2}\left(\frac{\mu_0}{a_0}[4+3(8c-3c^2)^{\frac{1}{2}}+3c]/(2-3c)^2\right) \quad \text{…(iv)}$$

and $\mu_c=\mu_o/(1-qc)$;

where $q=0.07\exp[2.49c+1107^0 K/T\exp(-1.69c)]$, and c denotes the volume friction density of particle which is assumed to be constant, and absolute temperature is expressed in terms of T, μ_0 is the fluid (plasma) viscosity and a_0 is the radius of particle.

The solutions of equations (ii) and (iii) subjected to the boundary conditions:

$$\left.\begin{array}{lll} u_1=0 & at & r=R(z) \\ \dfrac{\partial u_1}{\partial r}=\dfrac{\partial u_2}{\partial r} & at & r=0 \end{array}\right\} \quad \text{… (v)}$$

are

$$u_1=\frac{a^2}{4(1-c)\mu_0}\frac{dp}{dz}\left[\left(\frac{r}{a}\right)^2-\left(\frac{R}{a}\right)^2\right] \quad \text{… (vi)}$$

$$u_2 = \frac{a^2}{4(1-c)\mu_c}\frac{dp}{dz}\left[\left(\frac{r}{a}\right)^2 - \left(\frac{R}{a}\right)^2 - \frac{4(1-c)\mu_c}{Da^2}\right] \quad \text{…(vii)}$$

Now the flux of blood flow in the artery may be obtained through the following expression:

$$Q = 2\pi\left[(1-c)\int_0^R r\,u_1 dr + c\int_0^R r\,u_2 dr\right]$$

$$= -\frac{\pi a^4}{8(1-c)\mu_c}\frac{dp}{dz}\left[\eta^2\left(\frac{R}{a}\right)^2 + \left(\frac{r}{a}\right)^2\right] \quad \text{…(viii)}$$

where $\eta = \frac{2}{a}\sqrt{\frac{2c(1-c)\mu_c}{D}}$ is the suspension parameter.

Integrating equation (viii) and using the conditions $P = P_i\ at\ z = 0\ and\ P = P_0\ at\ z = L$, we get:

$$P_i - P_0 = \frac{8(1-c)\mu_c Q}{\pi a^4}\int_0^L \frac{dz}{\eta^2 (R/a)^2 + (R/a)^4} \quad \text{…(ix)}$$

The resistance to flow, λ may be defined as:

$$\lambda = \frac{P_i - P_o}{Q} \quad \text{… (x)}$$

Substitute the value of $p_i - p_0$ from equations (ix) to (x), we get

$$\lambda = \frac{8(1-c)\mu_c}{\pi a^4}\int_0^L \frac{dz}{\eta^2 (R/a)^2 + (R/a)^4}$$

or we may write it as:

$$\lambda = \frac{8(1-c)\mu_c}{\pi a^4}\left[\int_0^d \frac{dz}{1+\eta^2} + \int_d^{d+L_0} \frac{dz}{\eta^2 (R/a)^2 + (R/a)^4} + \int_{d+L_0}^L \frac{dz}{1+\eta^2}\right] \quad \text{…(xi)}$$

because in the first and last part of this integrals, the value of $R/R_0 = 1$.

In case of no stenosis ($\delta = 0$ or $R = a$), the resistance to flow, λ_N (normal artery) is expressed as:

$$\lambda_N = \frac{8\mu_0 L}{\pi a^4} \quad \text{…(xii)}$$

From eqns. (xi) and (xii), the ratio $\frac{\lambda}{\lambda_N}$ may be expressed as:

$$\frac{\lambda}{\lambda_N} = (1-c)\mu'\left\{\frac{1}{1+\eta^2}\left(1 - \frac{L_0}{L}\right) + \frac{1}{L}\int_d^{d+L_0}\frac{dz}{(R/a)^4 + \eta^2 (R/a)^2}\right\} \quad \text{…(xiii)}$$

where $\mu' = \mu_c/\mu_0$

The final expression for the wall shear stress τ_w and shear stress at the stenosis throat τ_s, is:

$$\bar{\tau}_w = \frac{(1-c)\mu'}{\left[\eta^2 (R/a) + (R/a)^3\right]} \quad \text{...(xiv)}$$

$$\bar{\tau}_s = \frac{(1-c)\mu'}{\left[\eta^2 (1-\delta/a) + (1-\delta/a)^3\right]} \quad \text{...(xv)}$$

where $\tau_w = -(R/z)\frac{dp}{dz}$ and $\bar{\tau}_s = \tau_s/\tau_0$.

III Numerical results and discussion

The variations of λ/λ_N given by equation (viii) with δ/a are plotted through fig.-3 for different values of particle concentration *c*. From fig. 5(c), we can say that the flow resistance $\bar{\lambda}$ is always greater than 1which increases with particle concentration. The ratio of shearing stress at the wall $\bar{\tau}$ and ratio of shearing stress at the stenosis throat posses the character similar to $\bar{\lambda}$.

It is also notice that $\bar{\tau}_w$ steeply increases in the upstream from its approached value (i.e. *z/l =0*) to the peak value at the throat and decreases in the downstream of the throat to end of constriction profile which approached to a value *z / l =1*. It is found that $\bar{\tau}_w$ increases in the diverging zone while decreases in the converging zone but there is no effect if stenosis geometry on $\bar{\tau}_s$ in the similar situation. We use numerical technique to solve the analytical result of this model of considering the temperature at $25^0 C$.

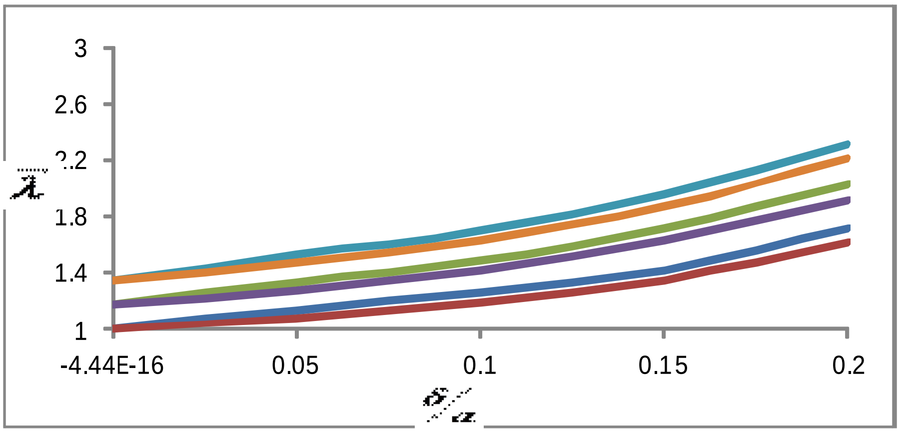

Fig.3 Variation of dimensionless resistance to flow, $\bar{\lambda}$ *with* δ/a *for different values of c*

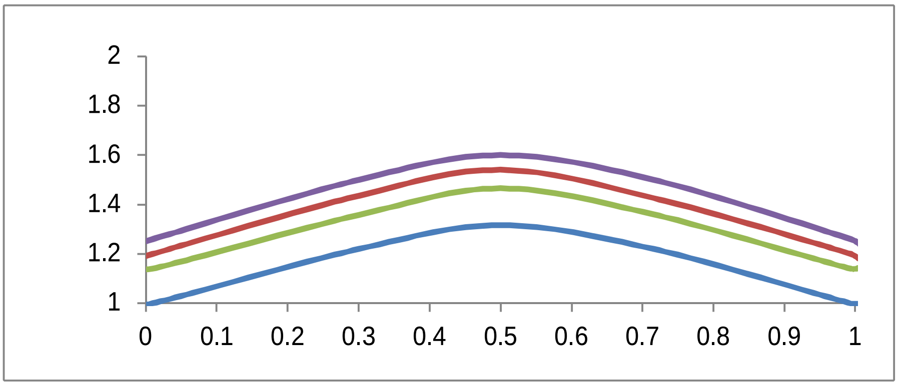

Fig.4 Dimensionless wall shear stress $\bar{\tau}_w$ *distribution in the stenotic region with* z/l *for different values of c*

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