MAX-NORM Q-FUZZY INTERVAL VALUED SUBGROUPS OF NEAR RINGS

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Abstract: In this paper we introduce the notion of Interval Max norm Q-fuzzy R-subgroup of near rings and investigate some of their properties. Using Lower level set, we give a characterization of Max^i Q-fuzzy right R- subgroup. Finally we establish the idea of the homomorphic image and the inverse image.

Key words: Interval number, Interval max norm, Q-fuzzy set, Homomorphism , Lower level cut, near rings.

Section 1: Introduction

Zadeh [18] made an extension of the concept of fuzzy sets by an interval valued set.. Abou-zoid [1] , introduced the notion of a fuzzy sub near-ring and studied fuzzy ideals of near-ring. The notion of intuitionistic Q- fuzzy semi primality in a semi group is given by Kim [3]. Roy and Biswas [12] studied the interval valued fuzzy relations and applied these in seaxhez's approach for medical diagnosis, and Jun and Kim [5] discussed interval valued fuzzy subalgebra's in BCK'S algebra's. Gor. Zalczany[2] studied the interval valued fuzzy sets for approximate reasoning. A.Solairaju and R.Nagarajan introduced the concept of Structures of Q- fuzzy groups [[13],[14],[15],[16],[17]]. In this paper, we introduce the notion of Interval Max norm Q-fuzzy R-subgroup of near rings and investigate some of their properties. Using Lower level set, we give a characterization of Max^i Q-fuzzy right R- subgroup. Finally we establish the idea of the homomorphic image and the inverse image.

Section 2: Preliminaries

We first recall some basic concept which are used to present the paper.

An interval number on [0,1], say $\bar{a}$ is a closed subinterval of [0,1], (ie) $\bar{a}=[a^-,a^+]$ where $0 \le a^- \le a^+ \le 1$.

For any interval numbers $\bar{a} = [a^-, a^+]$ and $\bar{b}= [b^-, b^+]$ on [0,1] , we define

(i) $\bar{a} \le \bar{b}$ if and only if $\bar{a} \le \bar{b}$ and $a^+ \le b^+$

(ii) $\bar{a} = \bar{b}$ if and only if $\bar{a} = \bar{b}$ and $a^+ = b^+$

(iii) $\bar{a} + \bar{b} = [a^- + b^-, a^+ + b^+]$, whenever $\bar{a}+ \bar{b} \le 1$ and $a^+ + b^+ \le 1$

Let X be a set. A mapping $A : X \rightarrow [0,1]$ is called a fuzzy set in X. Let A be a fuzzy set in X and $\alpha \in [0,1]$. Define L(A : α) as follows

$L(A: \alpha) = \{ x \in X / A(x) \le \alpha\}$. Then L (A:$\alpha$) is called the Lower level cut of A.

Let X be a set. A mapping $\bar{A} : X \rightarrow D[0,1]$ is called on interval-valued fuzzy set (briefly i-v fuzzy set) of X, where D[0,1] denotes the family of all closed sub intervals of [0,1], and $\bar{A}(x) = [A^-(x), A^+(x)]$, $\forall x \in X$, where $\bar{A}$ and A^+ are fuzzy sets in X.

For an i-v fuzzy set $\bar{A}$ of a set X and $(\alpha, \beta) \in D[0,1]$ define $L(\bar{A}: [\alpha, \beta])$ as follows

$\bar{L}(A:[\alpha, \beta]) = \{x \in X / \bar{A}(x) \le [\alpha,\beta]\}$which is called the Level sub set of $\bar{A}$.

Section 3: Interval Max-Norm Q-fuzzy R-sub groups.

The notion of an Interval Max-Norm was introduced by Jun and Kim as follows.

Definition 3.1: A mapping

$Max^i : D[0,1] \times D[0,1] \rightarrow D[0,1]$ given by

$Max^i(\bar{a}, \bar{b})=[\max (\bar{a},\bar{b}), \max(a^+,b^+)]$, $\forall \bar{a},\bar{b} \in D[0,1]$ is called an Interval Max-Norm.

Proposition 3.1: Let Max^i be an Interval Max-Norm on D[0,1] then

(i) $Max^i (\bar{a}, \bar{b})= \bar{a}$, $\forall \bar{a} \in D [0,1]$

(ii) $Max^i (\bar{a}, \bar{b})= Max^i (\bar{b} , \bar{a})$, $\forall \bar{a}, \bar{b} \in D [0,1]$

(ii) If $\bar{a} \le \bar{b}$ in D[0,1], then $Max^i(\bar{a},\bar{c}) \le Max^i (\bar{b}, \bar{c})$, $\forall \bar{c} \in D [0,1]$

Definition 3.2 : An interval valued fuzzy set $\bar{A}$(i-v) is a near ring R is called an interval valued right (respectively left) R-subgroup of R with respect to the interval Max-norm Max^i (briefly a Max^i Q-fuzzy Right (respectively left)) R-subgroup if

(IVR1): $\bar{A}(x-y, q) \leq Max^i(\bar{A}(x, q), \bar{A}(y, q))$

(IVR2): $\bar{A}(xr,q) \leq \bar{A}(x)$ (respectively)

$\bar{A}(rx) \geq \bar{A}(x)$ for all x, y, r $\in$ R.

Example 3.1: Let R={a,b,c,d} be a set with two binary operators as follows

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	b	c	d

Then (R,+,.) is a near ring we define a Q-fuzzy set $\bar{A}:R\times Q\rightarrow[0,1]$ by

$\bar{A}(c,q)= \bar{A}(d,r)=[0.3,0.4] > \bar{A}(b,q)=[0.2,0.3] > \bar{A}(a,q)=[0.1,0.2]$, then $\bar{A}$ is a Max^i Q-fuzzy Right R-subgroup of R.

Proposition 3.2 : If $\bar{A}$ is a Max^i Q-fuzzy Right (respectively left) R-subgroup of a near ring R, then $\bar{A}(0, q) \leq \bar{A}(x, q)$ for all $x \in R$, $q \in Q$.

Proof: For every $x \in R$, $q \in Q$,

$\bar{A}(0, q) = \bar{A}(x-x, q) \leq Max^i\{(\bar{A}(x, q), \bar{A}(x, q)\}$

$= [max((A^-(x, q), A^-(x, q)),$

$max((A^+(x, q),A^+(x, q))]$

$=[A^-(x,q), A^+(x,q)]=[A(x,q)]$

completing the proof. In what follows, the notion of Q-fuzzy (resp. Max^i Q-fuzzy) R subgroup means the notion of Q-fuzzy (rsp. Max^i Q-fuzzy)right R-subgroup.

Proposition 3.3: If $\{\bar{A}_i / i\in\wedge\}$ is a family of Max^i Q-fuzzy R-subgroups of a near-ring R, Then so is $\wedge\bar{A}_i$ where $\wedge$ is any index set.

Proof: Let $x, y \in R$, $q\in Q$, we have $(\wedge \bar{A}_i)(x-y,q)=\sup\{\bar{A}_i(x-y, q): i\in\wedge\}$

$\leq \sup\{Max^i\{\bar{A}_i(x,q), \bar{A}_i(y,q)\} : i\in\wedge\}$

$=Max^i\{\sup\{\bar{A}_i(x,q):i\in\wedge\},\sup\{ \bar{A}_i(y,q): i\in\wedge \}\}$

$=Max^i\{ (\wedge\bar{A}_i)_{i\in\wedge}(x,q), (\wedge \bar{A}_i)_{i\in\wedge}(y,q)\}$

And for every r, x $\in$ R ,we have $(\wedge \bar{A}_i)(xr,q)=\sup\{\bar{A}_i(xr,q) : i\in\wedge\}$

$\leq\sup \{\bar{A}(x r, q) : i\in\wedge\}$

$=(\wedge\bar{A}_i)(x,q)$

Hence $\wedge\bar{A}_i$ is a Max^i Q-fuzzy R-subgroup of R.

Proposition 3.4: Let R be a near-ring. An i-v Q-fuzzy set $\bar{A}$ in R Max^i Q-fuzzy R-subgroup of R if and only if A^+ and A^- are Q-fuzzy R-subgroup of R.

Proof: Assume that A^+ and A^- are Q-fuzzy R-subgroup of R and let x,y$\in$R and q$\in$Q. Then

$\bar{A}(x- y, q) = [A^-(x-y, q), A^+(x-y, q)]$

$\leq Max^i \{(\bar{A}(x , q), \bar{A}(y ,a)\} , Max^i\{A^+(x,q), A^+(y,q)\}$

$= Max^i (\bar{A}(x, q), \bar{A}(y, q))$

and for x, r$\in$ R

$\bar{A}(xr, q) = [A^-(xr, q), A^+(xr,q)]$

$\leq [A^-(x, q), A^+(x, q)] =\bar{A}(x, q)$

Here $\bar{A}$ is a Max^i Q-fuzzy R-subgroup of R.

Conversely,

$[A^-(x-y,q),A^+(x-y,q)]=\bar{A}(x-y,q)\leq Max^i(\bar{A}(x, q),\bar{A}(y,q))$

$= Max^i ([A^-(x,q), A^+(x, q)], [A^-(y, q), A^+(y, q)]) \quad = [Max (A^-(x,q),(A^-(y, q)) , Max(A^+(x ,q), A^+(y ,q))]$

It follows that $A^{-}(x\text{-}y,q) \leq$ Max($A^{-}(x,q)$, $A^{-}(y,q)$) and

$$A^{+}(x\text{-}y,q) \leq \text{Max}(A^{+}(y,q), A^{+}(y,q))$$

For any xr $\in$ R we have

$[A^{-}(xr,q),A^{+}(xr,q)]=\bar{A}(xr,q)\leq\bar{A}(x,q)=[A^{-}(x,q),A^{+}(x,q)]$

And So $A^{-}(xr,q) \leq A^{-}(x,q)$ and $A^{+}(xr,q) \leq A^{+}(x,q)$

How A^{-} and A^{+} are Q-fuzzy R subgroups of R.

Proposition 3.5: Every R subgroup of a near-ring R can be realized as an lower level R subgroup of a Max^{i} Q-fuzzy R subgroup of R.

Proof: Let H be an R subgroup of a near-ring R and let $\bar{A}$ be on i-v Q-fuzzy set in R defined by

$$\bar{A}(x,q) = \begin{cases} \bar{a} & \text{if } x \in H \\ \bar{0} & \text{otherwise} \end{cases}$$

Where $\bar{a}$ $(\neq \bar{0})$ $\in$ D[0,1]. It is clear that $\bar{L}(\bar{A},\bar{a}) = H$. We will show that A is a Max^{i} Q-fuzzy R-subgroup of R. If x,y$\in$ H, then x-y$\in$H and so

$\bar{A}(x\text{-}y,q) = \bar{a} = \text{Max}^{i}(\bar{a},\bar{a}) = \text{Max}^{i}(\bar{A}(x,q), \bar{A}(x,q))$.

If x,y does not belongs to H ,then $\bar{A}(x,q)= \bar{0} = \bar{A}(y,q)$ and thus

$\bar{A}(x - y ,q)\leq = \text{Max}^{i}(\bar{0} , \bar{0})= \text{Max}^{i}(\bar{A}(x,q), \bar{A}(x,q))$

Suppose that the only one of x, y belongs to H, say x. then

$\bar{A}(x - y)\leq \bar{0}= \text{Max}^{i}(\bar{a},0) = \text{Max}^{i}(\bar{A}(x,q), \bar{A}(y,q))$. Now if x$\in$ R/H , then $\bar{A}(x, q)=\bar{0}$ and so $\bar{A}(xr,q) \leq \bar{0}= \bar{A}(x)$ for all r$\in$R. If x$\in$H, then xr$\in$H which implies that $\bar{A}(xr,q)= \bar{a} = \bar{A}(x ,q)$ for all r$\in$ R. Hence $\bar{A}$ is a Max^{i} Q-fuzzy R-subgroups of R.

Proposition 3.6: Let H be a subset of a near-ring R. A function $\tau_H : R \times Q \to D[0,1]$ Defined by

$$\tau_H(x,q) = \begin{cases} \overline{1} & \text{if } x \in H \\ \overline{0} & \text{otherwise} \end{cases}$$

for all $x \in R$ is a Max^i Q-fuzzy R-subgroup of R. If and only if H is an R-subgroup of R.

Proof: Let H be an R-subgroup. Using the same inference of Proposition 3.4.

We know that τ_H is a Max^i Q-fuzzy R-subgroup of R. Conversely, suppose that τ_H is a

Max^i Q-fuzzy R-subgroup of R.

Let $x,y \in H$ then $\tau_H(x,q) = 1 = \tau_H(y,q)$, and so

$\tau_H(x-y,q) \leq Max^i\{\tau_H(x,q),\tau_H(y,q)\} = Max^i(1,1) = 1.$

It follows that $\tau_H(x-y,q) = 1$ so that $x-y \in H$.

Let $r \in R$ and $x \in H$ then we have $\tau_H(xr,q) \leq \tau_H(x,q) = 1$ and So $xr \in H$. Hence H is an R-subgroup of R.

Proposition 3.7: If $\bar{A}$ is a Max^i Q-fuzzy R-subgroup of R, then the set

$R_{\bar{A}} := \{ x \in R / \bar{A}(x,q) = \bar{A}(0,q)\}$ is an R-subgroup of R.

Proof: Let $x, y \in R_{\bar{A}}$ and $q \in Q$ then

$\bar{A}(x,q) = \bar{A}(0,q) = \bar{A}(y,q)$ and so

$\bar{A}(x-y,q) < Max^i(\bar{A}(x,q),\bar{A}(y,q)) = Max^i(\bar{A}(0,q),\bar{A}(0,q)$

$= \bar{A}(0,q)$

It follows from Proposition 1.1 that $\bar{A}(x-y,q) = \bar{A}(0,q)$ So that $x-y \in R_{\bar{A}}$.

Let $r \in R$ and $x \in R_{\bar{A}}$ then we have

$\bar{A}(xr,q) \leq \bar{A}(x,q) = \bar{A}(0,q)$.

Hence $\bar{A}(xr,q) = \bar{A}(x,q)$ and so $xr \in R_{\bar{A}}$.

Definition 3.3: Let R and R' be near-rings. A map $f:R\rightarrow R'$ is called a (near-ring) homomorphism if $f(x+y) = f(x)+f(y)$ and $f(xy)= f(x) f(y)$ for all $x, y \in R$.

Let X and Y be sets. A mapping $f : X\rightarrow Y$ induces two mappings.

$F_f : IF(x)\rightarrow IF(y)$ and $F_f^{-1} : IF(y)\rightarrow IF(x)$

We define $F_f(\bar{A})(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \bar{A}(x) & f^{-1}(y) \neq 0 \\ \bar{0} \text{ otherwise} \end{cases}$

and $F_f^{-1}(\bar{B})(x) = \bar{B}f(x)\ \forall x \in X$ where $\bar{A}$ = IF(x) , $\bar{B}$ IF(y) and

$F^{-1}(y) = \{x \in X / f(x) = y\}$

Proposition 3.8: An i-v Q-fuzzy set $\bar{A}$ is a near-ring R is a Max^i Q-fuzzy R-subgroup of R if and only if the non-empty lower level set $\bar{L}(\bar{A}:[\alpha, \beta])$ is an R-subgroup of R for $\alpha,\beta \in D[0,1]$

Proof: Assume that $\bar{A}$ is a Max^i Q-fuzzy R-subgroup of R and Let $[\alpha, \beta]\in D[0,1]$ be such that

$x, y\in L(\bar{A}:[\alpha, \beta])$ Then $\bar{A}(x-y,a)\leq Max^i(\bar{A}(x,q),\bar{A}(y,q))$

$\leq Max^i([\alpha,\beta],[\alpha,\beta])=[\alpha, \beta]$,

and so $x-y \in \bar{L}(\bar{A};[\alpha,\beta])$, let $r\in R$ Then we have

$\bar{A}(xr,q) \leq \bar{A}(x,q) = [\alpha,\beta]$ for every $x \in \bar{L}(\bar{A};[\alpha,\beta])$ and so $xr\in\bar{L}(\bar{A};[\alpha, \beta])$

Thus $\bar{L}(\bar{A};[\alpha,\beta])$ is on R-subgroup of R.

Conversely, Assume that $\bar{L}(\bar{A};[\alpha,\beta])(\neq 0)$ is an R-subgroup of R for every $[\alpha,\beta]\in D[0,1]$.

Suppose that there exist $x_0, y_0 \in R$ such that $\bar{A}(x_0 - y_0, q) > Max^i(\bar{A}(x_0,q), \bar{A}(y_0,q))$

Let $\bar{A}(x_0,q) = [\alpha_1, \beta_1]$, $\bar{A}(y_0,q) = [\alpha_2, \beta_2]$ and $\bar{A}(x_0 - y_0,q) = [\sigma_1, \sigma_2]$

Then $[\sigma_1, \sigma_2] > Max^i([[\alpha_1,\beta_1],[[\alpha_2,\beta_2]) = [Max^i(\alpha 1,\alpha 2), Max^i([\alpha_2,\beta_2)]$

Using the definition of the interval numbers with respect to the order '>'without loss of generality we may assume that $\delta_1 > \max(\alpha_1,\alpha_2)$ and $\delta_2 > \max(\beta_1,\beta_2)$,

Let $[\lambda_1, \lambda_2] = ½(\bar{A}(x_0-y_0,q) + Max^i(\bar{A}(x_0,q), \bar{A}(y_0, q))$

Then $[\lambda_1, \lambda_2] = ½([\delta_1, \delta_2] + [\max(\alpha_1, \alpha_2).\min(\beta_1, \beta_2)]) = [½(\delta_1 + \max(\alpha_1,\alpha_2)), ½(\delta_2 + \max(\beta_1, \beta_2))]$

It follows that $\max(\alpha_1,\alpha_2) < \lambda_1 = ½(\sigma_1 + \max(\alpha_1,\alpha_{2)}) < \delta_1$

$\max(\beta_1, \beta_2) < \lambda_2 = ½(\sigma_2 + \max(\beta_1, \beta_2)) < \delta_2$

So that

$[\min(\alpha_1,\alpha_2),\min(\beta_1,\beta_2)] < [\lambda_1,\lambda_2] < [\delta_1,\delta_2] = \bar{A}(x_0-y_0,q)$

Therefore $x_0,y_0 \in \bar{L}(\bar{A}:[\lambda_1,\lambda_2])$

On other hand, noticing that

$\max(\alpha_1,\alpha_2) \geq \alpha_1$, $\max(\alpha_1,\alpha_2) \geq \alpha_2$,

$\max(\beta_1, \beta_2) \geq \beta_1$, $\max(\beta_1, \beta_2) \geq \beta_2$ and we get

$\bar{A}(x_0,q) = [\alpha_1,\beta_1] \leq [\max(\alpha_1,\alpha_2),\max(\beta_1, \beta_2)] < [\lambda_1,\lambda_2]$

And

$\bar{A}(y_0,q) = [\alpha_2,\beta_2] \leq [\max(\alpha_1,\alpha_2),\max(\beta_1, \beta_2)] < [\lambda_1,\lambda_2]$

And so $x_0,y_0 \in \bar{L}(\bar{A};[\lambda_1,\lambda_2])$ It contradicts that $\bar{L}(\bar{A};[\lambda_1,\lambda_2])$ is on R-subgroup of R. Also suppose that there exists $x_0,y_0 \in R$ such that $\bar{A}(x_0 r_0, q) > \bar{A}(x_0,q)$

Let $\bar{A}(x_0,q)=[\alpha_1,\beta_1]$ and $\bar{A}(x_0r_0, q)=[\delta_1, \delta_2]$

Then $[\delta_1, \delta_2] > [\alpha_1,\beta_1]$

Let $[\lambda_1,\lambda_2] =½(\bar{A}(x_0r_0, q)+\bar{A}(x_0,q))$

Then $[\lambda_1,\lambda_2] = ½([\delta_1, \delta_2]+[\alpha_1,\beta_1])=½(\delta_{1+} \alpha_1 , \delta_{2+} \beta_1)$

it follows that $\alpha_1 < \lambda_1 = ½(\delta_{1+} \alpha_1) < \delta_1$ and

$$\beta_1 < \lambda_2 = ½(\delta_{2+} \beta_1) < \delta_2$$

So that $[\alpha_1,\beta_1] < [\lambda_1,\lambda_2] < [\delta_1, \delta_2]=\bar{A}(x_0r_0,q)$

Therefore $x_0r_0 \in \overline{L}(\bar{A};[\lambda_1,\lambda_2])$. On the other hand ,we get $\bar{A}(x_0, q)=[\alpha_1,\beta_1]<[\lambda_1,\lambda_2]$ and so $x_0 \in \overline{L}(\bar{A};[\lambda_1,\lambda_2])$. It contradicts that $\overline{L}(\bar{A};[\lambda_1,\lambda_2])$ is an R-subgroup of R. Hence $\bar{A}$ is a Max^i Q-fuzzy R-subgroup of R. This completes the proof.

Proposition 3.9: Let f: R→R' be an onto homomorphism of near-ring. If $\bar{A}$ is a Max^i Q-fuzzy R-subgroup of R. Then $F_f(\bar{A})$ is a Max^i Q-fuzzy R-subgroup of R.

Proof: For any $y_1, y_2 \in R'$, let $S_1=f^{-1}(y_1)$

$S_2=f^{-1}(y_2)$ and $S_{12}=f^{-1}(y_1-y_2)$

Consider the set

$S_1 - S_2 = \{ x\in R / x=a_1 - a_2 \text{ for some } a_1\in S,a_2 \in S\}$

If $x \in S_1 - S_2$, then $x = x_1- x_2$ for $x_1 \in S$ and $x_2 \in S_2$ and so $f(x) =f(x_1- x_2) = f(x_1) –f(x_2) = y_1- y_2$

(ie) $x \in f^{-1}(y_1- y_2,q) = S_{12}$. Thus $S_1 - S_2 \subseteq S_{12}$ it follows that

$$F_f(\bar{A}) (y_1- y_2,q) = \sup_{x \in f^{-1}(y_1- y_2)} \bar{A}(x ,q)$$

$= \textbf{Sup}\ \bar{A}(x ,q)$

$x \in S_{12}$

$\leq \textbf{Sup}\ \bar{A}(x,q)$

$x \in S_1 - S_2$

$= \textbf{Sup}\ \bar{A}(x_1 - x_2 ,q)$

$x \in S,\ x_2 \in S2$

$< \textbf{Sup}\ Max^i\ (\bar{A}(x_1,q) ,\bar{A}(x_2 ,q))$

$x_1 - x_2 \in S$

$< Max^i\ (\textbf{Sup}\ \bar{A}(x_1,q) , \textbf{Sup}\ \bar{A}(x_2 ,q))$

$x_1 \in S \qquad x_2 \in S$

$= Max^i\ (F_f(\bar{A})(y_1,q),F_f(\bar{A})(y_2 ,q))$

Also, for any $x,r \in R$. Let $T_1 = f^{-1}(x)$, $T_2 = f^{-1}(r)$ and $T_{12} = f^{-1}(xr)$. Consider the set

$T_1T_2 = \{y \in R/\ y = t_1t_2 \text{ for } t_1 \in T_1 \text{ and } t_2 \in T_2\}$. If $y \in T_1T_2$, then

$y = x'r'$ for some $x' \in T_1$ and $r' \in T_2$ and so

$F(y) = f(x'r') = f(x')f(r') = xr$

(ie) $y \in f^{-1}(x_2) = T_{12}$ thus $T_1T_2 \subseteq T_{12}$

It follows that $F_f(\bar{A})(xr,q) = \textbf{Sup}\ \bar{A}(y ,q)$

$y \in f^{-1}(xr)$

$$= \mathbf{Sup}_{y \in T_{12}} \bar{A}(y,q)$$

$$< \mathbf{Sup}_{y \in T_1T_2} \bar{A}(y,q)$$

$$= \mathbf{Sup}_{x' \in T_1, r' \in T_2} \bar{A}(x'r',q)$$

$$\leq \mathbf{Sup}_{x' \in T_1 = f^{-1}(x)} \bar{A}(x',q)$$

$$= F_f(\bar{A})(x,q)$$

Similarly , $F_f(\bar{A})(r'x') \leq Ff^{-1}(\bar{A})(x,q)$

Proposition 3.10: Let f: R→ R' be a homomorphism of near rings. If B is a Max^i Q-fuzzy R-subgroup of R ,then $F_f^{-1}(\overline{B})$ is a Max^i Q-fuzzy R subgroup of R.

Proof: Let x,y∈R, then $F_f^{-1}(\overline{B})(x-y,q) = \overline{B}(f(x-y,q))$

$$= \overline{B}(f(x,q)-f(y,q)) \leq Max^i(\overline{B}(f(x,q),\overline{B}(f(y,q))$$

$$= Max^i(F_f^{-1}(\overline{B})(x,q), F_f^{-1}(\overline{B})(y,q))$$

Let r∈R, then we have

$$Ff^{-1}(\overline{B})(xr,q) = \overline{B}(f(xr,q)) = \overline{B}(f(x,q),f(r,q))$$

$$\leq \overline{B}(f(x,q)) = F_f^{-1}(\overline{B})(x,q)$$

Hence $F_f^{-1}(B)$ is a Max^i Q-fuzzy R subgroup of R.

Conclusion : Y.B.Jun [6] introduced the concept of interval valued fuzzy R-subgroup of near ring. In this paper we introduce the notion of Interval Max norm Q-fuzzy R-subgroup of near rings and investigate some of their properties. Using Lower level set, we give a characterization of Max^{i} Q-fuzzy right R- subgroup. Finally we establish the idea of the homomorphic image and the inverse image.

Future work: One can obtain the similar idea in interval valued soft fuzzy subgroup of near rings with suitable mathematical tool and characterize the images of soft fuzzy groups.

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