Eulerian integral associated with product of two multivariable Aleph-functions, a generalized Lauricella function and a class of polynomials

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ABSTRACT
The present paper is evaluated a new Eulerian integral associated with the product of two multivariableAleph-functions, a generalized Lauricella function and a class of multivariable polynomials with general arguments . Several particular cases are given .

Keywords: Eulerian integral, multivariable Aleph-function, generalized Lauricella function of several variables, multivariable I-function, generalized hypergeometric function, class of polynomials.

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1. Introduction

In this paper, we evaluate a new Eulerian integral of most general characters associated with the products of two multivariable Aleph-functions and a class of polynomials with general arguments. Several particular cases are given.
The Aleph-function of several variables generalize the multivariable I-function defined by Sharma and Ahmad [2] , itself is an a generalisation of G and H-functions of several variables defined by Srivastava et al [6]. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array}\right|$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]$

$\left.[(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\right)$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \quad (1.1)$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \quad (1.2)$$

$$\text{and }\ \theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \quad (1.3)$$

Suppose , as usual , that the parameters

$a_j, j = 1, \cdots, p; b_j, j = 1, \cdots, q;$

$c_j^{(k)}, j = 1, \cdots, n_k; c_{ji^{(k)}}^{(k)}, j = n_k + 1, \cdots, p_{i^{(k)}};$

$d_j^{(k)}, j = 1, \cdots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \cdots, q_{i^{(k)}};$

with $k = 1 \cdots, r, i = 1, \cdots, R \ , i^{(k)} = 1, \cdots, R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \qquad (1.4)$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to n and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1 \cdots, r, i = 1, \cdots, R \ , i^{(k)} = 1, \cdots, R^{(k)} \qquad (1.5)$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(y_1, \cdots, y_r) = 0(|y_1|^{\alpha_1}, \cdots, |y_r|^{\alpha_r}), max(|y_1|, \cdots, |y_r|) \rightarrow 0$

$\aleph(y_1, \cdots, y_r) = 0(|y_1|^{\beta_1}, \cdots, |y_r|^{\beta_r}), min(|y_1|, \cdots, |y_r|) \rightarrow \infty$

where $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R \ ; \ V = m_1, n_1; \cdots ; m_r, n_r \tag{1.6}$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.7}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.8}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots, \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.9}$$

$$C = \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \cdots, \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.10}$$

$$D = \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \cdots, \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.11}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots, z_r) = \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} z_1 & \mathrm{A} : \mathrm{C} \\ \cdot & \cdots \\ \cdot & \\ \cdot & \\ z_r & \mathrm{B} : \mathrm{D} \end{array}\right) \tag{1.12}$$

Consider the Aleph-function of s variables

$$\aleph(z_1, \cdots, z_s) = \aleph^{0,N:M_1,N_1,\cdots,M_s,N_s}_{P_i,Q_i,\iota_i;r':P_{i^{(1)}},Q_{i^{(1)}},\iota_{i^{(1)}};r^{(1)};\cdots;P_{i^{(s)}},Q_{i^{(s)}};\iota_{i^{(s)}};r^{(s)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\mathrm{u}_j; \mu_j^{(1)}, \cdots, \mu_j^{(r')})_{1,N}], [\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots, \mu_{ji}^{(r')})_{N+1,P_i}] : \\ \cdot & \cdots\cdots, [\iota_i(v_{ji}; \upsilon_{ji}^{(1)}, \cdots, \upsilon_{ji}^{(r')})_{M+1,Q_i}] : \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & \end{array}\right.$$

$$\left.\begin{array}{c} [(\mathrm{a}_j^{(1)}); \alpha_j^{(1)})_{1,N_1}], [\iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_i^{(1)}}]; \cdots; [(\mathrm{a}_j^{(s)}); \alpha_j^{(s)})_{1,N_s}], [\iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)}; \alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_i^{(s)}}] \\ [(\mathrm{b}_j^{(1)}); \beta_j^{(1)})_{1,M_1}], [\iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)}; \beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_i^{(1)}}]; \cdots; [(\mathrm{b}_j^{(s)}); \beta_j^{(s)})_{1,M_s}], [\iota_{i^{(s)}}(b_{ji^{(s)}}^{(s)}; \beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_i^{(s)}}] \end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L_1} \cdots \int_{L_s} \zeta(t_1, \cdots, t_s) \prod_{k=1}^{s} \phi_k(t_k) z_k^{t_k} \, \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.13}$$

with $\omega = \sqrt{-1}$

$$\zeta(t_1, \cdots, t_s) = \frac{\prod_{j=1}^{N} \Gamma(1 - u_j + \sum_{k=1}^{s} \mu_j^{(k)} t_k)}{\sum_{i=1}^{r'} [\iota_i \prod_{j=N+1}^{P_i} \Gamma(u_{ji} - \sum_{k=1}^{s} \mu_{ji}^{(k)} t_k) \prod_{j=1}^{Q_i} \Gamma(1 - \upsilon_{ji} + \sum_{k=1}^{s} \upsilon_{ji}^{(k)} t_k)]} \tag{1.14}$$

and $$\phi_k(t_k) = \frac{\prod_{j=1}^{M_k} \Gamma(b_j^{(k)} - \beta_j^{(k)} t_k) \prod_{j=1}^{N_k} \Gamma(1 - a_j^{(k)} + \alpha_j^{(k)} s_k)}{\sum_{i^{(k)}=1}^{r^{(k)}} [\iota_{i^{(k)}} \prod_{j=M_k+1}^{Q_{i^{(k)}}} \Gamma(1 - b_{ji^{(k)}}^{(k)} + \beta_{ji^{(k)}}^{(k)} t_k) \prod_{j=N_k+1}^{P_{i^{(k)}}} \Gamma(a_{ji^{(k)}}^{(k)} - \alpha_{ji^{(k)}}^{(k)} s_k)]} \tag{1.15}$$

Suppose , as usual , that the parameters

$u_j, j = 1, \cdots, P; v_j, j = 1, \cdots, Q;$

$a_j^{(k)}, j = 1, \cdots, N_k; a_{ji^{(k)}}^{(k)}, j = n_k + 1, \cdots, P_{i^{(k)}};$

$b_{ji^{(k)}}^{(k)}, j = m_k + 1, \cdots, Q_{i^{(k)}}; b_j^{(k)}, j = 1, \cdots, M_k;$

with $k = 1 \cdots, s, i = 1, \cdots, r', i^{(k)} = 1, \cdots, r^{(k)}$

are complex numbers , and the $\alpha' s, \beta' s, \gamma' s$ and $\delta' s$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{N} \mu_j^{(k)} + \iota_i \sum_{j=N+1}^{P_i} \mu_{ji}^{(k)} + \sum_{j=1}^{N_k} \alpha_j^{(k)} + \iota_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \alpha_{ji^{(k)}}^{(k)} - \iota_i \sum_{j=1}^{Q_i} \upsilon_{ji}^{(k)} - \sum_{j=1}^{M_k} \beta_j^{(k)}$$

$$-\iota_{i^{(k)}} \sum_{j=M_k+1}^{Q_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.16}$$

The reals numbers τ_i are positives for $i = 1, \cdots, r$, $\iota_{i^{(k)}}$ are positives for $i^{(k)} = 1 \cdots r^{(k)}$

The contour L_k is in the t_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(b_j^{(k)} - \beta_j^{(k)} t_k)$ with $j = 1$ to M_k are separated from those of $\Gamma(1 - u_j + \sum_{i=1}^{s} \mu_j^{(k)} t_k)$with $j = 1$ to N and $\Gamma(1 - a_j^{(k)} + \alpha_j^{(k)} t_k)$ with $j = 1$ to N_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z_k| < \frac{1}{2} B_i^{(k)} \pi$, where

$$B_i^{(k)} = \sum_{j=1}^{N} \mu_j^{(k)} - \iota_i \sum_{j=N+1}^{P_i} \mu_{ji}^{(k)} - \iota_i \sum_{j=1}^{Q_i} \upsilon_{ji}^{(k)} + \sum_{j=1}^{N_k} \alpha_j^{(k)} - \iota_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \alpha_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{M_k} \beta_j^{(k)} - \iota_{i^{(k)}} \sum_{j=M_k+1}^{q_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1, \cdots, s, i = 1, \cdots, r, i^{(k)} = 1, \cdots, r^{(k)} \tag{1.17}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_s) = 0(|z_1|^{\alpha'_1}, \cdots, |z_s|^{\alpha'_s}), max(|z_1|, \cdots, |z_s|) \rightarrow 0$

$\aleph(z_1, \cdots, z_s) = 0(|z_1|^{\beta'_1}, \cdots, |z_s|^{\beta'_s}), min(|z_1|, \cdots, |z_s|) \rightarrow \infty$

where $k = 1, \cdots, z : \alpha'_k = min[Re(b_j^{(k)} / \beta_j^{(k)})], j = 1, \cdots, M_k$ and

$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, N_k$

We will use these following notations in this paper

$$U' = P_i, Q_i, \iota_i; r' ; \; V' = M_1, N_1; \cdots ; M_s, N_s \tag{1.18}$$

$$W' = P_{i^{(1)}}, Q_{i^{(1)}}, \iota_{i^{(1)}}; r^{(1)}, \cdots , P_{i^{(r)}}, Q_{i^{(r)}}, \iota_{i^{(s)}}; r^{(s)} \tag{1.19}$$

$$A' = \{(u_j; \mu_j^{(1)}, \cdots , \mu_j^{(s)})_{1,N}\}, \{\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots , \mu_{ji}^{(s)})_{N+1,P_i}\} \tag{1.20}$$

$$B' = \{\iota_i(v_{ji}; v_{ji}^{(1)}, \cdots , v_{ji}^{(s)})_{M+1,Q_i}\} \tag{1.21}$$

$$C' = (a_j^{(1)}; \alpha_j^{(1)})_{1,N_1} , \iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_{i^{(1)}}}, \cdots , (a_j^{(s)}; \alpha_j^{(s)})_{1,N_s} , \iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)}; \alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_{i^{(s)}}} \tag{1.22}$$

$$D' = (b_j^{(1)}; \beta_j^{(1)})_{1,M_1}, \iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)}; \beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_{i^{(1)}}}, \cdots , (b_j^{(s)}; \beta_j^{(s)})_{1,M_s}, \iota_{i^{(s)}}(\beta_{ji^{(s)}}^{(s)}; \beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_{i^{(s)}}} \tag{1.23}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots , z_s) = \aleph_{U':W'}^{0,N:V'} \left(\begin{array}{c|c} z_1 & \text{A' : C'} \\ \cdot & \cdots \\ \cdot & \text{B' : D'} \\ z_s & \end{array} \right) \tag{1.24}$$

Srivastava and Garg [4] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_v}[z_1, \cdots , z_v] = \sum_{R_1,\cdots,R_v=0}^{h_1R_1+\cdots h_vR_v \leqslant L} (-L)_{h_1R_1+\cdots+h_vR_v} B(E; R_1, \cdots , R_v) \frac{z_1^{R_1} \cdots z_r^{R_v}}{R_1! \cdots R_v!} \tag{1.25}$$

2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [5 ,page 39 eq .30]

$$\frac{\prod_{j=1}^P \Gamma(A_j)}{\prod_{j=1}^Q \Gamma(B_j)} {}_PF_Q \left[(A_P); (B_Q); -(x_1 + \cdots + x_r)\right]$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \frac{\prod_{j=1}^P \Gamma(A_j + s_1 + \cdots + s_r)}{\prod_{j=1}^Q \Gamma(B_j + s_1 + \cdots + s_r)} \Gamma(-s_1) \cdots \Gamma(-s_r) x_1^{s_1} \cdots x_r^{s_r} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j + s_1 + \cdots + s_r)$ are separated from those of $\Gamma(-s_j), j = 1, \cdots , r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j = 1, \cdots , r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1} \prod_{j=1}^{l} \left[1 - \tau_j(t-a)^{h_i}\right]^{-\lambda_j} \prod_{j=1}^{k} (f_jt + g_j)^{\sigma_j} \mathrm{d}t = (b-a)^{\alpha+\beta-1} B(\alpha, \beta) \prod_{j=1}^{k} (af_j + g_j)^{\sigma_j}$$

$$F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right) \tag{2.2}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i,\tau_j,h_j\in\mathbb{C},\lambda_j\in\mathbb{R}^+(i=1,\cdots,k;j=1,\cdots,l)$

$$min(Re(\alpha),Re(\beta))>0,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1,\ \max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,$$

and $F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust[3,page 454] and [4] given by :

$$F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\prod_{j=1}^{l}\Gamma(\lambda_j)\prod_{j=1}^{k}\Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}}\int_{L_1}\cdots\int_{L_{l+k}}\frac{\Gamma\left(\alpha+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}{\Gamma\left(\alpha+\beta+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}\prod_{j=1}^{l}\Gamma(\lambda_j+s_j)\prod_{j=1}^{k}\Gamma(-\sigma_j+s_{l+j})$$

$$\prod_{j=1}^{l+k}\Gamma(-s_j)z_1^{s_1}\cdots z_l^{s_l}z_{l+1}^{s_{l+1}}\cdots,z_{l+k}^{s_{l+k}}\,\mathrm{d}s_1\cdots\mathrm{d}s_{l+k} \tag{2.3}$$

Here the contour $L_j's$ are defined by $L_j=L_{w\zeta_j\infty}(Re(\zeta_j)=v_j'')$ starting at the point $v_j''-\omega\infty$ and terminating at the point $v_j''+\omega\infty$ with $v_j''\in\mathbb{R}(j=1,\cdots,l)$ and each of the remaining contour $L_{l+1},\cdots,L_{l+k}$ run from $-\omega\infty$ to $\omega\infty$

(2.2) can be easily established by expanding $\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.4}$$

integrating term by term with the help of the integral given by Saigo and Saxena [1, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [3, page 454].

3. Eulerian integral

In this section , we note :

$$\theta_i = \prod_{j=1}^{l} \left[1-\tau_j (t-a)^{h_i}\right]^{-\zeta_j^{(i)}}, \zeta_j^{(i)} > 0 (i=1,\cdots,r)\,;\, \theta_i' = \prod_{j=1}^{l} \left[1-\tau_j (t-a)^{h_i}\right]^{-\zeta_j'^{(i)}}, \zeta_j'^{(i)} > 0 (i=1,\cdots,s)$$

$$\theta_i'' = \prod_{j=1}^{l} \left[1-\tau_j (t-a)^{h_i}\right]^{-\zeta_j''^{(i)}}, \zeta_j''^{(i)} > 0 (i=1,\cdots,u) \tag{3.1}$$

$$K_1 = (1-\alpha-\sum_{i=1}^{u} R_i a_i; \mu_1,\cdots,\mu_r,\mu_1',\cdots,\mu_s',h_1,\cdots,h_l,1,\cdots,1) \tag{3.2}$$

$$K_2 = (1-\beta-\sum_{i=1}^{u} R_i b_i; \rho_1,\cdots,\rho_r,\rho_1',\cdots,\rho_s',0,\cdots,0,0\cdots,0) \tag{3.3}$$

$$K_j = [1-\lambda_j-\sum_{i=1}^{u} R_i \zeta_j''^{(i)}; \zeta_j^{(1)},\cdots,\zeta_j^{(r)},\zeta_j'^{(1)}\cdots,\zeta_j'^{(s)},0,\cdots,\underset{\text{j}}{1},\cdots,0,0\cdots,0]_{1,l} \tag{3.4}$$

$$K_j' = [1+\sigma_j-\sum_{i=1}^{u} R_i \lambda_j''^{(i)}; \lambda_j^{(1)},\cdots,\lambda_j^{(r)},\lambda_j'^{(1)}\cdots,\lambda_j'^{(s)},0,\cdots,0,0\cdots,\underset{\text{j}}{1},\cdots,0]_{1,k} \tag{3.5}$$

$$L_1 = (1-\alpha-\beta-\sum_{i=1}^{u} R_i (a_i+b_i); \mu_1+\rho_1,\cdots,\mu_r+\rho_r,\mu_1'+\rho_1',\cdots,\mu_r'+\rho_r',$$

$$h_1,\cdots,h_l,1,\cdots,1) \tag{3.6}$$

$$L_j = [1-\lambda_j-\sum_{i=1}^{u} R_i \zeta_j''^{(i)}; \zeta_j^{(1)},\cdots,\zeta_j^{(r)},\zeta_j'^{(1)}\cdots,\zeta_j'^{(s)},0,\cdots,0,0\cdots,0]_{1,l} \tag{3.7}$$

$$L_j' = [1+\sigma_j-\sum_{i=1}^{u} R_i \lambda_j''^{(i)}; \lambda_j^{(1)},\cdots,\lambda_j^{(r)},\lambda_j'^{(1)}\cdots,\lambda_j'^{(s)},0,\cdots,0,0,\cdots,0]_{1,k} \tag{3.8}$$

$$V_1 = V;V';1,0;\cdots;1,0;1,0;\cdots;1,0\ ;W_1 = W;W';0,1;\cdots;0,1;0,1;\cdots;0,1 \tag{3.9}$$

$$C_1 = C;C';(1,0);\cdots;(1,0);(1,0);\cdots;(1,0)\,;D_1 = D;D';(0,1);\cdots;(0,1);(0,1);\cdots;(0,1) \tag{3.10}$$

$$P_1 = (b-a)^{\alpha+\beta-1}\left\{\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}\right\} \tag{3.11}$$

$$P_u = (b-a)^{\sum_{i=1}^{u}(a_i+b_i)R_i}\left\{\prod_{j=1}^{h}(af_j+g_j)^{-\sum_{l=1}^{u}\lambda_j''^{(i)}R_i}\right\} \tag{3.12}$$

$$B_u = \frac{(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!} \tag{3.13}$$

We the following generalized Eulerian integral :

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_u}\left(\begin{array}{c} \mathrm{z}_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j''^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j''^{(u)}} \end{array}\right)$$

$$\aleph\left(\begin{array}{c} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array}\right)\aleph\left(\begin{array}{c} \mathrm{z}_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)$$

$$\mathrm{d}t = P_1\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\prod_{k=1}^{u} z_k''^{R_k}P_uB_u$$

$$\aleph_{U;U';l+k+2,l+k+1:W_1}^{0,n+N+l+k+2:V_1}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(1)}}} & \mathrm{A}\ ;\ \mathrm{A}\ ;\ \mathrm{K}_1,K_2,K_j,K_j' : C_1 \\ \cdots & . \\ \cdots & . \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(r)}}} & . \\ \frac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j'^{(1)}}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z_s'(b-a)^{\mu_s'+\rho_s'}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j'^{(s)}}} & . \\ \tau_1(b-a)^{h_1} & \\ \cdots & \\ \cdots & \\ \tau_l(b-a)^{h_l} & \mathrm{B}\ ;\mathrm{B'};\ \mathrm{L}_1,L_j,L_j' : D_1 \\ \frac{(b-a)f_1}{af_1+g_1} & \\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (3.14)$$

We obtain the Aleph-function of $r+s+k+l$ variables. The quantities $A, A', B, B', C, C', C_1, D_1, V_1$ and W_1 are defined above.

Provided that

(A) $a, b \in \mathbb{R}(a < b); \mu_i, \mu'_u, \rho_i, \rho'_u, \lambda_j^{(i)}, \lambda_j'^{(u)}, h_v \in \mathbb{R}^+, f_i, g_j, \tau_v, \sigma_j, \lambda_v \in \mathbb{C}$ $(i = 1, \cdots, r; j = 1, \cdots; k;$

$u = 1, \cdots, s; v = 1, \cdots, l), a_i, b_i, \zeta_j''^{(i)} \in \mathbb{R}^+, (i = 1, \cdots, u; j = 1, \cdots, k)$

(B) See the section 1

(C) $\max_{1 \leqslant j \leqslant k} \left\{ \left| \frac{(b-a)f_i}{af_i + g_i} \right| \right\} < 1, \ \max_{1 \leqslant j \leqslant l} \left\{ \left| \tau_j (b-a)^{h_j} \right| \right\} < 1$

(D) $$Re\big[\alpha + \sum_{j=1}^{r} \mu_j \min_{1 \leqslant k \leqslant m_i} \frac{d_k^{(j)}}{\delta_k^{(j)}} + \sum_{j=1}^{s} \mu'_i \min_{1 \leqslant k \leqslant M_i} \frac{b_k^{(j)}}{\beta_k^{(j)}}\big] > 0$$

$$Re\big[\beta + \sum_{j=1}^{r} \rho_j \min_{1 \leqslant k \leqslant m_i} \frac{d_k^{(j)}}{\delta_k^{(j)}} + \sum_{j=1}^{s} \rho'_i \min_{1 \leqslant k \leqslant M_i} \frac{b_k^{(j)}}{\beta_k^{(j)}}\big] > 0$$

(E) $$Re \left(\alpha + \sum_{i=1}^{u} R_i a_i + \sum_{i=1}^{r} \mu_i s_i + \sum_{i=1}^{s} t_i \mu'_i \right) > 0 \, ; Re \left(\beta + \sum_{i=1}^{u} R_i b_i + \sum_{i=1}^{r} v_i s_i + \sum_{i=1}^{s} t_i \rho'_i \right) > 0$$

$$Re \left(\lambda_j + \sum_{i=1}^{u} R_i \lambda_j''^{(i)} + \sum_{i=1}^{r} s_i \zeta_j^{(i)} + \sum_{i=1}^{s} t_i \zeta_j'^{(i)} \right) > 0 (j = 1, \cdots, l);$$

$$Re \left(-\sigma_j + \sum_{i=1}^{u} R_i \lambda_j''^{(i)} + \sum_{i=1}^{r} s_i \lambda_j^{(i)} + \sum_{i=1}^{s} t_i \lambda_j'^{(i)} \right) > 0 (j = 1, \cdots, k);$$

(F) $$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0$$

$$U_i'^{(k)} = \sum_{j=1}^{N} \mu_j^{(k)} + \iota_i \sum_{j=N+1}^{P_i} \mu_{ji}^{(k)} + \sum_{j=1}^{N_k} \alpha_j^{(k)} + \iota_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \alpha_{ji^{(k)}}^{(k)} - \iota_i \sum_{j=1}^{Q_i} \upsilon_{ji}^{(k)} - \sum_{j=1}^{M_k} \beta_j^{(k)}$$

$$-\iota_{i^{(k)}} \sum_{j=M_k+1}^{Q_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} \leqslant 0$$

(G) $A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$

$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} - \sum_{l=1}^{k} \lambda_j^{(i)} - \sum_{l=1}^{l} \zeta_j^{(i)} - \mu_k - \rho_k > 0,$ with $k = 1 \cdots, r,$

$i = 1, \cdots, R \ , i^{(k)} = 1, \cdots, R^{(k)}$

$B_i^{(k)} = \sum_{j=1}^{N} \mu_j^{(k)} - \iota_i \sum_{j=N+1}^{P_i} \mu_{ji}^{(k)} - \iota_i \sum_{j=1}^{Q_i} \upsilon_{ji}^{(k)} + \sum_{j=1}^{N_k} \alpha_j^{(k)} - \iota_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \alpha_{ji^{(k)}}^{(k)}$

$+ \sum_{j=1}^{M_k} \beta_j^{(k)} - \iota_{i^{(k)}} \sum_{j=M_k+1}^{q_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} - \sum_{l=1}^{k} \lambda_j'^{(i)} - \sum_{l=1}^{l} \zeta_j'^{(i)} - \mu_k' - \rho_k' > 0,$ with $k = 1, \cdots, s,$

$i = 1, \cdots, r \ , i^{(k)} = 1, \cdots, r^{(k)}$

(H) $\left| arg \left(z_i \prod_{j=1}^{l} \left[1 - \tau_j (t-a)^{h_i}\right]^{-\zeta_j^{(i)}} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j^{(i)}} \right) \right| < \frac{1}{2} A_i^{(k)} \pi \quad (a \leqslant t \leqslant b; i = 1, \cdots, r)$

$\left| arg \left(z_i' \prod_{j=1}^{l} \left[1 - \tau_j' (t-a)^{h_i'}\right]^{-\zeta_j'^{(i)}} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'^{(i)}} \right) \right| < \frac{1}{2} B_i^{(k)} \pi \quad (a \leqslant t \leqslant b; i = 1, \cdots, s)$

Proof

To prove (3.24), first, we express in serie a class of multivariable polynomials defined by Srivastava et al [4], $S_L^{h_1, \cdots, h_u}[.]$ with the help of (1.25), espressing the Aleph-function of r variables by the Mellin-Barnes contour integral with the help of the equation (1.1), the Aleph-function of s variables by the Mellin-Barnes contour integral with the help of the equation (1.13). Now collect the power of $\left[1 - \tau_j (t-a)^{h_i}\right]$ with $(i = 1, \cdots, r; j = 1, \cdots, l)$ and collect the power of $(f_j t + g_j)$ with $j = 1, \cdots, k$. Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the $(r+s+k+l)$ dimensional Mellin-Barnes integral in multivariable Aleph-function, we obtain the equation (3.14).

Remarks

If a) $\rho_1 = \cdots, \rho_r = \rho_1' = \cdots, \rho_s' = 0$; b) $\mu_1 = \cdots, \mu_r = \mu_1' = \cdots, \mu_s' = 0$, we obtain the similar formulas that (3.24) with the corresponding simplifications.

4. Particular cases

a) If $\iota_i, \iota_{i^{(1)}}, \cdots, \iota_{i^{(s)}} \rightarrow 1$, the multivariable Aleph-function of s-variables reduces to multivariable I-function of s-variables defined by Sharma and al [2] and we have

$$\int_a^b (t-a)^{\alpha-1} (b-t)^{\beta-1} \prod_{j=1}^{l} \left[1 - \tau_j (t-a)^{h_i}\right]^{-\lambda_j} \prod_{j=1}^{k} (f_j t + g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_u}\left(\begin{array}{c} z''_1\theta''_1(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda''^{(1)}_j} \\ . \\ . \\ . \\ z''_u\theta''_u(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^k(f_jt+g_j)^{-\lambda''^{(u)}_j} \end{array}\right)$$

$$I\left(\begin{array}{c} z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda^{(1)}_j} \\ . \\ . \\ . \\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^k(f_jt+g_j)^{-\lambda^{(r)}_j} \end{array}\right)$$

$$I\left(\begin{array}{c} z'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda'^{(1)}_j} \\ . \\ . \\ . \\ z'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^k(f_jt+g_j)^{-\lambda'^{(s)}_j} \end{array}\right)\mathrm{d}t = P_1\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\prod_{k=1}^{u} z''^{R_k}_k P_u B_u$$

$$I^{0,n+N+l+k+2:V_1}_{U;U';l+k+2,l+k+1:W_1}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(1)}}} & \\ \cdots & \\ \cdots & \mathrm{A}\ ;\mathrm{K}_1,K_2,K_j,K'_j,\mathfrak{A},\mathfrak{A}';\mathfrak{A}_1 \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(r)}}} & . \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^k(af_j+g_j)^{\lambda'^{(1)}_j}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^k(af_j+g_j)^{\lambda'^{(s)}_j}} & . \\ \tau_1(b-a)^{h_1} & . \\ \cdots & . \\ \cdots & \\ \tau_l(b-a)^{h_l} & \mathrm{B}\ ,\mathrm{L}_1,L_j,L'_j,\mathfrak{B},\mathfrak{B}';\mathfrak{B}_1 \\ \frac{(b-a)f_1}{af_1+g_1} & \\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad \textbf{(4.1)}$$

under the same notations and conditions that (3.14) with $\iota_i,\iota_{i^{(1)}},\cdots,\iota_{i^{(s)}}\to 1$

b) If $B(L;R_1,\cdots,R_u) = \dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_u\theta_j^{(u)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi'_j+\cdots+m_u\psi_j^{(u)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}}$ (4.2)

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [3].

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\text{-L});\mathrm{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}]:[(b');\phi'];\cdots;[(b^{(u)});\phi^{(u)}] \\ \cdots & \\ \cdots & [(\mathrm{c});\psi',\cdots,\psi^{(u)}]:[(d');\delta'];\cdots;[(d^{(u)});\delta^{(u)}] \\ \mathrm{z}_u & \end{array}\right) \quad (4.3)$$

and we have the following formula

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}}\left(\begin{array}{c} \mathrm{z}''_1\theta''_1(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda''^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}''_u\theta''_u(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda''^{(u)}_j} \end{array}\right.$$

$$\left.\begin{array}{c} [(\text{-L});\mathrm{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}]:[(b');\phi'];\cdots;[(b^{(u)});\phi^{(u)}] \\ [(\mathrm{c});\psi',\cdots,\psi^{(u)}]:[(d');\delta'];\cdots;[(d^{(u)});\delta^{(u)}] \end{array}\right)$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array}\right)$$

$$\aleph_{U':W'}^{0,N:V'}\left(\begin{array}{c} z'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{\prime(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{\prime(s)}} \end{array}\right)\mathrm{d}t =$$

$$= P_1 \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L} \prod_{k=1}^{u} z_k^{\prime\prime R_k} P_u B'_u$$

$$\aleph_{U;U';l+k+2,l+k+1:W_1}^{0,n+N+l+k+2:V_1}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \\ \cdot\ \cdot\ \cdot & \\ \cdot\ \cdot\ \cdot & \mathrm{A}\ ;\ \mathrm{A}\ ;\ \mathrm{K}_1, K_2, K_j, K'_j : C_1 \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{\prime(1)}}} & \cdot \\ \cdot\ \cdot\ \cdot & \cdot \\ \cdot\ \cdot\ \cdot & \cdot \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{\prime(s)}}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdot\ \cdot\ \cdot & \cdot \\ \cdot\ \cdot\ \cdot & \\ \tau_l(b-a)^{h_l} & \mathrm{B}\ ;\mathrm{B'};\ \mathrm{L}_1, L_j, L'_j : D_1 \\ \frac{(b-a)f_1}{af_1+g_1} & \\ \cdot\ \cdot\ \cdot & \\ \cdot\ \cdot\ \cdot & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad \textbf{(4.4)}$$

under the same conditions that (3.14)

and $B'_u = \dfrac{(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!}$; $B(L;R_1,\cdots,R_u)$ is defined by (4.2)

Remark:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable Aleph-functions and a class of multivariable polynomials defined by Srivastava et al [4].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable Aleph-functions and a class of multivariable polynomials defined by Srivastava et al [4] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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