Anti Q-Fuzzy M-Subgroups Of Near Rings

A. Solairaju[1], P. Sarangapani[2] , R. Nagarajan[3] & P.Muruganantham[4]

[1]AssociateProfessor, PG & Research Department of Mathematics, Jamal Mohamed college, Triuchirappalli-20, Tamilnadu, India
[2]Assistant Professor, Department of Computer Science, Kurinji College of Arts & Science, Tiruchirappalli-02, Tamilnadu, India
[3]Associate Professor, Department of Mathematics, J J College of Engineering and Technology, Tiruchirappalli-09, Tamilnadu, India
[4]Assistant Professor, Department of Mathematics, Kurinji College of Arts & Science, Tiruchirappalli-02, Tamilnadu, India

Abstract: In this paper, we introduce the notion of Q-fuzzification of M-subgroups in a near-ring and investigate some related properties. Characterization of Anti Q-fuzzy M- subgroups with respect to s-norm is given.

Keywords: Q-fuzzy set, Q- fuzzy M-subgroup (sub near rings), anti Q-fuzzy M-subgroups, s-norm.

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Section - 1: Introduction

The theory of fuzzy sets which was introduced by Zadeh [9] is applied to many mathematical branches. Abou-zoid [1], introduced the notion of a fuzzy sub near-ring and studied fuzzy ideals of near-ring. This concept discussed by many researchers among Cho, Davvaz, Dudek, Jun, Kim [2]. In [5], considered the intuitionistic fuzzification of a right (respectively left) R-subgroup in a near-ring. A.Solairaju and R.Nagarajan [7] introduced the new structures of Q-fuzzy groups and then they investigate the notion Q-fuzzy left R-subgroups of near rings with respect to T-norms in [6]. Also Cho.at.al in [2] the notion of normal intuitionistic fuzzy R-subgroup in a near-ring is introduced and related properties are investigated. The notion of intuitionistic Q-fuzzy semi-primality in a semi group is given by Kim [3]. In this paper, we introduce the notion of Q- fuzzification of left M-subgroups in a near ring and investigate some related properties. Characterizations of anti Q-fuzzy M- subgroups are given.

Section-2 : Preliminaries

Definition 2.1: A non empty set R with two binary operations '+' and '.' is called a near-ring if it satisfies the following axioms

(i) (R,+) is a group.
(ii) (R,.) is a semi group.
(iii) $x.(y+z) = x.y + x.z$ for all $x,y,z \in R$. Precisely speaking it is a left near-ring. Because it satisfies the left distributive law.

As R – subgroup of a near- ring S is a subset H of R such that

(i) (H , +) is a subgroup of (R, +)
(ii) $RH \subset H$
(iii) $HR \subset H$. If H satisfies (i) and (ii) then it is called left N- subgroup of R and if N satisfies (i) and (iii) then it is called a right N- subgroup of R.

A map $f : R \to S$ is called homomorphism if $f(x+y) = f(x) + f(y)$ for all x, y in R.

Definition 2.2 : Let M is a left operator sets of group G, N is right operator sets of group G.

Definition 2.3 : Let G and G^1 both be M- groups. $f : G \to G^1$ be a homomorphism's, If $f(mx) = mf(x)$ and $f(x) = f(x)$ for all $x \in G$, $m \in M$, then f is called M -homomorphism.

Definition 2.4: Let R be a near ring. A fuzzy set 'μ' in R is called Q- fuzzy sub near ring in R if

(i) $\mu(x-y,q) \geq \min\{\mu(x,q), \mu(y,q)\}$

(ii) μ(xy,q) ≥ min{μ(x,q) , μ(y,q)} for all x, y in R.

Definition 2.5: A Q-fuzzy set μ is called a Anti Q-fuzzy M-subgroup of R over Q if μ satisfies

(i) μ(m(x-y), q) ≤ max{μ(mx,q), μ(my,q)}

(ii) μ(x,q) ≤ μ(x,q) for all x,y,m ∈ R and q ∈ Q.

Definition 2.6 : By a s-norm S , we mean a function S: [0,1]× [0,1]→ [0,1] satisfying the following conditions

(S1) S(x ,0) = x

(S2) S(x,y) ≤ S(x,z) if y ≤ z

(S3) S(x,y) = S(y,x)

(S4) S(x, S(y,z)) = S(S(x,y),z), for all x,y,z ∈ [0,1].

Proposition 2.1: For a s-norm, then the following statement holds

S(x,y) ≥ max{x,y}, for all x,y ∈ [0,1].

Definition 2.8: Let S be a s-norm. A fuzzy set A in R is said to be sensible with respect to S if Im (A) c Δs, where Δs = { s(α, α) = α / α ∈ [0,1] }

Section 3 : Properties of Anti Q- Fuzzy M-Subgroups

Proposition 3.1: Let S be a s-norm. Then every imaginable anti Q- fuzzy M- subgroup μ of a near ring R is a Q-fuzzy M-subgroup of R.

Proof: Assume μ is imaginable anti Q- fuzzy left M- subgroup of R, then we have

μ (m(x-y) , q) ≤ S{μ(mx,q), μ(my,q)}

and μ (x, q) ≤ μ (x,q) for all x,y in R.

Since μ is imaginable, we have

max{μ(mx,q) ,μ(my,q)} = S{max{μ(mx,q), μ(my,q)}, max{ μ(mx,q) , μ(my,q) }}

≥ S {μ(mx,q) , μ(my,q)}

≥ max {μ(mx,q) , μ(my,q)}

And so S{μ(mx,q) ,μ(my,q)} = max { μ(mx,q) , μ(my,q) }

It follows that μ(m(x-y), q) ≤ S{μ(mx,q) , μ(my,q)}

= max {μ(mx,q) , μ(my,q)} for all x,y ∈ R

Hence μ is a Q-fuzzy M- subgroup of R.

Proposition 3.2: If μ is anti Q-fuzzy M-subgroups of a near ring R and Θ is an endomorphism of R, then μ[Θ] is a anti Q- fuzzy M-sub group of R

Proof: For any x,y ∈ R, we have

(i) μ[Θ](m(x-y),q) = μ(Θ(m(x-y),q))

= μ (Θ(mx,q),Θ(my,q))

≤ S{μ(Θ(mx,q)),μ(Θ(my,q))}

= S{μ[Θ] (mx,q),μ[Θ](my,q)}

(ii) μ[Θ](x,q) = μ(Θ(x,q))

≤ μ(Θ(x,q))

≤ μ[Θ](x,q)

Hence μ[Θ] is a anti Q-fuzzy M-subgroup of R.

Proposition 3.3: An onto homomorphism's of anti Q- fuzzy M-subgroup of near ring R is anti Q-fuzzy M-subgroup.

Proof: Let f : R→ R^1 be an onto homomorphism of near rings and let ξ be anti Q-fuzzy M-subgroup of R^1 and μ be the pre image of ξ under f, then we have

(i) $\mu(m(x-y), q) = \xi(f(m(x-y),q))$
$= \xi(f(mx,q),f(my,q))$
$\leq S(\xi(f(mx,q)),\xi(f(my,q)))$
$\leq S(\mu(mx,q),\mu(my,q))$
(ii) $\mu(x,q) = \xi(f(x,q))$
$\leq \xi(f(x,q))$
$\leq \mu(x,q)$

Proposition 3.4: An onto homomorphic image of a anti Q-fuzzy M-subgroup with the **inf** property is anti Q-fuzzy M-subgroup.

Proof: Let $f: R \to R^1$ be an onto homomorphism of near rings and let μ be a **inf** property of anti Q-fuzzy M- subgroups of R.
Let $x^1, y^1 \in R^1$, and $x_0 \in f^{-1}(x^1)$, $y_0 \in f^{-1}(y^1)$ be such that
$\mu(x_0,q) = \inf_{(h,q) \in f^{-1}(x^1)} \mu(h,q)$, $\mu(y_0,q) = \inf_{(h,q) \in f^{-1}(y^1)} \mu(h,q)$
Respectively, then we can deduce that
(i) $\mu^f(m(x^1-y^1), q) = \inf_{(mz,q) \in f^{-1}(m(x^1-y^1),q)} \mu(mz,q)$
$\leq \max\{\mu(mx_0,q), \mu(my_0,q)\}$
$= \max\{\inf_{(h,q) \in f^{-1}(x^1,q)} \mu(mh,q), \inf_{(h,q) \in f^{-1}(y^1,q)} \mu(mh,q)\}$
$= \max\{\mu^f(mx^1,q), \mu^f(my^1,q)\}$
(ii) $\mu^f(x,q) = \inf_{(z,q) \in f^{-1}(r^1x^1,q)} \mu(z,q)$
$\leq \mu(y_0, q)$
$= \inf_{(h,q) \in f^{-1}(y^1,q)} \mu(h,q)$
$= \mu^f(y^1,q)$
Hence μ^f is a anti Q-fuzzy M-subgroup of R^1.

Proposition 3.5: Let S be a continuous s-norm and let f be a homomorphism on a near ring R. If μ is anti Q-fuzzy M-subgroup of R, then μ^f is anti Q-fuzzy M-subgroup of f(R).

Proof: Let $A_1 = f^{-1}(y_1,q)$, $A_2 = f^{-1}(y_2,q)$ and $A_{12} = f^{-1}(n(y_1-y_2), q)$ where $y_1,y_2 \in f(S)$, $q \in Q$ Consider the set
$A_1 - A_2 = \{ x \in S / (x,q) = (a_1,q) - (a_2,q) \}$ for some $(a_1,q) \in A_1$ and $(a_2,q) \in A_2$.
If $(x,q) \in A_1-A_2$, then $(x,q) = (x_1,q) - (x_2,q)$ for some $(x_1,q) \in A_1$ and $(x_2,q) \in A_2$ so that we have
$f(x,q) = f(x_1,q) - f(x_2,q)$
$= y_1 - y_2$
$(x,q) \in f^{-1}((y_1,q) - (y_2,q))$
$= f^{-1}(n(y_1-y_2), q) = A_{12}$
Thus $A_1-A_2 \subset A_{12}$
It follows that
(i) $\mu^f(m(y_1-y_2), q)$
$= \inf\{\mu(mx,q)/(mx,q) \in f^{-1}(my_1,q)-(my_2,q))\}$
$= \inf\{\mu(mx,q) / (x,q) \in A_{12}\}$
$\geq \inf\{\mu(mx,q)/ (x,q) \in A_1-A_2\}$
$\geq \inf\{\mu((mx_1,q)-(mx_2,q)) / (x_1,q) \in A_1 \text{ and } (x_2,q) \in A_2\}$
$\geq \inf\{S(\mu(mx_1,q), \mu(mx_2,q))/ (x_1,q) \in A_1 \text{ and } (x_2,q) \in A_2\}$
Since S is continuous. For every $\varepsilon > 0$, we see that if
$\inf\{\mu(mx_1,q) / (x_1,q) \in A_1\} - (mx_1^*, q) \} \geq \delta$ and
$\inf\{\mu(mx_2,q) / (x_2,q) \in A_2\} - (mx_2^*,q)\} \geq \delta$

$S\{\mathbf{inf}\{\mu(mx_1,q) / (x_1,q) \in A_1\}$, $\mathbf{inf}\{ \mu(mx_{2,}q) / (x_2,q) \in A_2 \}$ - $S((mx_1^*,q), (mx_2^*,q) \geq \varepsilon$
Choose $(a_1,q) \in A_1$ and $(a_2,q) \in A_2$ such that
$\mathbf{inf}\{\mu(mx_1,q) / (x_1.q) \in A_1\}$ - $\mu(ma_1,q) \geq \delta$ and
$\mathbf{inf}\{\mu(mx_2,q) / (x_2,q) \in A_2\}$ - $\mu(ma_2,q) \geq \delta$.
Then we have
$S\{\mathbf{inf}\{\mu(mx_1,q)/(x_1,q)\in A_1\}, \mathbf{inf}\{\mu(mx_2,q)/(x_2,q) \in A_2\} - S(\mu(ma_1,q),\mu(ma_2,q) \geq \varepsilon$
consequently, we have $\mu^f(m(y_1\text{-}y_{),}\ q) \leq \mathbf{inf}\{ S(\mu(mx_1,q), \mu(x_2,q)) / (x_1,q) \in A_1 ,(x_2,q) \in A_2\}$
$\leq S(\mathbf{inf}\{\mu(mx_1,q) / (x_1,q) \in A_1\}, \mathbf{inf}\{\mu(mx_2,q) / (x_2,q) \in A_2\}$
$\leq S\{(\mu^f(my_1,q) , \mu^f(my_2,q)\}$
Similarly we can show $\mu^f(x,q) \leq \mu^f(y,q)$
Hence μ^f is anti Q-fuzzy M-subgroup of f(R).

Proposition 3.6: Let μ be anti Q-fuzzy M-subgroup of R. Then the Q-fuzzy subset $<\mu>$ is a anti Q-fuzzy M-subgroup of S generated by μ. More over $<\mu>$ is the smallest anti Q-fuzzy M-subgroup containing μ.

Proof: Let $x,y \in N$ and let $\mu(x,q) = t$, $\mu(y,q) = t_2$ and $\mu(m(x\text{-}y),q) = t$
Let it possible $t = <\mu>(m(x\text{-}y),q) \geq S\{<\mu>(mx,q),<\mu>(my,q)\}$
$S\{t_1,t_2\} = t_1$ (say)
Then $t_1 = <\mu>(mx,q) = \mathbf{inf}\{k/x \in <\mu_k> \} \leq t$, therefore there exist k_1, such that $x \in <\mu_{k1}>$.
Also $t_2 = <\mu>(my,q) = \sup\{k/y \in <\mu_k> \}\ t_1 \leq t$. Therefore there exists $k_2 \leq t$ such that $y \in <\mu_k>$ without loss of generality, we may assume that $k_1\ k_2$, so that $<\mu_{k1}> \subset <\mu_{k2}>$. Then $x,y \in <\mu_k>$ that is x-y which is a contradiction since $k_2 \leq t$. therefore $t \leq t_1$.
Consequently, $\mu(m(x\text{-}y),q) \leq S\{<\mu>(mx,q) ,<\mu>(my,q)\}$ ------ (1)
Now let , if possible, $t_3 = <\mu> (x,q) \leq <\mu> (x,q) = t_1$
Then $t_1 = <\mu> (x,q) = \mathbf{inf}\ \{k / x \in <\mu_k>\} \leq t_3$, therefore there exists k such that $x \in <\mu_k>$
and $t_1 \leq k \leq t_3$ so that $x \in <\mu_k> \subset \{\mu_{t1}\}$ which is a contradiction.
Hence $t_3 = <\mu> (x,q) \leq <\mu> (x,q) = t_1$ ------ (2)
Consequently conditions (1) and (2) yield that $<\mu>$ is a anti Q- fuzzy M- subgroup of R. Finally, to show that $<\mu>$ is the smallest anti Q-fuzzy M-subgroup containing μ, let us assume that θ to be anti Q-fuzzy M-subgroup of R such that $\mu \subset \theta$ and show that $<\mu> \subset \theta$.
Let it possible, $t = <\mu> (x,q) \geq \theta (x,q)$ for some $x \in N$, $q \in Q$. Let $\varepsilon > 0$ be given, then
$t = \mu_t = \sup \{ k / x \in <\mu_k>\}$. Therefore there exists k such that $x \in<\mu_k>$ and $t\text{-}\varepsilon \geq k \geq t$ so that $x \in <\mu_k> \subset <\mu_t\text{-}\varepsilon >$, for all $\varepsilon >0$.
Now $x = ά_1x_1 + ά_2\ x_2+ \ldots\ldots\ldots ά_nx_n$, $ά_i \in N$, x_i belongs to t-ε. $x_i\ \varepsilon\ \mu_{t\text{-}\varepsilon}$ implies $\mu (x_i,q) \leq t\text{-}\varepsilon$, that is $\theta(x_i,q) \leq t\text{-}\varepsilon$ for all $\varepsilon > 0$.
Therefore $\theta(x,q) \leq S \{\theta(x_1,q) ,\theta(x_2,q) \ldots \theta(x_n,q)\}$
$\leq t\text{-}\varepsilon$ for $\varepsilon > 0$
Hence $\theta(x, q) = t$ which is a contradiction to our assumption.

Proposition3.7: Let μ be a anti Q-fuzzy M-subgroup of a near ring R and let μ^* be a Q-fuzzy set in N defined by $\mu^*(x,q) = \mu(x,q) +1\text{-}\mu(0,q)$ for all $x \in N$. Then μ^* is a normal anti Q-fuzzy M-subgroup of R containing μ.

Proof : For any $x, y \in R$ and $q \in Q$ we have
$\mu^*(m(x\text{-}y),q) = \mu(m(x\text{-}y),q) +1 - \mu(0,q) \leq S(\mu(mx,q)+1\text{-}\ \mu(0,q), (\mu(my,q)+1\text{-}\ \mu(0,q))$
$= T (\mu^*(mx,q) , \mu^*(my,q))$.
$\mu^*(x,q) = \mu(x,q) +1 - \mu(0,q)$
$\leq \mu(x,q) +1\text{-}\ \mu(0,q)$
$= \mu(x,q)$

Proposition 3.8: Let μ be anti Q- fuzzy M- subgroup of near ring R. Let μ^+ be a fuzzy α–cut set in R defined by $\mu^+(x,q) = \mu(x,q) +1\text{-}\ \mu(0,q)$ for $x \in R$, $q \in Q$. Then μ^+ is α- cut normal anti Q-fuzzy M- subgroup of R which contains μ.

Proof: For any $x,y \in R$, we have $\mu^+(x,q) + 1 - \mu(0,q)$ and $\mu^+(x,q) \leq \alpha$ for all $x \in R$, $m \in M$.
$\mu^+(m(x\text{-}y), q) = \mu (m(x\text{-}y), q) +1 - \mu(0,q)$

$\leq \max\{\mu(m_x,q), \mu(m_y,q)\} + 1 - \mu(0,q)$
$= \max\{\mu(m_x,q)+1-\mu(0,q),\mu(m_y,q)+1-\mu(0,q)\}$
$= \max\{\mu^+(m_x,q), \mu^+(m_y,q)\}$
$\leq \max\{\alpha, \alpha\} \leq \alpha$
$\mu^+(x,q) = \mu(x,q) + 1 - \mu(0,q)$
$\leq \mu(x,q) + 1 - \mu(0,q)$
$= \mu^+(x,q)$
$\leq \alpha$
Therefore, μ^+ is a α-cut normal Anti Q-fuzzy M-subgroup of R.

Definition 3.9: Let u and v be Q-fuzzy subsets in R. Then the S-product of u and v written as $[u,v]_S(x,q) = S(u(x,q),v(x,q))$ for all $x \in R$, $q \in Q$.

Proposition 3.10 : If u and v be Anti Q-fuzzy M-subgroups of R, then the S-product of Anti Q-fuzzy M-subgroups of R is Anti Q-fuzzy M-subgroups of R.

Proof: For any $x,y \in R$, $q \in Q$
$[u,v]_S(m(x-y),q) = S\{u(m(x-y),q),v(m(x-y),q)\}$
$\leq S\{\max\{u(m_x,q),u(m_y,q)\},\max\{v(m_x,q), v(m_y,q)\}\}$
$\leq \max\{S\{u(m_x,q),v(m_x,q)\},S\{v(m_y,q), v(m_y,q)\}\}$
$\leq \max\{[u,v]_S(m_x,q),[u,v]_S(m_y,q)\}$
$[u,v]_S(x_n,q) = S\{u(x_n,q), v(x_n,q)\}$
$\leq S\{u(x,q), v(x,q)\}$
$\leq [u,v]_S(x,q)$
Hence S-product of Anti Q-fuzzy M-subgroups of R is Anti Q-fuzzy left M-subgroups of R.

Definition3.11: Anti Q-fuzzy M- subgroup near ring R is said to Anti Q-fuzzy characteristic, if $A^f(x,q) = A(x,q)$ for all $x \in R$, $q \in Q$.

Proposition 3.12 : Let $f : R\rightarrow R'$ be an epimorphism of A, A is an anti Q-fuzzy M-subgroups of R then A^f is anti Q-fuzzy M-subgroups of R'.
Proof: Let $x,y \in R$ and $q \in Q$
$A^f(m(x-y),q) = A\, f(m(x-y),q)$
$= A\,(f(mx) - f(my), q)$
$\leq \max\{A\,(f(mx),q), A\,(f(my),q)\}$
$\leq \max\{A^f(mx,q), A^f(my,q)\}$
$A^f(x,q) = Af(x,q)$
$\leq Af(x,q)$
$\leq A^f(x,q)$
Therefore, A^f is anti Q-fuzzy M-N subgroup of R'.

Proposition 3.13 : Let $f : R\rightarrow R'$ be epimorphism. If A^f is anti Q-fuzzy M-subgroup of R', then A is anti Q-fuzzy M-subgroup of R.

Proof: Let $x,y \in R$, $q \in Q$, then there exists $a,b \in X$ such that $f(a,q) = x$ and $f(b,q) = y$.
It follows that $A(x,q) = A\, f(a,q) = A^f(a,q)$
$A(m(x-y),q) = A\, f(a,q) = A^f(a,q) \leq \max\{A^f(a,q), A^f(b,q)\}$
$= \max\{A(a,q), A(b,q)\}$
$\leq \max\{A(x,q), A(y,q)\}$
$A(xn,q) = A\, f(a,q) = A^f(a,q) \leq A\, f(a,q) \leq A\, f(x,q)$
Therefore A is anti Q-fuzzy M- subgroup of R.

Conclusion: Osman kazanci , Sultanyamark and Serifeyilmaz introduced the intutionistic Q-fuzzy R-subgroups of near rings. A.Solairaju and R.Nagarajan investigate the notion of Q-fuzzy left R-subgroup of near rings with respect to T-norms. In this paper we investigate the notion of anti Q-fuzzy M-subgroup of near ring with respect to s-norm and the characterization of them.

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