General Bulk Service Queueing System with Multiple Working Vacation

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***Abstract*: This paper analyses a single server with bulk service queue with general arrival pattern and multiple working vacation period. The model is analyzed by using Embedded Markov Chain technique. The steady state probability distribution at pre arrival epoch and arbitrary epoch are derived and measures like mean queue length are calculated. Finally, through some numerical examples, the parametric effect on the performance measures are discussed and presented graphically.**

***Keywords*:** Working vacation, Bulk service queue, and multiple vacations.

1. INTRODUCTION

Tian, Li and Zhang (2009) provided a survey of the results of working vacation queues and demonstrated that the matrix analytic methods developed by Neuts (1995) are powerful tools for analyzing the WVQ's. The survey also shows that neither bulk input nor bulk service WVQs are considered in the existing literature. Later Xu et al. (2009) and Julia Rose Mary and Afthab Begum (2010) studied the bulk input Markovian $M^X / M / 1$ queue with working vacations and presented the PGF of the stationary queue length.

There are situations, particularly in transportation systems, where the service provided is such that a group (batch) of customers can be served simultaneously. The theory of batch service queues originated with the works of Bailey (1954). He considered a queue with Poisson arrival and fixed size service. Later many authors have investigated a variety of extensions of the basic model. The bulk service rule introduced by Neuts (1967) is the most general one and this has been further investigated by Medhi (1984), Borthakur et al. (1987). Julia Rose Mary and Afthab Begum (2009) have analyzed the Markovian M/M(a,b)/1 queueing model under multiple working vacation and derived the steady-state probability distribution and the mean queue length for the model.

In some practical situations such as production systems and distribution systems, the input to the queueing system may not be a Poisson process so that a more general arrival process should be used. Baba (2005) have analyzed GI/M/1 queue under multiple working vacation as an extension of /M/1 working vacation introduced by Servi and Finn (2002). In this paper, GI/M(a,b)/1 multiple working vacation model analyzed by assuming that the inter arrivals form an independently identically distributed sequence of random variables having a general distribution function and the customers are served in batches following General Bulk Service Rule introduced by Neuts (1967). The results of GI/M/1 multiple working vacation model and M/M(a,b)/1 multiple working vacation are derived as special cases. It is also proved that when the arrival pattern is a Markov process, then the steady state probability distribution at pre arrival epoch and at arbitrary epoch coincide.

2. MATHEMATICAL ANALYSIS

We consider a batch service queueing system GI/M(a,b)/1 under multiple working vacation. In this system, it is assumed that the inter arrival times (A) form an independently identically distributed sequence of random variables having a general distribution function $A(t) = \Pr(A \le t)$. The server processes the customers in batches according to the GBSR introduced by Neuts (1967). The service time of batches of size x ($a \le x \le b$) (a: minimum number of units, b: maximum number of units) is assumed to be an independently identically distributed sequence of random variables with exponential distribution of parameter μ_b.

Whenever the server completes a service and finds less than "a" customers in the queue then he begins a vacation which is an exponentially distributed random variable V with parameter η. After completing a vacation, if the system length is still less than "a", then he takes another vacation and the vacations are repeated until the server finds at least "a " customers in the queue.

Suppose during vacation, if the queue size becomes at least "a" then the server starts a service under General Bulk Service Rule but with the service rate μ_v which is strictly less than the regular service rate μ_b. When the vacation ends, the server switches his service rate from μ_v to μ_b. It is assumed that, the size of service batch that being served remains unchanged when the server enters into the regular busy period. In either case (working vacation or regular busy period), the service rates are assumed to be independent of the size of the batch in service ($a \le x \le b$).

The Queueing system is formulated as an embedded two dimensional Markov chain by choosing arrival epochs as embedded points. The steady state distribution for the number of customers in the queue at arrival epochs is derived by analyzing the embedded Markov chain defined. Using the theory of semi Markov process, the steady state distribution for the number of customers in the queue at arbitrary epochs is also derived.

Case 1 : Pre-arrival Epoch

Let t_n, n = 1, 2, . . . ($t_0 = 0$) be the arrival epoch at which the n^{th} customer arrives. The system is examined at time ($t_n - 0$). The interarrival times $\{T_n, n \ge 1\}$ are independent and identically distributed with a general distribution function denoted by A(t) with mean $1/\lambda$. The LST of A(t) is given by $A^*(\theta) = \int_0^\infty e^{-\theta t} dA(t)$. The working vacation times, the service times during regular service period and the service times during working vacation are all exponentially distributed with rates η, μ_b and μ_v respectively.

Let W(t) denote the number of customers in the queue at time t, $W_n = W(t_n - 0)$ and

$$J_n = \begin{cases} 0, & \text{if the } n^{th} \text{ arrival occurs during idle vacation period} \\ 1, & \text{if the } n^{th} \text{ arrival occurs during working vacation period} \\ 2, & \text{if the } n^{th} \text{ arrival occurs during regular busy period} \end{cases}$$

Since, the working vacation times, the service times during regular and working vacation are all exponentially distributed, the process $\{(W_n, J_n), n \ge 1\}$ is an embedded Markov chain with state space $\Omega = \{(n, j); n \ge 0, j = 1,2\} \cup \{(n,0); 0 \le n \le a-1\}$.

The steady-state queue size probabilities are defined by $R_n = \lim_{k\to\infty} \Pr(W(t_k - 0) = n, J_k = 0), 0 \le n \le a-1$

$Q_n = \lim_{k\to\infty} \Pr(W(t_k - 0) = n, J_k = 1), n \ge 0$ and $P_n = \lim_{k\to\infty} \Pr(W(t_k - 0) = n, J_k = 2), n \ge 0$.

Then R_n, Q_n and P_n respectively denote the probability that the queue contains n customers and the server is idle in vacation state, is busy in vacation state and regular busy state at pre arrival epochs. During idle vacation period, the number of customers in the system and queue are the same, whereas in working vacation period and in regular busy period n denotes the number of customers in the queue and the system will contain (n + k), ($a \le k \le b$) customers.

Let b_k denote the probability that k batches are served at regular service rate μ_b during an interarrival time. Then

$$b_k = \int_0^\infty e^{-\mu_b t} \frac{(\mu_b t)^k}{k!} dA(t), k \ge 0 \tag{1}$$

and
$$\sum_{k=0}^\infty b_k z^{kb} = B(z^b) = A^*(\mu_b(1-z^b)) \tag{2}$$

Let c_k denote the probability that, the working vacation time is greater than an inter arrival time and k batches are served at rate μ_v during an inter arrival time.

Then $$c_k = \int_0^\infty e^{-\eta t}\, e^{-\mu_v t} \frac{(\mu_v t)^k}{k!} \, dA(t), k \ge 0 \tag{3}$$

and $$\sum_{k=0}^{\infty} c_k z^{kb} = C(z^b) = A^*(\eta + \mu_v(1-z^b)) \tag{4}$$

Let d_k denote the probability that, the server returns from vacation in an inter arrival time and k service completions occur in an inter arrival time. Then

$$d_k = \int_0^\infty \sum_{i=0}^{k} \left\{ \int_0^t \eta\, e^{-\eta x} \frac{(\mu_v x)^i}{i!} e^{-\mu_v x} \frac{(\mu_b(t-x))^{k-i}}{(k-i)!} e^{-\mu_b(t-x)} \, dx \right\} dA(t),\ k \ge 0 \tag{5}$$

i.e., k services in an inter arrival time can occur in such a way that, i ($0 \le i \le k$) service completions occur at rate μ_v (till the server returns from vacation) and the remaining (k – i) service completions occur at rate μ_b

and $$\sum_{k=0}^{\infty} d_k z^{kb} = D(z^b) = \frac{\eta\,[A^*(\mu_b(1-z^b)) - A^*(\eta + \mu_v(1-z^b))]}{\eta + (\mu_v - \mu_b)(1-z^b)} \tag{6}$$

The steady-state queue size equations at pre-arrival epochs, are obtained by noting the transitions between the states of the Markov chain and are given by :

$$Q_n = \sum_{k=0}^{\infty} Q_{kb+n-1} c_k, \qquad n \ge 1 \tag{7}$$

$$Q_0 = \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} Q_{(k-1)b+j} c_k + R_{a-1} Q_0 \tag{8}$$

$$P_n = \sum_{k=0}^{\infty} P_{kb+n-1} b_k + \sum_{k=0}^{\infty} Q_{kb+n-1} d_k \qquad n \ge 1 \tag{9}$$

$$P_0 = \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} P_{(k-1)b+j} b_k + \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} Q_{(k-1)b+j} d_k + R_{a-1} d_0 \tag{10}$$

$$R_n = R_{n-1} + \sum_{k=0}^{\infty} Q_{kb+n-1}(1 - \sum_{i=0}^{k}(c_i + d_i)) + \sum_{k=0}^{\infty} P_{kb+n-1}(1 - \sum_{i=0}^{k} b_i),\ 1 \le n \le a-1 \tag{11}$$

$$R_0 = \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} Q_{(k-1)b+j}(1 - \sum_{i=0}^{k}(c_i + d_i))$$
$$+ \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} P_{(k-1)b+j}(1 - \sum_{i=0}^{k} b_i) + R_{a-1}(1 - (d_0 + c_0)) \tag{12}$$

By operator technique, equation (7) and (9) becomes

$$\left(E - \sum_{k=0}^{\infty} c_k E^{kb}\right) Q_n = 0, \qquad n \ge 0 \tag{13}$$

and $$\left(E - \sum_{k=0}^{\infty} b_k E^{kb}\right) P_n = \sum_{k=0}^{\infty} Q_{kb+n} d_k, \ n \ge 0 \tag{14}$$

The characteristic equation $z = C(z^b) = A^*(\eta + \mu_v(1-z^b))$, $\eta > 0$ of the homogeneous difference equation (13) has a unique root r_1 inside (0, 1).

For $\psi(z) = A^*(\eta + \mu_v(1 - z^b))$, satisfies the inequality

$0 < \psi(0) = A^*(\eta + \mu_v) < \psi(1) = A^*(\eta) < 1$, and for $0 < z < 1$,

$$\psi'(z) = b\,\mu_b \int_0^\infty t\, e^{-(\eta + \mu_b(1 - z^b))t}\, dA(t) > 0$$

$$\psi''(z) = (b\,\mu_b)^2 \int_0^\infty t^2\, e^{-(\eta + \mu_b(1 - z^b))t}\, dA(t) > 0$$

Therefore, $r_1 = A^*(\eta + \mu_v(1 - r_1^{\,b}))$ with $0 < r_1 < 1$ (Baba (2005)) (15)

Thus homogeneous difference equation (13) has solution

$$Q_n = r_1^{\,n}\, Q_0, \quad n \geq 0 \tag{16}$$

The characteristic equation of the non-homogeneous difference equation (14) is $z = B(z^b) = A^*(\mu_b(1 - z^b))$ and $B(z^b)$ is the pgf of b_k's with $B(1) = 1$. Hence following the arguments of Gross and Harris (1998), the characteristic equation $z = B(z^b)$ has a unique root in (0, 1), if $B'(1) > 1$, i.e., $\rho_b = \frac{\lambda}{b\,\mu_b} < 1$. Thus under the condition $\rho_b < 1$, the solution of the non-homogeneous equation (14) is given by

$$P_n = (A_d\, r^n + B_d\, r_1^{\,n})\, Q_0, \quad n \geq 0 \tag{17}$$

where $B_d = \dfrac{-\eta}{\eta + (\mu_v - \mu_b)(1 - r_1^{\,b})}$

To find the remaining probabilities, the equations (11) and (12) are used.

$$R_n = \left[A_d \frac{r^{a-1} - r^n}{(1 - r^b)} + (B_d + 1)\left(\frac{r_1^{a-1} - r_1^n}{1 - r_1^b}\right) + \frac{1}{r_1^b\,(1 - r_1)}\left(\frac{r_1^b - r_1^a}{c_0} + r_1^{a-1} - r_1^b\right)\right] Q_0$$

for $0 \leq n \leq a-1$

After some algebraic manipulation, the steady-state queue size probabilities at arrival epochs are given by

$$Q_n = r_1^{\,n}\, Q_0, \quad n \geq 0$$

$$P_n = (A_d\, r^n + B_d\, r_1^{\,n})\, Q_0, \quad n \geq 0$$

$$R_n = \left[A_d\, h_n(r) + (B_d + 1)\, h_n(r_1) + \frac{1}{(r_1^b\,(1 - r_1))}\left(\frac{r_1^b - r_1^a}{c_0} + r_1^{a-1} - r_1^b\right)\right] Q_0, \quad \text{for } 0 \leq n \leq a-1 \tag{18}$$

where $h_n(x) = \dfrac{x^{a-1} - x^n}{1 - x^b}$, $A_d\, f(r) + B_d\, f(r_1) = \dfrac{d_0}{c_0}\left(\dfrac{r_1^a - r_1^b}{r_1^b\,(1 - r_1)}\right)$ with

$$f(x) = \frac{x^a - x^b}{x^b\,(1 - x)} + b_0\,\frac{(x^b - x^{a-1})}{x^b\,(1 - x)} \quad \text{and} \quad B_d = \frac{-\eta}{\eta + (\mu_v - \mu_b)(1 - r_1^b)}$$

and by using the normalizing condition

$$Q_0^{-1} = A_d\, g(r) + (B_d + 1)\, g(r_1) + \frac{a}{r_1^b\,(1 - r_1)}\left(\frac{r_1^b - r_1^a}{c_0} + (r_1^{a-1} - r_1^b)\right) \tag{19}$$

where $g(x) = \dfrac{1}{(1 - x^b)}\left(\dfrac{x^a - x^b}{1 - x} + a\, x^{a-1}\right)$

The mean queue length L_q is calculated as

$$L_q = \left[A_d\, H(r) + (B_d + 1)\, H(r_1) + \frac{a\,(a-1)}{2 r_1^b\,(1 - r_1)}\left(\frac{r_1^b - r_1^a}{c_0} + r_1^{a-1} - r_1^b\right)\right] Q_0$$

where $H(x) = \frac{x}{(1-x)^2} + \frac{1}{(1-x^b)}\left[\frac{a(a-1)}{2}x^{a-1} + \frac{ax^a(1-x)-x(1-x^a)}{(1-x)^2}\right]$

Performance Measures at Pre-Arrival Epochs

Let P_v, P_I and P_{busy} denote that the server is busy in vacation, idle in vacation and in regular busy state respectively at pre-arrival epochs. Then,

(i) $\quad P_v = \sum_{n=0}^{\infty} r_1^n Q_0 = \frac{Q_0}{(1-r_1)}$

(ii) $\quad P_I = \left[\left(\frac{A_d r^{a-1} a}{(1-r^b)} + \frac{(B_d+1) r_1^{a-1} a}{(1-r_1^b)} + \frac{a}{r_1^b(1-r_1)}\left(\frac{r_1^b - r_1^a}{c_0} + r_1^{a-1} - r_1^b\right)\right) - \frac{A_d(1-r^a)}{(1-r^b)(1-r)} - \frac{(B_d+1)(1-r_1^a)}{(1-r_1^a)(1-r_1)}\right] Q_0$

(iii) $\quad P_{busy} = \sum_{n=0}^{\infty}(A_d r^n + B_d r_1^n) Q_0 = \left(\frac{A}{(1-r)} + \frac{B_d}{(1-r_1)}\right) Q_0$

Case 2 : Random Epoch

To obtain the limiting probabilities of queue size at random epochs, the system is examined at some time t, proceding an arrival epoch. Then using the relation between the two sequences, $(W(t), J(t))$, $(t_n \le t < t_{n+1})$ and (W_n, J_n), $(n = 0, 1, 2, \ldots)$, the steady-state equations satisfied by the steady-state queue size probabilities

$(R_n^*, Q_n^*, P_n^*) = \lim_{t\to\infty} \Pr(W(t) = n, J(t) = (0,1,2))$ at arbitrary epochs are obtained :

$$Q_n^* = \sum_{k=0}^{\infty} Q_{kb+n-1} c_k^*, \quad n \ge 1 \tag{20}$$

$$Q_0^* = \sum_{k=1}^{\infty}\sum_{j=a-1}^{b-1} Q_{(k-1)b+j} c_k^* + R_{a-1} c_0^* \tag{21}$$

$$P_n^* = \sum_{k=0}^{\infty} P_{kb+n-1} b_k^* + \sum_{k=0}^{\infty} Q_{kb+n-1} d_k^*, \quad n \ge 1 \tag{22}$$

$$P_0^* = \sum_{k=1}^{\infty}\sum_{j=a-1}^{b-1} P_{(k-1)b+j} b_k^* + \sum_{k=1}^{\infty}\sum_{j=a-1}^{b-1} Q_{(k-1)b+j} d_k^* + R_{a-1} d_0^* \tag{23}$$

$$R_n^* = R_{n-1} + \sum_{k=0}^{\infty} Q_{kb+n-1}\left(1 - \sum_{i=0}^{k}(c_i^* + d_i^*)\right) + \sum_{k=0}^{\infty} P_{kb+n-1}\left(1 - \sum_{i=0}^{k} b_i^*\right), \quad 1 \le n \le a-1 \tag{24}$$

$$R_0^* = \sum_{k=1}^{\infty}\sum_{j=a-1}^{b-1} Q_{(k-1)b+j}\left(1 - \sum_{i=0}^{k}(c_i^* + d_i^*)\right) + \sum_{k=1}^{\infty}\sum_{j=a-1}^{b-1} P_{(k-1)b+j}\left(1 - \sum_{i=0}^{k} b_i^*\right) + R_{a-1}(1 - c_0^* - d_0^*) \tag{25}$$

where b_k^*, c_k^* and d_k^* are the corresponding quantities for b_k, c_k and d_k respectively, where the interarrival time A is replaced by T – the period of time between a random epoch and the preceding arrival epoch. The distribution function $F_T(t)$ and the density function $f_T(t)$ of T are given by $F_T(t) = \lambda \int_0^t (1-A(x))\,dx$, $f_T(t) = \lambda(1 - A(t))\,dt$. Thus we have,

$$b_k^* = \int_0^\infty \frac{(\mu_b t)^k}{k!} e^{-\mu_b t} \lambda(1-A(t))\,dt, \quad c_k^* = \int_0^\infty e^{-\eta t} \frac{(\mu_v t)^k}{k!} e^{-\mu_v t} \lambda(1-A(t))\,dt$$

$$d_k^* = \int_0^\infty \sum_{r=0}^{k} \left(\int_0^t \eta \frac{e^{-\eta x} (\mu_v x)^r}{r!} e^{-\mu_v x} \frac{(\mu_b (t-x))^{k-r}}{(k-r)!} e^{-\mu_b (t-x)}\,dx \right) \lambda(1-A(t))\,dt$$

i.e., b_k^* denotes the probability that k customers are served at regular service rate μ_b in an interval of time T and similarly c_k^* and d_k^* can be interpreted.

Substituting for the steady-state probabilities at pre-arrival epochs, the limiting probability distribution at arbitrary epochs are

$$Q_0^* = \left[\frac{r_1^{a-1} - r_1^b}{r_1^b(1-r_1)} \left(\frac{\lambda(1-r_1)}{\eta + \mu_v(1-r_1^b)} \right) + \frac{r_1^b - r_1^a}{r_1^b(1-r_1)} \frac{c_0^*}{c_0} \right] Q_0$$

$$P_0^* = \frac{Q_0 A_d (r^{a-1} - r^b)}{(1-r) r^b} \left[\frac{\lambda(1-r)}{\mu_b(1-r^b)} - b_0^* \right]$$

$$+ \frac{B_d Q_0 (r_1^{a-1} - r_1^b)}{(1-r_1) r_1^b} \left[\frac{\lambda(1-r_1)}{\eta + \mu_v(1-r_1^b)} - b_0^* \right] + \frac{d_0^*}{c_0} \left[\frac{r_1^b - r_1^a}{r_1^b(1-r_1)} \right] Q_0$$

$$R_0^* = \left[A_d h_0^*(r, \mu_b(1-r^b)) + (B_d + 1) h_0^*(r_1, \eta + \mu_v(1-r_1^b)) \right.$$

$$\left. + \frac{1}{r_1^b(1-r_1)} \left(\frac{(r_1^b - r_1^a)(1 - c_0^* - d_0^*)}{c_0} + (r_1^{a-1} - r_1^b)(1 - b_0^*) \right) \right] Q_0$$

where $h_0^*(x, y) = \dfrac{x^{a-1} - x^b}{(1-x) x^b} \left[\dfrac{x^b}{1-x^b} - \left(\dfrac{\lambda(1-x)}{(1-x^b) y} - b_0^* \right) \right]$

and the other queue size probabilities at arbitrary epochs are

$$Q_n^* = \frac{\lambda(1-r_1) r_1^{n-1} Q_0}{\eta + \mu_v(1-r_1^b)}, \qquad n \ge 1$$

$$P_n^* = \left[A_d r^{n-1} \frac{\lambda(1-r)}{\mu_b(1-r^b)} + B_d r_1^{n-1} \frac{\lambda(1-r_1)}{\eta + \mu_v(1-r_1^b)} \right] Q_0, \qquad n \ge 1$$

$$R_n^* = \left[A_d h_n^*(r, \mu_b(1-r^b)) + (B_d + 1) h_n^*(r_1, \eta + \mu_v(1-r_1^b)) \right.$$

$$\left. + \frac{1}{r_1^b(1-r_1)} \left(\frac{r_1^b - r_1^a}{c_0} + r_1^{a-1} - r_1^b \right) \right] Q_0, \quad 1 \le n \le a-1$$

The mean queue length L_q^* at arbitrary epoch is calculated as

$$L_q^* = \left[A_d\, H(r, \mu_b(1-r^b)) + (B_d+1)\, H(r_1, \eta + \mu_v(1-r_1^b)) + \left(\frac{r_1^b - r_1^a}{c_0} + (r_1^{a-1} - r_1^b) \right) \frac{a(a-1)}{2r_1^b(1-r_1)} \right] Q_0$$

where $H(x, y) = \dfrac{\lambda}{(1-x)\,y} + \dfrac{1}{(1-x^b)} \left[\dfrac{x^{a-1}\, a(a-1)}{2} + \dfrac{\lambda\,(a\,x^{a-1}(1-x) - (1-x^a))}{y\,(1-x)} \right]$

3. PARTICULAR CASES

I. M / M(a, b) / 1 / MWV: If the inter arrival time follows exponential distribution, then the limiting probabilities at arbitrary epochs and at pre-arrival epochs coincide and give the steady-state queue size probabilities of the Markovian M / M(a, b) / 1 / MWV. These results coincide with the corresponding results of Julia Rose Mary and Afthab Begum (2009) with the following identifications

$A(t) = (1 - e^{-\lambda t})$, $A^*(\theta) = \dfrac{\lambda}{\lambda+\theta}$, and r and r_1 respectively satisfy the equations $\dfrac{\lambda}{\mu_b(1-r^b)} = \dfrac{r}{1-r}$

and $\dfrac{\lambda}{\eta + \mu_v(1-r_1^b)} = \dfrac{r_1}{1-r_1}$. And also for every k, $b_k^* = b_k = \left(\dfrac{\lambda}{\lambda+\mu_b}\right)\left(\dfrac{\mu_b}{\mu_b+\lambda}\right)^k$,

$c_k^* = c_k = \left(\dfrac{\lambda}{\lambda+\mu_v+\eta}\right)\left(\dfrac{\mu_v}{\lambda+\mu_v+\eta}\right)^k$ and $d_0^* = d_0 = \left(\dfrac{\lambda\,\eta}{(\lambda+\mu_b)(\lambda+\eta+\mu_v)}\right)$.

II. GI / M / 1 / MWV: When a = b = 1, the steady-state queue size probabilities coincide with the corresponding results of Baba (2005) by using the following relations.

$$\frac{A}{r} = \frac{-B_d}{r_1},\ -B_d = \frac{\eta}{\eta - (\mu_b - \mu_v)(1-r_1)} = \alpha \text{ and } \frac{Q_0}{r_1} = (1-r)\,\beta$$

where $\beta = \dfrac{\eta - (\mu_b - \mu_v)(1-r_1)}{\eta - (\mu_b - \mu_v)(1-r)}$

4. NUMERICAL ANALYSIS

To demonstrate the influence of the system parameters on (i) the waiting line (L_q), (ii) steady state queue size probabilities and (iii) the average number of customers waiting in the queue, when the system is in different states, we consider different distributions for the interarrival time are considered.

In Table 1, the mean queue size both at arbitrary epochs (L_{arb}) and at pre-arrival epochs ($L_{arrival}$) are presented for different values of the vacation parameter (η) and vacation service rate (μ_v) corresponding to different interarrival time distributions (Erlang-k = 1, 3, 5, 10 and deterministic) to know how the expected queue length changes with the parameters. It is shown that

(i) both L_{arb} and $L_{arrival}$ decrease as μ_v or η increases.
(ii) The performance measures at arbitrary epoch and pre-arrival epochs coincide for Markovian interarrival time distribution.
(iii) The smaller values of η significantly affect the queue size.
(iv) When $\mu_v = \mu_b$, the queue length of working vacation model and classical non-vacation model coincide.

The graphical representation of the effect of μ_v and η on the mean queue length (L_{arb}) for Deterministic and Erlang-3 type inter arrival type can be seen in Figures 1a and 1b respectively.

Table 1 Expected queue size ($L_{arrival}$ and L_{arb}) with respect to η and μ_v
(a, b, λ, μ_b, ρ_b) = (5, 15, 7, 0.9, 0.5)

μ_v	η	D/M(a, b)/1	M/M(a, b)/1	E_3/M(a, b)/1	E_5/M(a, b)/1	E_{10}/M(a, b)/1
0.005	0.05	104.748	135.263	134.696	134.504	134.235
		71.232	135.264	134.942	134.832	134.678
	0.1	54.493	67.071	66.508	66.319	66.053
		38.583	67.071	66.750	66.642	66.488
	0.2	29.545	34.118	33.569	33.384	33.124
		22.219	34.118	33.804	33.699	33.549
0.050	0.05	94.139	122.801	122.225	122.031	121.759
		60.762	122.801	122.470	122.358	122.200
	0.1	49.584	61.328	60.761	60.570	60.301
		34.495	61.328	61.002	60.891	60.735
	0.2	27.449	31.665	31.124	30.939	30.679
		20.718	31.665	31.359	31.253	31.102
0.500	0.05	19.510	26.223	25.538	25.309	24.987
		15.834	26.223	25.770	25.618	25.405
	0.1	17.608	19.285	18.713	18.521	18.252
		14.387	19.285	18.938	18.821	18.658
	0.2	15.322	14.654	14.143	13.975	13.739
		12.742	14.654	14.362	14.267	14.134
0.900	0.05	7.130	9.382	9.000	8.872	8.694
		7.894	9.382	9.192	9.129	9.042
	0.1	7.130	9.382	9.00	8.872	8.694
		7.894	9.382	9.192	9.129	9.042
	0.2	7.130	9.382	9.000	8.872	8.694
		7.894	9.382	9.192	9.129	9.042

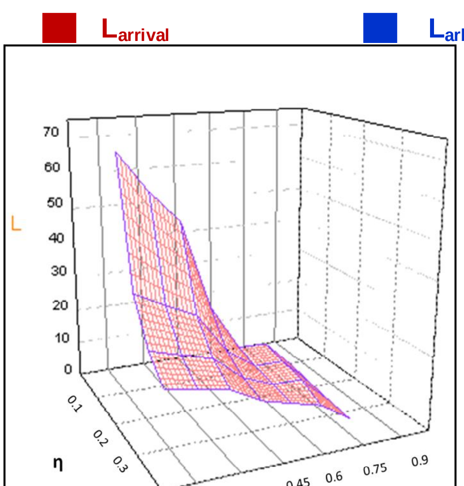


Figure 1a D/M(a, b)/1/MWV

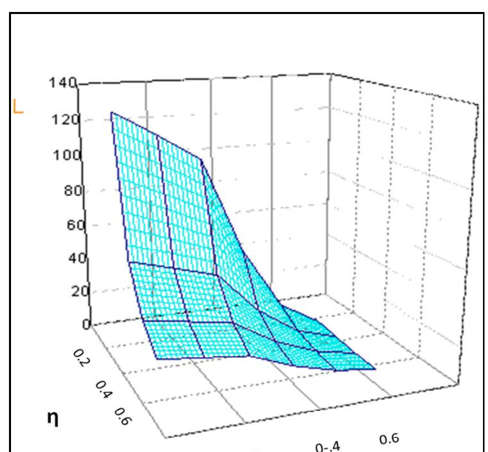


Figure 1b E_3/M(a, b)/1/MWV

In Table 2, the values of the expected queue size at arbitrary and prearrival epochs are presented for different values of arrival rate λ and for different vacation service rates μ_v for two different values of regular service rate μ_b for Erlang-3 interarrival distribution. The values show that mean queue size increases with the arrival rate λ and decreases as the service rate increases.

Table 2 Mean queue size with respect to λ and μ_v for μ_b = 1 and μ_b = 1.5

λ	μ_v	0.1	0.2	0.3	0.4	0.5
5	$\mu_b = 1$	34.508	24.641	17.238	12.381	9.390
		34.817	24.946	17.531	12.660	9.656
	$\mu_b = 1.5$	34.425	24.478	16.989	12.063	9.033
		34.729	24.779	17.276	12.336	9.290
6	$\mu_b = 1$	43.918	33.017	24.044	17.459	13.098
		44.233	33.332	24.348	17.754	13.382
	$\mu_b = 1.5$	43.663	32.697	23.638	16.967	12.541
		43.973	33.008	23.937	17.257	12.818
7	$\mu_b = 1$	53.727	42.085	31.916	23.758	17.862
		54.046	42.406	32.228	24.064	18.160
	$\mu_b = 1.5$	53.096	41.420	31.187	22.950	16.978
		53.411	41.737	31.495	23.250	17.270

Prearrival epochs **Arbitrary epochs**

The influence of λ and μ_v on the mean queue length at arbitrary epochs is graphically represented in Figures 3a and 3b for Deterministic and Erlang–3 inter arrival time distribution. Table 3 gives the data for the graphs.

Table 3 Mean queue size with respect to λ and μ_v
(μ_b, η, a, b) = (0.9, 0.1, 5, 15)

λ	ρ_b \ μ_v	0.1	0.3	0.5	0.7
2	0.15	2.106	2.088	2.062	2.047
		9.936	4.462	3.032	2.507
3	0.22	3.981	3.543	3.252	2.789
		17.411	7.654	4.661	3.485
4	0.29	4.916	4.573	4.211	4.101
		25.872	12.033	6.910	4.893
5	0.37	7.881	7.526	7.380	7.063
		34.950	17.713	9.873	6.736

D / M(a,b) / 1 / MWV **E_3 / M(a, b) / 1 / MWV**

Figure 3a D / M(a, b) / 1 / MWV

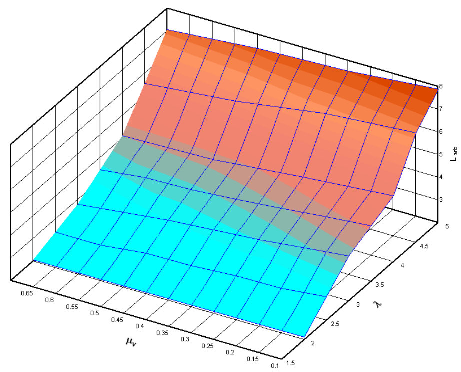
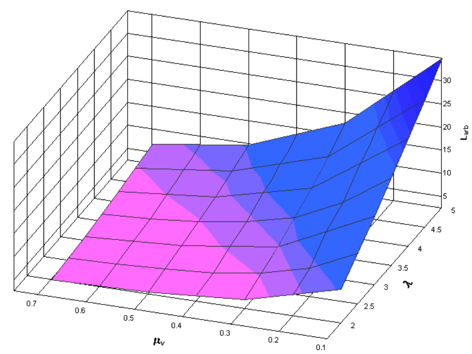


Figure 3b E_3 / M(a, b) / 1 / MWV



5. CONCLUSION

In this paper, a Non Markovian bulk service queue GI/M(a,b)/1 is investigated using Embedded Markov chain technique and the steady state queue size probabilities at pre arrival epochs as well as arbitrary epochs are obtained and few existing models are proved as particular cases. The expected queue length is calculated numerically and presented graphically.

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