Eulerian integral associated with product of two multivariable I-functions, generalized Lauricella function and a class of polynomials and the multivariable I-function defined by Nambisan I

$F.Y.\ AYANT^1$

1 Teacher in High School , France

ABSTRACT
The present paper is evaluated a new Eulerian integral associated with the product of two multivariable I-functions defined by Prasad [1] a generalized Lauricella function , a class of multivariable polynomials and Multivariable I-function defined by Nambisan [2] with general arguments . We will study the case concerning the multivariable H-function defined by Srivastava et al [7]. .

Keywords: Eulerian integral, multivariable I-function, generalized Lauricella function of several variables, multivariable H-function, generalized hypergeometric function, class of polynomials

2010 Mathematics Subject Classification. 33C60, 82C31

1. Introduction

In this paper, we evaluate a new Eulerian integral of most general characters associated with the products of two multivariable I-functions defined by Prasad [1] , a expansion serie of multivariable I-function defined by Nambisan et al [2] and a class of polynomials with general arguments.
First time, we define the multivariable I-function by :

$$\bar{I}(z_1,\cdots,z_v)=\bar{I}^{0,N:M_1,N_1;\cdots;M_v,N_v}_{P,Q:P_1,Q_1;\cdots;P_v,Q_v}\left(\begin{array}{c|c} \mathrm{z}_1''' & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(v)};A_j)_{N+1,P}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_v''' & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(v)};B_j)_{M+1,Q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};1)_{1,N_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{N_1+1,P_1};\cdots;(c_j^{(v)},\gamma_j^{(v)};1)_{1,N_u},(c_j^{(v)},\gamma_j^{(v)};C_j^{(v)})_{N_v+1,P_v} \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};1)_{1,M_1},(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{M_1+1,Q_1};\cdots;(d_j^{(v)},\delta_j^{(v)};1)_{1,M_v},(d_j^{(v)},\delta_j^{(v)};D_j^{(v)})_{M_v+1,Q_v} \end{array}\right) \tag{1.1}$$

$$=\frac{1}{(2\pi\omega)^v}\int_{L_1}\cdots\int_{L_v}\phi_1(s_1,\cdots,s_v)\prod_{i=1}^{v}\theta_i(s_i)z_i''^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_v \tag{1.2}$$

where $\phi_1(s_1,\cdots,s_v),\theta_i(s_i),i=1,\cdots,v$ are given by :

$$\phi_1(s_1,\cdots,s_v)=\frac{1}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{v}\alpha_j^{(i)}s_j\right)\prod_{j=M+1}^{Q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{v}\beta_j^{(i)}s_j\right)} \tag{1.3}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{N_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}-\delta_j^{(i)}s_i\right)} \tag{1.4}$$

$i = 1, \cdots, v$

Serie representation

If $z_i''' \neq 0; i = 1, \cdots, v$

$\delta_{h_i}^{(i)}(d_j^{(i)}+k_i) \neq \delta_j^{(i)}(\delta_{h_i}^{(i)}+\eta_i) for j \neq h_i, j, h_i = 1, \cdots, m_i(i=1,\cdots,v)$,$k_i, \eta_i = 0,1,2,\cdots (i=1,\cdots,v)$, then

$$\bar{I}(z_1''',\cdots,z_v''') = \sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\left[\phi_1\left(\frac{dh_1^{(1)}+k_1}{\delta h_1^{(1)}},\cdots,\frac{dh_v^{(v)}+k_v}{\delta h_v^{(v)}}\right)\right]\prod_{j\neq h_i i=1}^{r}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\frac{dh_i+k_i}{\delta h_i}} \tag{1.5}$$

This result can be proved on computing the residues at the poles :

$$s_i = \frac{dh_i^{(i)}+k_i}{\delta h_i^{(i)}}, (h_i = 1,\cdots,m_i, k_i = 0,1,2,\cdots) for i = 1,\cdots,v \tag{1.6}$$

We may establish the the asymptotic expansion in the following convenient form :

$\bar{I}(z_1''',\cdots,z_v''') = 0(\,|z_1'''|^{\alpha_1},\cdots,|z_v'''|^{\alpha_v}\,), max(\,|z_1'''|,\cdots,|z_v'''|\,) \rightarrow 0$

$I(z_1''',\cdots,z_v''') = 0(\,|z_1'''|^{\beta_1},\cdots,|z_v'''|^{\beta_u}\,), min(\,|z_1'''|,\cdots,|z_v'''|\,) \rightarrow \infty$

where $k = 1,\cdots,v : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1,\cdots,m_k$ and

$$\beta_k = max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j = 1,\cdots,n_k$$

We will note $\eta_{h_i,k_i} = \dfrac{dh_i^{(i)}+k_i}{\delta h_i}$, $(h_i = 1,\cdots,m_i, k_i = 0,1,2,\cdots) for i = 1,\cdots,v$ (1.7)

The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,\cdots,z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c|c} z_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)})_{1,p_r} : (a_j^{(1)},\alpha_j^{(1)})_{1,p^{(1)}};\cdots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta_{rj}^{(1)},\cdots,\beta_{rj}^{(r)})_{1,q_r} : (b_j^{(1)},\beta_j^{(1)})_{1,q^{(1)}};\cdots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}} \end{array}\right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1, \cdots, s_r) \prod_{i=1}^{r} \theta_i(s_i) z_i^{s_i} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.9}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i \pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)} \right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)} \right) \tag{1.10}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}), max(|z_1|, \cdots, |z_r|) \rightarrow 0$$

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}), min(|z_1|, \cdots, |z_r|) \rightarrow \infty$$

where $k = 1, \cdots, r : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

Condider a second multivariable I-function defined by Prasad [1]

$$I(z'_1, z'_2, ...z'_s) = I_{p'_2,q'_2,p'_3,q'_3;\cdots;p'_s,q'_s:p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)}}^{0,n'_2;0,n'_3;\cdots;0,n'_r:m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)}} \left(\begin{array}{c|c} z'_1 & (a'_{2j}; \alpha'^{(1)}_{2j}, \alpha'^{(2)}_{2j})_{1,p'_2}; \cdots ; \\ \cdot & \\ \cdot & \\ \cdot & \\ z'_s & (b'_{2j}; \beta'^{(1)}_{2j}, \beta'^{(2)}_{2j})_{1,q'_2}; \cdots ; \end{array} \right.$$

$$\left. \begin{array}{c} (a'_{sj}; \alpha'^{(1)}_{sj}, \cdots, \alpha'_{sj}{}^{(s)})_{1,p'_s} : (a'^{(1)}_j, \alpha'^{(1)}_j)_{1,p'^{(1)}}; \cdots ; (a'_j{}^{(s)}, \alpha'_j{}^{(s)})_{1,p'^{(s)}} \\ \\ (b'_{sj}; \beta'^{(1)}_{sj}, \cdots, \beta'_{sj}{}^{(s)})_{1,q'_s} : (b'^{(1)}_j, \beta'^{(1)}_j)_{1,q'^{(1)}}; \cdots ; (b'_j{}^{(s)}, \beta'_j{}^{(s)})_{1,q'^{(s)}} \end{array} \right) \tag{1.11}$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\psi(t_1,\cdots,t_s)\prod_{i=1}^{s}\xi_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.12}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

where $|argz_i'| < \frac{1}{2}\Omega_i'\pi$,

$$\Omega_i' = \sum_{k=1}^{n'^{(i)}}\alpha_k'^{(i)} - \sum_{k=n'^{(i)}+1}^{p'^{(i)}}\alpha_k'^{(i)} + \sum_{k=1}^{m'^{(i)}}\beta_k'^{(i)} - \sum_{k=m^{(i)}+1}^{q'^{(i)}}\beta_k'^{(i)} + \left(\sum_{k=1}^{n_2'}\alpha_{2k}'^{(i)} - \sum_{k=n_2+1}^{p_2'}\alpha_{2k}'^{(i)}\right)$$

$$+\cdots+\left(\sum_{k=1}^{n_s'}\alpha_{sk}'^{(i)} - \sum_{k=n_s'+1}^{p_s'}\alpha_{sk}'^{(i)}\right) - \left(\sum_{k=1}^{q_2'}\beta_{2k}'^{(i)} + \sum_{k=1}^{q_3'}\beta_{3k}'^{(i)} + \cdots + \sum_{k=1}^{q_s'}\beta_{sk}'^{(i)}\right) \tag{1.13}$$

where $i = 1,\cdots,s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1',\cdots,z_s') = 0(\,|z_1'|^{\alpha_1'},\cdots,|z_s'|^{\alpha_s'}\,)\,, max(\,|z_1'|\,,\cdots,|z_s'|\,) \to 0$$

$$I(z_1',\cdots,z_s')\ = 0(\,|z_1'|^{\beta_1'}\,,\cdots,|z_s'|^{\beta_s'}\,)\,, min(\,|z_1'|\,,\cdots,|z_s'|\,) \to \infty$$

where $k = 1,\cdots,z : \alpha_k'' = min[Re(b_j'^{(k)}/\beta_j'^{(k)})], j = 1,\cdots,m_k'$ and

$$\beta_k'' = max[Re((a_j'^{(k)} - 1)/\alpha_j'^{(k)})], j = 1,\cdots,n_k'$$

where $k = 1,\cdots,z : \alpha_k'' = min[Re(b_j'^{(k)}/\beta_j'^{(k)})], j = 1,\cdots,m_k'$ and

$$\beta_k'' = max[Re((a_j'^{(k)} - 1)/\alpha_j'^{(k)})], j = 1,\cdots,n_k'$$

Srivastava [4] introduced and defined a general class of multivariable polynomials as follows

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[y_1,\cdots,y_u] = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!}$$

$$A[N_1,K_1;\cdots;N_u,K_u]y_1^{K_1}\cdots y_u^{K_u} \tag{1.14}$$

where $M_1,\cdots,M_u$ are arbitrary positive integers and the coefficients are $A[N_1,K_1;\cdots;N_u,K_u]$ arbitrary constants, real or complex.

2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [6 ,page 39 eq .30]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^{P}\Gamma(A_j+s_1+\cdots+s_r)}{\prod_{j=1}^{Q}\Gamma(B_j+s_1+\cdots+s_r)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j+s_1+\cdots+s_r)$ are separated from those of $\Gamma(-s_j), j=1,\cdots,r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j=1,\cdots,r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b(t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}\mathrm{d}t\ =(b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}$$

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right) \tag{2.2}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i,\tau_j,h_j\in\mathbb{C},\lambda_j\in\mathbb{R}^+(i=1,\cdots,k;j=1,\cdots,l)$

$$min(Re(\alpha),Re(\beta))>0,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1,\ \max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,$$

and $F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust[3,page 454] given by :

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\prod_{j=1}^{l}\Gamma(\lambda_j)\prod_{j=1}^{k}\Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}}\int_{L_1}\cdots\int_{L_{l+k}}\frac{\Gamma\left(\alpha+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}{\Gamma\left(\alpha+\beta+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}\prod_{j=1}^{l}\Gamma(\lambda_j+s_j)\prod_{j=1}^{k}\Gamma(-\sigma_j+s_{l+j})$$

$$\prod_{j=1}^{l+k}\Gamma(-s_j)z_1^{s_1}\cdots z_l^{s_l}z_{l+1}^{s_{l+1}}\cdots,z_{l+k}^{s_{l+k}}\,\mathrm{d}s_1\cdots\mathrm{d}s_{l+k} \tag{2.3}$$

Here the contour $L_j's$ are defined by $L_j=L_{w\zeta_j\infty}(Re(\zeta_j)=v_j'')$ starting at the point $v_j''-\omega\infty$ and terminating at the point $v_j''+\omega\infty$ with $v_j''\in\mathbb{R}(j=1,\cdots,l)$ and each of the remaining contour $L_{l+1},\cdots,L_{l+k}$ run from $-\omega\infty$ to $\omega\infty$

(2.2) can be easily established by expanding $\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.4}$$

integrating term by term with the help of the integral given by Saigo and Saxena [3, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [5, page 454].

3. Eulerian integral

In this section , we note :

$$\theta_i=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}},\zeta_j^{(i)}>0(i=1,\cdots,r)\,;\,\theta_i'=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'^{(i)}},\zeta_j'^{(i)}>0(i=1,\cdots,s)$$

$$\theta_i''=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j''^{(i)}},\zeta_j''^{(i)}>0(i=1,\cdots,u)$$

$$\theta_i'''=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'''^{(i)}},\zeta_j'''^{(i)}>0(i=1,\cdots,v) \tag{3.1}$$

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};p_2',q_2';p_3',q_3';\cdots;p_{s-1}',q_{s-1}';0,0;\cdots;0,0;0,0;\cdots;0,0 \tag{3.2}$$

$$V=0,n_2;0,n_3;\cdots;0,n_{r-1};0,n_2';0,n_3';\cdots;0,n_{s-1}';0,0;\cdots;0,0;0,0;\cdots;0,0 \tag{3.3}$$

$$X=m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)};m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)};1,0;\cdots;1,0;1,0;\cdots;1,0 \tag{3.4}$$

$$Y=p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)};p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)};0,1;\cdots;0,1;0,1;\cdots;0,1 \tag{3.5}$$

$$A=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)});\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)})\,;(a_{2k}';\alpha_{2k}'^{(1)},\alpha_{2k}'^{(2)});\cdots;$$

$$(a_{(s-1)k}';\alpha_{(s-1)k}'^{(1)},\alpha_{(s-1)k}'^{(2)},\cdots,\alpha_{(s-1)k}'^{(s-1)}) \tag{3.6}$$

$$;(b_{2k}';\beta_{2k}'^{(1)},\beta_{2k}'^{(2)});\cdots;B=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)});\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)})$$

$$(b'_{(s-1)k};\beta'^{(1)}_{(s-1)k},\beta'^{(2)}_{(s-1)k},\cdots,\beta'^{(s-1)}_{(s-1)k}) \tag{3.7}$$

$$\mathfrak{A}=(a_{rk};\alpha^{(1)}_{rk},\alpha^{(2)}_{rk},\cdots,\alpha^{(r)}_{rk},0,\cdots,0,0\cdots,0,0,\cdots,0) \tag{3.8}$$

$$\mathfrak{A}'=(a'_{sk};0,\cdots,0,\alpha'^{(1)}_{sk},\alpha'^{(2)}_{sk},\cdots,\alpha'^{(s)}_{sk},0,\cdots,0,0,\cdots,0) \tag{3.9}$$

$$\mathfrak{B}=(b_{rk};\beta^{(1)}_{rk},\beta^{(2)}_{rk},\cdots,\beta^{(r)}_{rk},0,\cdots,0,0,\cdots,0,0,\cdots,0) \tag{3.10}$$

$$\mathfrak{B}'=(b'_{sk};0,\cdots,0,\beta'^{(1)}_{sk},\beta'^{(2)}_{sk},\cdots,\beta'^{(s)}_{sk},0,\cdots,0,0,\cdots,0) \tag{3.11}$$

$$\mathfrak{A}_1=(a^{(1)}_k,\alpha^{(1)}_k)_{1,p^{(1)}};\cdots;(a^{(r)}_k,\alpha^{(r)}_k)_{1,p^{(r)}};(a'^{(1)}_k,\alpha^{(1)}_k)_{1,p^{(1)}};\cdots;(a'^{(s)}_k,\alpha'^{(s)}_k)_{1,p'^{(s)}};$$

$$(1,0);\cdots;(1,0);(1.0);\cdots;(1.0) \tag{3.12}$$

$$\mathfrak{B}_1=(b^{(1)}_k,\beta^{(1)}_k)_{1,q^{(1)}};\cdots;(b^{(r)}_k,\beta^{(r)}_k)_{1,q^{(r)}};(b'^{(1)}_k,\beta'^{(1)}_k)_{1,q'^{(1)}};\cdots;(b'^{(s)}_k,\beta'^{(s)}_k)_{1,q'^{(s)}};$$

$$(0,1);\cdots;(0,1);(0,1);\cdots;(0,1) \tag{3.13}$$

$$K_1=(1-\alpha-\sum_{i=1}^{u}K_ia_i-\sum_{i=1}^{v}\eta_{G_i,g_i}a'_i;\mu_1,\cdots,\mu_r,\mu'_1,\cdots,\mu'_s,h_1,\cdots,h_l,1,\cdots,1) \tag{3.14}$$

$$K_2=(1-\beta-\sum_{i=1}^{u}K_ib_i-\sum_{i=1}^{v}\eta_{G_i,g_i}b'_i;\rho_1,\cdots,\rho_r,\rho'_1,\cdots,\rho'_s,0,\cdots,0,0\cdots,0) \tag{3.15}$$

$$K_j=[1-\lambda_j-\sum_{i=1}^{u}K_i\zeta''^{(i)}_j-\sum_{i=1}^{v}\eta_{G_i,g_i}\zeta'''^{(i)}_j;\zeta^{(1)}_j,\cdots,\zeta^{(r)}_j,\zeta'^{(1)}_j\cdots,\zeta'^{(s)}_j,$$

$$0,\cdots,\underset{\text{j}}{1},\cdots,0,0\cdots,0]_{1,l} \tag{3.16}$$

$$K'_j=[1+\sigma_j-\sum_{i=1}^{u}K_i\lambda''^{(i)}_j-\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}_j;\lambda^{(1)}_j,\cdots,\lambda^{(r)}_j,\lambda'^{(1)}_j\cdots,\lambda'^{(s)}_j,$$

$$0,\cdots,0,0\cdots,\underset{\text{j}}{1},\cdots,0]_{1,k} \tag{3.17}$$

$$L_1=(1-\alpha-\beta-\sum_{i=1}^{u}K_i(a_i+b_i)-\sum_{i=1}^{v}(a'_i+b'_i)\eta_{G_i,g_i};\mu_1+\rho_1,\cdots,\mu_r+\rho_r,\mu'_1+\rho'_1,\cdots,\mu'_r+\rho'_r,$$

$$h_1,\cdots,h_l,1,\cdots,1) \tag{3.18}$$

$$L_j = [1-\lambda_j - \sum_{i=1}^{u} K_i \zeta_j''^{(i)} - \sum_{i=1}^{s} \zeta_j'''^{(i)} \eta_{G_i,g_i}; \zeta_j^{(1)}, \cdots, \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots, \zeta_j'^{(s)}, 0, \cdots, 0, 0 \cdots, 0]_{1,l} \quad (3.19)$$

$$L_j' = [1+\sigma_j - \sum_{i=1}^{u} K_i \lambda_j''^{(i)} - \sum_{i=1}^{v} \lambda_j'''^{(i)} \eta_{G_i,g_i}; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)}, 0, \cdots, 0, 0, \cdots, 0]_{1,k} \quad (3.20)$$

$$P_1 = (b-a)^{\alpha+\beta-1} \left\{ \prod_{j=1}^{h} (af_j+g_j)^{\sigma_j} \right\} \quad (3.21)$$

$$B_{u,v} = (b-a)^{\sum_{i=1}^{v}(a_i'+b_i')\eta_{G_i,g_i} + \sum_{i=1}^{u}(a_i+b_i)K_i} \left\{ \prod_{j=1}^{h} (af_j+g_j)^{\sum_{i=1}^{v} \lambda_i''' \eta_{g_i,h_i} + \sum_{i=1}^{u} \lambda_i'' K_i} \right\} \quad (3.22)$$

$$A_u = \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_u)_{M_uK_u}}{K_u!} A[N_1,K_1;\cdots;N_u,K_u] \quad (3.23)$$

We have the general Eulerian integral.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1} \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j} \prod_{j=1}^{k} (f_jt+g_j)^{\sigma_j}$$

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u} \left(\begin{array}{c} \mathrm{z}_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1} \prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u} \prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(u)}} \end{array} \right)$$

$$\bar{I} \left(\begin{array}{c} \mathrm{z}_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'} \prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'} \prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{array} \right)$$

$$I \left(\begin{array}{c} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1} \prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r} \prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array} \right)$$

$$I\begin{pmatrix} z'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ . \\ . \\ . \\ z'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{pmatrix}\mathrm{d}t =$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{K_k k}A_uB_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$I^{V;0,n_r+n'_s+l+k+2;X}_{U:p_r+p'_s+l+k+2,q_r+q'_s+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathrm{A}\ ;\mathrm{K}_1,K_2,K_j,K'_j,\mathfrak{A},\mathfrak{A}';\mathfrak{A}_1 \\ \cdots & . \\ \cdots & . \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & . \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & . \\ \tau_1(b-a)^{h_1} & \\ \cdots & \\ \cdots & \\ \tau_l(b-a)^{h_l} & \mathrm{B}\ ,\mathrm{L}_1,L_j,L'_j,\mathfrak{B},\mathfrak{B}';\mathfrak{B}_1 \\ \frac{(b-a)f_1}{af_1+g_1} & \\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (3.24)$$

We obtain the I-function of $r+s+k+l$ variables. The quantities $U,V,X,Y,A,B,K_1,K_2,K_j,K'_j,\mathfrak{A},\mathfrak{A}',\mathfrak{A}_1,L_1,L_j,L'_j$, $\mathfrak{B},\mathfrak{B}',P_1,B_u,B_{u,v}$ and $\mathfrak{B}_1$ are defined above.

Provided that

(A) $a,b\in\mathbb{R}(a<b);\mu_i,\mu'_u,\rho_i,\rho'_u,\lambda_j^{(i)},\lambda_j'^{(u)},h_v\in\mathbb{R}^+,f_i,g_j,\tau_v,\sigma_j,\lambda_v\in\mathbb{C}\ (i=1,\cdots,r;j=1,\cdots;k;$

$u=1,\cdots,s;v=1,\cdots,l),a_i,b_i,\lambda_j''^{(i)},\zeta_j''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,u;j=1,\cdots,k)$

$a'_i,b'_i,\lambda_j'''^{(i)},\zeta_j'''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,v;j=1,\cdots,k)$

(B) $a_{ij},b_{ik},\in\mathbb{C}\ (i=1,\cdots,r;j=1,\cdots,p_i;k=1,\cdots,q_i);a_j^{(i)},b_j^{(k)}\in\mathbb{C}$

$(i=1,\cdots,r;j=1,\cdots,p^{(i)};k=1,\cdots,q^{(i)})$

$a'_{ij},b'_{ik},\in\mathbb{C}\,(i=1,\cdots,s;j=1,\cdots,p'_i;k=1,\cdots,q'_i);\,a'^{(i)}_j,b'^{(k)}_j,\in\mathbb{C}$

$(i=1,\cdots,r;j=1,\cdots,p'^{i)};k=1,\cdots,q'^{(i)})$

$\alpha^{(k)}_{ij},\beta^{(k)}_{ij}\in\mathbb{R}^+\,(\,(i=1,\cdots,r,j=1,\cdots,p_i,k=1,\cdots,r)\,;\alpha^{(i)}_j,\beta^{(i)}_i\in\mathbb{R}^+\,(i=1,\cdots,r;j=1,\cdots,p_i)$

$\alpha'^{(k)}_{ij},\beta'^{(k)}_{ij}\in\mathbb{R}^+\,(\,(i=1,\cdots,s,j=1,\cdots,p'_i,k=1,\cdots,s)\,;\alpha'^{(i)}_j,\beta'^{(i)}_i\in\mathbb{R}^+\,(i=1,\cdots,s;j=1,\cdots,p'_i)$

(C) $\max_{1\leqslant j\leqslant k}\left\{\left|\dfrac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1$

(D) $$Re\big[\alpha+\sum_{j=1}^{r}\mu_j\min_{1\leqslant k\leqslant m^{(i)}}\frac{b^{(j)}_k}{\beta^{(j)}_k}+\sum_{j=1}^{s}\mu'_i\min_{1\leqslant k\leqslant m'^{(i)}}\frac{b'^{(j)}_k}{\beta'^{(j)}_k}\big]>0$$

$$Re\big[\beta+\sum_{j=1}^{r}\rho_j\min_{1\leqslant k\leqslant m^{(i)}}\frac{b^{(j)}_k}{\beta^{(j)}_k}+\sum_{j=1}^{s}\rho'_j\min_{1\leqslant k\leqslant m'^{(i)}}\frac{b'^{(j)}_k}{\beta'^{(j)}_k}\big]>0$$

(E) $$Re\left(\alpha+\sum_{i=1}^{v}\eta_{G_i,g_i}a'_i+\sum_{i=1}^{u}K_ia_i+\sum_{i=1}^{r}\mu_is_i+\sum_{i=1}^{s}t_i\mu'_i\right)>0$$

$$Re\left(\beta+\sum_{i=1}^{v}\eta_{G_i,g_i}b'_i+\sum_{i=1}^{u}K_ib_i+\sum_{i=1}^{r}v_is_i+\sum_{i=1}^{s}t_i\rho'_i\right)>0$$

$$Re\left(\lambda_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}_j+\sum_{i=1}^{u}K_i\lambda''^{(i)}_j+\sum_{i=1}^{r}s_i\zeta^{(i)}_j+\sum_{i=1}^{s}t_i\zeta'^{(i)}_j\right)>0(j=1,\cdots,l);$$

$$Re\left(-\sigma_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}+\sum_{i=1}^{u}K_i\lambda''^{(i)}_j+\sum_{i=1}^{r}s_i\lambda^{(i)}_j+\sum_{i=1}^{s}t_i\lambda'^{(i)}_j\right)>0(j=1,\cdots,k);$$

(F) $$\Omega_i=\sum_{k=1}^{n^{(i)}}\alpha^{(i)}_k-\sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha^{(i)}_k+\sum_{k=1}^{m^{(i)}}\beta^{(i)}_k-\sum_{k=m^{(i)}+1}^{q^{(i)}}\beta^{(i)}_k+\left(\sum_{k=1}^{n_2}\alpha^{(i)}_{2k}-\sum_{k=n_2+1}^{p_2}\alpha^{(i)}_{2k}\right)+\cdots+$$

$$\left(\sum_{k=1}^{n_s}\alpha^{(i)}_{sk}-\sum_{k=n_s+1}^{p_s}\alpha^{(i)}_{sk}\right)-\left(\sum_{k=1}^{q_2}\beta^{(i)}_{2k}+\sum_{k=1}^{q_3}\beta^{(i)}_{3k}+\cdots+\sum_{k=1}^{q_s}\beta^{(i)}_{sk}\right)-\mu_i-\rho_i$$

$$-\sum_{l=1}^{k}\lambda^{(i)}_j-\sum_{l=1}^{l}\zeta^{(i)}_j>0\quad(i=1,\cdots,r)$$

$$\Omega_i' = \sum_{k=1}^{n'^{(i)}} \alpha_k'^{(i)} - \sum_{k=n'^{(i)}+1}^{p'^{(i)}} \alpha_k'^{(i)} + \sum_{k=1}^{m'^{(i)}} \beta_k'^{(i)} - \sum_{k=m^{(i)}+1}^{q'^{(i)}} \beta_k'^{(i)} + \left(\sum_{k=1}^{n_2'} \alpha_{2k}'^{(i)} - \sum_{k=n_2+1}^{p_2'} \alpha_{2k}'^{(i)} \right) +$$

$$\cdots + \left(\sum_{k=1}^{n_s'} \alpha_{sk}'^{(i)} - \sum_{k=n_s'+1}^{p_s'} \alpha_{sk}'^{(i)} \right) - \left(\sum_{k=1}^{q_2'} \beta_{2k}'^{(i)} + \sum_{k=1}^{q_3'} \beta_{3k}'^{(i)} + \cdots + \sum_{k=1}^{q_s'} \beta_{sk}'^{(i)} \right) - \mu_i' - \rho_i'$$

$$- \sum_{l=1}^{k} \lambda_j'^{(i)} - \sum_{l=1}^{l} \zeta_j'^{(i)} > 0 \quad (i = 1, \cdots, s)$$

(G) $$\left| arg \left(z_i \prod_{j=1}^{l} \left[1 - \tau_j (t-a)^{h_i}\right]^{-\zeta_j^{(i)}} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j^{(i)}} \right) \right| < \frac{1}{2} \Omega_i \pi \quad (a \leqslant t \leqslant b; i = 1, \cdots, r)$$

$$\left| arg \left(z_i' \prod_{j=1}^{l} \left[1 - \tau_j' (t-a)^{h_i'}\right]^{-\zeta_j'^{(i)}} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'^{(i)}} \right) \right| < \frac{1}{2} \Omega_i' \pi \quad (a \leqslant t \leqslant b; i = 1, \cdots, s)$$

(H) The multiple series occuring on the right-hand side of (3.24) is absolutely and uniformly convergent.

Proof

To prove (3.24), first, we express in serie the multivariable I-function defined by Nambisan et al [2] with the help of (1.5), a class of multivariable polynomials defined by Srivastava $S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[.]$ in serie with the help of (1.14), the I-functions of r-variables and s-variables defined by Prasad [1] in terms of Mellin-Barnes type contour integral with the help of (1.8) and (1.11) respectively. Now collect the power of $\left[1 - \tau_j (t-a)^{h_i}\right]$ with $(i = 1, \cdots, r; j = 1, \cdots, l)$ and collect the power of $(f_j t + g_j)$ with $j = 1, \cdots, k$. Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the $(r+s+k+l)$ dimensional Mellin-Barnes integral in multivariable I-function defined by Prasad [1], we obtain the equation (3.24).

Remarks

If a) $\rho_1 = \cdots, \rho_r = \rho_1' = \cdots, \rho_s' = 0$; b) $\mu_1 = \cdots, \mu_r = \mu_1' = \cdots, \mu_s' = 0$, we obtain the similar formulas that (3.24) with the corresponding simplifications.

4. Particular cases

If $U = V = A = B = 0$, **the multivariable I-function defined by Prasad reduces to multivariable H-function defined by Srivastava et al [7] and we obtain :**

$$\int_a^b (t-a)^{\alpha-1} (b-t)^{\beta-1} \prod_{j=1}^{l} \left[1 - \tau_j (t-a)^{h_i}\right]^{-\lambda_j} \prod_{j=1}^{k} (f_j t + g_j)^{\sigma_j}$$

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u} \begin{pmatrix} z_1'' \theta_1'' (t-a)^{a_1} (b-t)^{b_1} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_u'' \theta_u'' (t-a)^{a_u} (b-t)^{b_u} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j''^{(u)}} \end{pmatrix}$$

$$\bar{I}\left(\begin{array}{c} \mathrm{z}_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{array}\right)$$

$$H\left(\begin{array}{c} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array}\right)$$

$$H\left(\begin{array}{c} \mathrm{z}_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t =$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{K_k k}A_uB_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$H^{0,n_r+n_s'+l+k+2;X}_{p_r+p_s'+l+k+2,q_r+q_s'+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathrm{K}_1, K_2, K_j, K_j', \mathfrak{A}, \mathfrak{A}'; \mathfrak{A}_1 \\ \cdots & . \\ \cdots & . \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & . \\ \frac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z_s'(b-a)^{\mu_s'+\rho_s'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & . \\ \tau_1(b-a)^{h_1} & \\ \cdots & \\ \cdots & \\ \tau_l(b-a)^{h_l} & \mathrm{L}_1, L_j, L_j', \mathfrak{B}, \mathfrak{B}'; \mathfrak{B}_1 \\ \frac{(b-a)f_1}{af_1+g_1} & \\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (4.1)$$

under the same notations and conditions that (3.24) with $U = V = A = B = 0$

Remark:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable I-functions defined by Prasad [1] and a class of multivariable polynomials defined by Srivastava [4].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable I-functions defined by Prasad [1],a expansion of multivariable I-function defined by Nambisan et al [2] and a class of multivariable polynomials defined by Srivastava [4] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

REFERENCES

[1] Y.N. Prasad , Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , page 231-237.

[2] Prathima J. Nambisan V. and Kurumujji S.K. A Study of I-function of Several Complex Variables, International Journalof Engineering Mathematics Vol(2014) , 2014 page 1-12

[3] Saigo M. and Saxena R.K. Unified fractional integral formulas for the multivariable H-function I. J.Fractional Calculus 15 (1999), page 91-107.

4] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), page183-191

[5] Srivastava H.M. and Daoust M.C. Certain generalized Neumann expansions associated with Kampé de Fériet function. Nederl. Akad. Wetensch. Proc. Ser A72 = Indag Math 31(1969) page 449-457.

[6] Srivastava H.M. and Karlsson P.W. Multiple Gaussian Hypergeometric series. Ellis.Horwood. Limited. New-York, Chichester. Brisbane. Toronto , 1985.

[7] H.M. Srivastava and R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.

Personal adress : 411 Avenue Joseph Raynaud
Le parc Fleuri , Bat B
83140 , Six-Fours les plages
Tel : 06-83-12-49-68
Department : VAR
Country : FRANCE