Generalized Presemi Closed Mappings in Intuitionistic Fuzzy Topological Spaces

[1]T. Sampoornam, [2]Gnanambal Ilango and [3]K. M. Arifmohammed

[1]*Associate Professor and Head, Department of Mathematics, Government Arts College, Coimbatore-641018, (T.N.), India.*

[2]*Assistant Professor, Department of Mathematics, Government Arts College, Coimbatore-641018, (T.N.), India.*

[3]*Department of Mathematics, NGM College, Pollachi-642001, Tamil Nadu, India.*

***Abstract*—In this paper we introduce intuitionistic fuzzy generalized presemi closed mappings, intuitionistic fuzzy generalized presemi open mappings and intuitionistic fuzzy i-generalized presemi closed mappings and we study some of their properties. We provide the relation between intuitionistic fuzzy i-generalized presemi closed mappings and intuitionistic fuzzy generalized presemi closed mappings.**

***Keywords*—Intuitionistic fuzzy topology, intuitionistic fuzzy generalized presemi closed mappings, intuitionistic fuzzy generalized presemi open mappings and intuitionistic fuzzy i-generalized presemi closed mappings.**

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I. INTRODUCTION

In 1965, Zadeh [13] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of this notion. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In 2000, Seok Jong Lee and Eun Pyo Lee [10] investigated the properties of continuous, open and closed maps in the intuitionistic fuzzy topological spaces. In this direction we introduce the notions of intuitionistic fuzzy generalized presemi closed mappings, intuitionistic fuzzy generalized presemi open mappings and intuitionistic fuzzy i-generalized presemi closed mappings and study some of their properties.

II. PRELIMINARIES

Definition 2.1: [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$ where the functions $\mu_A: X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x)\rangle / x \in X \}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \{ \langle x, \nu_A(x), \mu_A(x)\rangle / x \in X \}$,
(iv) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X \}$,
(v) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X \}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$. The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short)in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then

(i) $int(A) = \cup\{ G / G$ is an IFOS in X and $G \subseteq A\}$,
(ii) $cl(A) = \cap\{ K / K$ is an IFCS in X and $A \subseteq K\}$,
(iii) $cl(A^c) = (int(A))^c$,
(iv) $int(A^c) = (cl(A))^c$.

Definition 2.5: [4] An IFS A of an IFTS (X, τ) is an

(i) intuitionistic fuzzy semiclosed set (IFSCS for short) if $int(cl(A)) \subseteq A$,

(ii) intuitionistic fuzzy semiopen set (IFSOS for short) if $A \subseteq cl(int(A))$,
(iii) intuitionistic fuzzy preclosed set (IFPCS for short) if $cl(int(A)) \subseteq A$,
(iv) intuitionistic fuzzy preopen set (IFPOS for short) if $A \subseteq int(cl(A))$,
(v) intuitionistic fuzzy α-closed set (IFαCS for short) if $cl(int(cl(A))) \subseteq A$,
(vi) intuitionistic fuzzy α-open set (IFαOS for short) if $A \subseteq int(cl(int(A)))$.

Definition 2.6: [12] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy semipre closed set (IFSPCS for short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$,
(ii) intuitionistic fuzzy semipre open set (IFSPOS for short) if there exists an IFPOS B such that $B \subseteq A \subseteq cl(B)$.

Definition 2.7: [11] An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy w-closed set (IFWCS for short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ). An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy w-open set (IFWOS for short) if A^c is an IFWCS in (X, τ).

Definition 2.8: [7] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized presemi closed set (IFGPSCS for short) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ).
Every IFCS, IFWCS, IFαCS, IFPCS is an IFGPSCS and every IFGPSCS is an IFGSPCS, IFGSPRCS but the converses are not true in general.

Definition 2.9: [7] The complement A^c of an IFGPSCS A in an IFTS (X, τ) is called an intuitionistic fuzzy generalized presemi open set (IFGPSOS for short) in X.
The family of all IFGPSOSs of an IFTS (X, τ) is denoted by IFGPSO(X). Every IFOS, IFWOS, IFαOS, IFPOS is an IFGPSOS and every IFGPSOS is an IFGSPOS, IFGSPROS but the converses are not true in general.

Definition 2.10: [8] An IFS A is an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semipre closed set (IFGSPCS) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ). An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy generalized semipre open set (IFGSPOS for short) if A^c is an IFGSPCS in (X, τ).

Definition 2.11: [5] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semipre regular closed set (IFGSPRCS for short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in (X, τ). The complement A^c of an IFGSPRCS A in an IFTS (X, τ) is called an intuitionistic fuzzy generalized semipre regular open set (IFGSPROS for short) in X.

Definition 2.12: [8] Let A be an IFS in an IFTS (X, τ). Then
(i) spint (A) = ∪{ G / G is an IFSPOS in X and G ⊆A },
(ii) spcl (A) = ∩{ K / K is an IFSPCS in X and A ⊆K }.

Definition 2.13: [7] If every IFGPSCS in (X, τ) is an IFPCS in (X, τ), then the space can be called as an intuitionistic fuzzy presemi $T_{1/2}$ (IFPS$T_{1/2}$ for short) space.

Definition 2.14: [10] A map f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for each IFCS A in X.

Definition 2.15: [10] A map f : (X, τ) → (Y, σ) is called an
(i) intuitionistic fuzzy α-open mapping (IFαOM for short) if f(A) is an IFαOS in Y for each IFOS A in X.
(ii) intuitionistic fuzzy preopen mapping (IFPOM for short) if f(A) is an IFPOS in Y for each IFOS A in X.

Definition 2.16: [11] A mapping f: (X, τ) →(Y, σ) is intuitionistic fuzzy w-closed mapping (IFWCM for short) if image of every IFCS of X is an IFWCS in Y.

Definition 2.17: [9] A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy generalized semipre closed mapping (IFGSPCM for short) if f (A) is an IFGSPCS in Y for each IFCS A in X.

Definition 2.18: [6] A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy generalized semipre regular closed mapping (IFGSPRCM for short) if f (A) is an IFGSPRCS in Y for each IFCS A in X.

III. INTUITIONISTIC FUZZY GENERALIZED PRESEMI CLOSED MAPPINGS

In this section we introduce intuitionistic fuzzy generalized presemi closed mappings and intuitionistic fuzzy i- generalized presemi closed mappings and study some of their properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy generalized presemi closed mapping (IFGPSCM for short) if f(A) is an IFGPSCS in Y for each IFCS A in X.

For the sake of simplicity, we shall use the notation A= ⟨x, (µ, µ), (ν, ν)⟩ instead of A = ⟨x,(a/μ_a, b/μ_b), (a/ν_a, b/ν_b)⟩ in all the examples used in this paper. Similarly we shall use the notation B = ⟨x, (µ, µ), (ν, ν)⟩ instead of B = ⟨x,(u/μ_u, v/μ_v), (u/ν_u, v/ν_v)⟩ in the following examples.

Example 3.2: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.6, 0.7), (0.4, 0.2)⟩, G_2 = ⟨y, (0.5, 0.4), (0.5, 0.6)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGPSCM.

Theorem 3.3: Every IFCM is an IFGPSCM but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFCM. Let A be an IFCS in X. Then f(A) is an IFCS in Y. This implies that f(A) is an IFGPSCS in Y. Hence f is an IFGPSCM.

Example 3.4: In Example 3.2., f: (X, τ) → (Y, σ) is an IFGPSCM but not an IFCM.

Theorem 3.5: Every IFαCM is an IFGPSCM but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFαCM. Let A be an IFCS in X. Then f(A) is an IFαCS in Y. This implies that f(A) is an IFGPSCS in Y. Hence f is an IFGPSCM.

Example 3.6: In Example 3.2., f: (X, τ) → (Y, σ) is an IFGPSCM but not an IFαCM.

Theorem 3.9: Every IFWCM is an IFGPSCM but not conversely.

Proof: Assume that f : (X, τ) → (Y, σ) is an IFWCM. Let A be an IFCS in X. Then f(A) is an IFWCS in Y. This implies that f(A) is an IFGPSCS in Y. Hence f is an IFGPSCM.

Example 3.10: In Example 3.2., f: (X, τ) → (Y, σ) is an IFGPSCM but not an IFWCM.

Theorem 3.11: Every IFPCM is an IFGPSCM but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFPCM. Let A be an IFCS in X. Then f(A) is an IFPCS in Y. This implies that f(A) is an IFGPSCS in Y. Hence f is an IFGPSCM.

Example 3.12: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.1, 0.4), (0.9, 0.6)⟩, G_2 = ⟨y, (0.2, 0.1), (0.8, 0.9)⟩, G_3 = ⟨y, (0.5, 0.6), (0.5, 0.4)⟩. Then τ = {$0_{\sim}$, G_1, $1_{\sim}$} and σ = {$0_{\sim}$, G_2, G_3, $1_{\sim}$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGPSCM but not an IFPCM.

Theorem 3.7: Every IFGPSCM is an IFGSPCM but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFGPSCM. Let A be an IFCS in X. Then f(A) is an IFGPSCS in Y. This implies that f(A) is an IFGSPCS in Y. Hence f is an IFGSPCM.

Example 3.8: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5, 0.6), (0.5, 0.4)⟩, G_2 = ⟨y, (0.3, 0.2), (0.7, 0.8)⟩. Then τ = {$0_{\sim}$, G_1, $1_{\sim}$} and σ = {$0_{\sim}$, G_2, $1_{\sim}$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGSPCM but not an IFGPSCM.

Theorem 3.7: Every IFGPSCM is an IFGSPRCM but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFGPSCM. Let A be an IFCS in X. Then f(A) is an IFGPSCS in Y. This implies that f(A) is an IFGSPRCS in Y. Hence f is an IFGSPRCM.

Example 3.8: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5, 0.6), (0.5, 0.4)⟩, G_2 = ⟨y, (0.3, 0.2), (0.7, 0.8)⟩. Then τ = {$0_{\sim}$, G_1, $1_{\sim}$} and σ = {$0_{\sim}$, G_2, $1_{\sim}$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGSPRCM but not an IFGPSCM.

Definition 3.13: A mapping f: (X, τ) → (Y, σ) is said to be an intuitionistic fuzzy generalized presemi open mapping (IFGPS OM for short) if f(A) is an IFGPSOS in Y for each IFOS in X.

Definition 3.14: A mapping f: (X, τ) → (Y, σ) is said to be an intuitionistic fuzzy i-generalized presemi closed mapping (IFiGPSCM for short) if f(A) is an IFGPSCS in Y for every IFGPSCS A in X.

Definition 3.15: A mapping f: (X, τ) → (Y, σ) is said to be an intuitionistic fuzzy i-generalized presemi open mapping (IFiGPSOM for short) if f(A) is an IFGPSOS in Y for every IFGPSOS A in X.

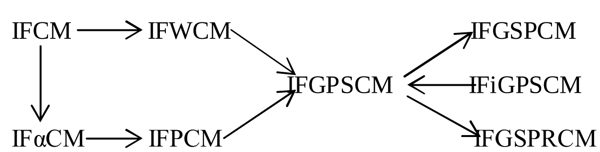
Example 3.16: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.3, 0.2), (0.7, 0.8)⟩, G_2 = ⟨y, (0.5, 0.4), (0.5, 0.6)⟩. Then τ = {$0_{\sim}$, G_1, $1_{\sim}$} and σ = {$0_{\sim}$, G_2, $1_{\sim}$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFiGPSCM.

Theorem 3.18: Every IFiGPSCM is an IFGPSCM but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFiGPSCM. Let A be an IFCS in X. Then A is an IFGPSCS in X. By hypothesis f(A) is an IFGPSCS in Y. Hence f is an IFGPSCM.

Example 3.19: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.8, 0.9), (0.2, 0.1)⟩, G_2 = ⟨y, (0.3, 0.2), (0.7, 0.8)⟩. Then τ = {$0_{\sim}$, G_1, $1_{\sim}$} and σ = {$0_{\sim}$, G_2, $1_{\sim}$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGPSCM but not an IFiGPSCM.

The relation between various types of intuitionistic fuzzy closed mappings is given by



The reverse implications are not true in general in the above diagram.

Theorem 3.20: Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping. Then the following statements are equivalent if Y is an $IFPST_{1/2}$ space:

(i) f is an IFGPSCM,
(ii) $pcl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.

Proof: (i) ⇒ (ii) Let A be an IFS in X. Then cl(A) is an IFCS in X. (i) implies that f(cl(A)) is an IFGPSCS in Y. Since Y is an $IFPST_{1/2}$ space, f(cl(A)) is an IFPCS in Y. Therefore $pcl(f(cl(A))) = f(cl(A))$. Now $pcl(f(A)) \subseteq pcl(f(cl(A))) = f(cl(A))$. Hence $pcl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.
(ii) ⇒ (i) Let A be any IFCS in X. Then cl(A) = A. (ii) implies that $pcl(f(A)) \subseteq f(cl(A)) = f(A)$. But $f(A) \subseteq pcl(f(A))$. Therefore pcl(f(A)) = f(A). This implies f(A) is an IFPCS in Y. Since every IFPCS is an IFGPSCS, f(A) is an IFGPSCS in Y. Hence f is an IFGPSCM.

Theorem 3.21: Let f: $(X, \tau) \to (Y, \sigma)$ be a bijection. Then the following statements are equivalent if Y is an $IFPST_{1/2}$ space:

(i) f is an IFGPSCM,
(ii) $pcl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.
(iii) $f^{-1}(pcl(B)) \subseteq cl(f^{-1}(B))$ for every IFS B of Y.

Proof: (i) ⇔ (ii) is obvious from Theorem 3.20.
(ii) ⇒ (iii) Let B be an IFS in Y. Then $f^{-1}(B)$ is an IFS in X. Since f is onto, $pcl(B) = pcl(f(f^{-1}(B)))$ and (ii) implies $pcl(f(f^{-1}(B))) \subseteq f(cl(f^{-1}(B)))$. Therefore $pcl(B) \subseteq f(cl(f^{-1}(B)))$. Now $f^{-1}(pcl(B)) \subseteq f^{-1}(f(cl(f^{-1}(B)))) = cl(f^{-1}(B))$, since f is one to one. Hence $f^{-1}(pcl(B)) \subseteq cl(f^{-1}(B))$.
(iii) ⇒ (ii) Let A be an IFS in X. Then f(A) is an IFS of Y. Since f is one to one. (iii) Implies that $f^{-1}(pcl(f(A))) \subseteq cl(f^{-1}(f(A))) = cl(A)$. Therefore $f(f^{-1}(pcl(f(A)))) \subseteq f(cl(A))$. Since f is onto $pcl(f(A)) = f(f^{-1}(pcl(f(A)))) \subseteq f(cl(A))$.

Theorem 3.22: If f: $(X, \tau) \to (Y, \sigma)$ is an IFCM and g: $(Y, \sigma) \to (Z, \eta)$ is an IFGPSCM then $g \circ f: (X, \tau) \to (Z, \eta)$ is an IFGPSCM.

Proof: Let A be any IFCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since g is an IFGPSCM, g(f(A)) is an IFGPSCS in Z. Therefore $g \circ f$ is an IFGPSCM.

Theorem 3.23: Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping where Y is an $IFPST_{1/2}$ space. Then the following statements are equivalent.

(i) f is an IFGPSCM,
(ii) f(B) is an IFGPSOS in Y for every IFOS B in X,
(iii) $f(int(B)) \subseteq int(cl(f(B)))$ for every IFS B in X.

Proof: (i) ⇔ (ii) is obvious.
(ii) ⇒ (iii) Let B be an IFS in X. Then int(B) is an IFOS in X. By hypothesis f(int(B)) is an IFGPSOS in Y. Since Y is an $IFPST_{1/2}$ space, f(int(B)) is an IFPOS in Y. Therefore $f(int(B)) \subseteq int(cl(f(int(B)))) \subseteq int(cl(f(B)))$.
(iii) ⇒ (i) Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis, $f(int(A^c)) = f(A^c) \subseteq int(cl(f(A^c)))$. That is $cl(int(f(A))) \subseteq f(A)$. This implies f(A) is an IFPCS in Y and hence an IFGPSCS in Y. Therefore f is an IFGPSCM.

Theorem 3.24: If f: $(X, \tau) \to (Y, \sigma)$ is a mapping. Then the following are equivalent if Y is an $IFPST_{1/2}$ space
(i) f is an IFGPSOM
(ii) $f(int(A)) \subseteq pint(f(A))$ for each IFS A of X
(iii) $int(f^{-1}(B)) \subseteq f^{-1}(pint(B))$ for every IFS B of Y.

Proof: (i) ⇒ (ii) Let f be an IFGPSOM. Let A be any IFS in X. Then int(A) is an IFOS in X. (i) implies that f(int(A)) is an IFGPSOS in Y. Since Y is an $IFPST_{1/2}$ space, f(int(A)) is an IFPOS in Y. Therefore pint(f(int(A))) = f(int(A)). Now $f(int(A)) = pint(f(int(A))) \subseteq pint(f(A))$.
(ii) ⇒ (iii) Let B be an IFS in Y. Then $f^{-1}(B)$ is an IFS in X. By (ii) $f(int(f^{-1}(B))) \subseteq pint(f(f^{-1}(B))) \subseteq pint(B)$. Now $int(f^{-1}(B)) \subseteq f^{-1}(f(int(f^{-1}(B)))) \subseteq f^{-1}(pint(B))$.
(iii) ⇒ (i) Let A be an IFOS in X. Then int(A) = A and f(A) is an IFS in Y. By (iii) $int(f^{-1}(f(A))) \subseteq f^{-1}(pint(f(A)))$. Now $A = int(A) \subseteq int(f^{-1}(f(A))) \subseteq f^{-1}(pint(f(A)))$. Therefore $f(A) \subseteq f(f^{-1}(pint(f(A)))) \subseteq pint(f(A)) \subseteq f(A)$. This implies pint(f(A)) = f(A) is an IFPOS in Y and hence an IFGPSOS in Y. Thus f is an IFGPSOM.

Theorem 3.25: A mapping f: $(X, \tau) \to (Y, \sigma)$ is an IFGPSOM if $f(pint(A)) \subseteq pint(f(A))$ for every $A \subseteq X$.

Proof: Let A be an IFOS in X. Then int(A) = A. Now $f(A) = f(int(A)) \subseteq f(pint(A)) \subseteq pint(f(A))$, by hypothesis. But $pint(f(A)) \subseteq f(A)$. Therefore f(A) is an IFPOS in Y. Then f(A) is an IFGPSOS in Y. Hence f is an IFGPSOM.

Theorem 3.26: A mapping f: $(X, \tau) \to (Y, \sigma)$ is an IFGPSOM if and only if $int(f^{-1}(B)) \subseteq f^{-1}(pint(B))$ for every IFS $B \subseteq Y$, where Y is an $IFPST_{1/2}$ space.

Proof: Necessity: Let B be an IFS in Y. Then $f^{-1}(B) \subseteq X$ and $int(f^{-1}(B))$ is an IFOS in X. By hypothesis, $f(int(f^{-1}(B)))$ is an IFGPSOS in Y. Since Y is an $IFPST_{1/2}$ space, $f(int(f^{-1}(B)))$ is an IFPOS in Y. Therefore $f(int(f^{-1}(B))) = pint(f(int(f^{-1}(B)))) \subseteq pint(B)$. This implies $int(f^{-1}(B)) \subseteq f^{-1}(pint(B))$.

Sufficiency: Let A be an IFOS in X. Therefore int(A) = A. Then $f(A) \subseteq Y$. By hypothesis $int(f^{-1}(f(A))) \subseteq f^{-1}(pint(f(A)))$. That is $int(A) \subseteq int(f^{-1}(f(A))) \subseteq f^{-1}(pint(f(A)))$. Therefore $A \subseteq f^{-1}(pint(f(A)))$. This implies $f(A) \subseteq pint(f(A)) \subseteq f(A)$. Hence f(A) is an IFPOS in Y and hence an IFGPSOS in Y. Thus f is an IFGPSOM.

Theorem 3.27: Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping where Y is an $IFPST_{1/2}$ space. Then the following statements are equivalent.

(i) f is an IFGPSCM,
(ii) $f(int(A)) \subseteq pint(f(A))$ for each IFS A of X,

(iii) $int(f^{-1}(B)) \subseteq f^{-1}(pint(B))$ for every IFS B of Y.

Proof: (i) ⇒ (ii) Let f be an IFGPSCM. Let A be any IFS in X. Then int(A) is an IFOS in X. Now f(int(A)) is an IFGPSOS in Y, by Theorem 3.23. Since Y is an $IFPST_{1/2}$ space, f(int(A)) is an IFPOS in Y. Therefore $pint(f(int(A))) = f(int(A))$. Now $f(int(A)) = pint(f(int(A))) \subseteq pint(f(A))$.
(ii) ⇒ (iii) Let B be an IFS in Y. Then $f^{-1}(B)$ is an IFS in X. By (ii) $f(int(f^{-1}(B))) \subseteq pint(f(f^{-1}(B))) \subseteq pint(B)$. Now $int(f^{-1}(B)) \subseteq f^{-1}(f(int(f^{-1}(B)))) \subseteq f^{-1}(pint(B))$.
(iii) ⇒ (i) Let A be an IFOS in X. Then int(A) = A and f(A) is an IFS in Y. By (iii) $int(f^{-1}(f(A))) \subseteq f^{-1}(pint(f(A)))$. Now $A = int(A) \subseteq int(f^{-1}(f(A))) \subseteq f^{-1}(pint(f(A)))$. Therefore $f(A) \subseteq f(f^{-1}(pint(f(A)))) \subseteq pint(f(A)) \subseteq f(A)$. Therefore pint(f(A)) = f(A) is an IFPOS in Y and hence an IFGPSOS in Y. Thus f is an IFGPSCM, by Theorem 3.23.

Theorem 3.28: A mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is an IFGPSCM if $f(pint(A)) \subseteq pint(f(A))$ for every $A \subseteq X$.

Proof: Let A be an IFOS in X. Then int(A) = A. Now $f(A) = f(int(A)) \subseteq f(pint(A)) \subseteq pint(f(A))$, by hypothesis. But $pint(f(A)) \subseteq f(A)$. Therefore f(A) is an IFPOS in Y. Then f(A) is an IFGPSOS in Y. Hence f is an IFGPSCM, by Theorem 3.23.

Theorem 3.29: If f: $(X, \tau) \rightarrow (Y, \sigma)$ is a mapping where X and Y are $IFPST_{1/2}$ space, then the following statements are equivalent:

(i) f is an IFiGPSCM,
(ii) f(A) is an IFGPSOS in Y for every IFGPSOS A in X,
(iii) $f(pint(B)) \subseteq pint(f(B))$ for every IFS B in X,
(iv) $pcl(f(B)) \subseteq f(pcl(B))$ for every IFS B in X.

Proof: (i) ⇒ (ii) is obvious.
(ii) ⇒ (iii) Let B be any IFS in X. Since pint(B) is an IFPOS, it is an IFGPSOS in X. Then by hypothesis, f(pint(B)) is an IFGPSOS in Y. Since Y is an $IFPST_{1/2}$ space, f(pint(B)) is an IFPOS in Y. Therefore $f(pint(B)) = pint(f(pint(B))) \subseteq pint(f(B))$.
(iii) ⇒ (iv) is obvious by taking complement in (iii).
(iv) ⇒ (i) Let A be an IFGPSCS in X. By Hypothesis, $pcl(f(A)) \subseteq f(pcl(A))$. Since X is an $IFPST_{1/2}$ space, A is an IFPCS in X. Therefore $pcl(f(A)) \subseteq f(pcl(A)) = f(A) \subseteq f(pcl(A))$. Hence f(A) is an IFPCS in Y and hence an IFGPSCS in Y. Thus f is an IFiGPSCM.

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