Eulerian integral associated with product of two multivariable I-functions, generalized Lauricella function and a class of polynomials and expansion of multivariable I-function I

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ABSTRACT
The present paper is evaluated a new Eulerian integral associated with the product of two multivariable I-functions defined by Nambisan et al [2] a generalized Lauricella function , a class of multivariable polynomials and a expansion of multivariable I-function defined by Nambisan et al [2] with general arguments . We will study the case concerning the multivariable H-function defined by Srivastava et al [7]. .

Keywords: Eulerian integral, multivariable I-function, generalized Lauricella function of several variables, multivariable H-function, generalized hypergeometric function, class of polynomials

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1. Introduction

In this paper, we evaluate a new Eulerian integral of most general characters associated with the products of two multivariable I-functions defined by nambisan et al [2] , a serie expansion of multivariable I-function defined by Nambisan et al [2] and a class of polynomials with general arguments.

First time, we define the multivariable $\bar{I}$-function by :

$$\bar{I}(z_1''',\cdots,z_v''') = \bar{I}^{0,N:M_1,N_1;\cdots;M_v,N_v}_{P,Q:P_1,Q_1;\cdots;P_v,Q_v}\left(\begin{array}{c|c} z_1''' & (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(v)};A_j)_{N+1,P} : \\ \cdot & \\ \cdot & \\ \cdot & \\ z_v''' & (b_j;\beta_j^{(1)},\cdots,\beta_j^{(v)};B_j)_{M+1,Q} : \end{array}\right.$$

$$\left.\begin{array}{c} (c_j^{(1)},\gamma_j^{(1)};1)_{1,N_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{N_1+1,P_1};\cdots;(c_j^{(v)},\gamma_j^{(v)};1)_{1,N_u},(c_j^{(v)},\gamma_j^{(v)};C_j^{(v)})_{N_v+1,P_v} \\ (\bar{d}_j^{(1)},\bar{\delta}_j^{(1)};1)_{1,M_1},(\bar{d}_j^{(1)},\bar{\delta}_j^{(1)};D_j^{(1)})_{M_1+1,Q_1};\cdots;(\bar{d}_j^{(v)},\bar{\delta}_j^{(v)};1)_{1,M_v},(\bar{d}_j^{(v)},\bar{\delta}_j^{(v)};D_j^{(v)})_{M_v+1,Q_v} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^v}\int_{L_1}\cdots\int_{L_v}\phi_1(s_1,\cdots,s_v)\prod_{i=1}^{v}\xi_i'(s_i)z_i'''^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_v \tag{1.2}$$

where $\phi_1(s_1,\cdots,s_v), \xi_i'(s_i), i=1,\cdots,v$ are given by :

$$\phi_1(s_1,\cdots,s_v) = \frac{1}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{v}\alpha_j^{(i)}s_j\right)\prod_{j=M+1}^{Q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{v}\beta_j^{(i)}s_j\right)} \tag{1.3}$$

$$\xi_i'(s_i)=\frac{\prod_{j=1}^{N_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{M_i}\Gamma\left(\bar{d}_j^{(i)}-\bar{\delta}_j^{(i)}s_i\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{D_j^{(i)}}\left(1-\bar{d}_j^{(i)}-\bar{\delta}_j^{(i)}s_i\right)} \tag{1.4}$$

$i=1,\cdots,v$

Serie representation

If $z_i''' \neq 0; i=1,\cdots,v$

$\delta_{h_i}^{(i)}(d_j^{(i)}+k_i)\neq\delta_j^{(i)}(\delta_{h_i}^{(i)}+\eta_i) for j\neq h_i, j, h_i=1,\cdots,m_i(i=1,\cdots,v)$,$k_i,\eta_i=0,1,2,\cdots(i=1,\cdots,v)$, then

$$\bar{I}(z_1''',\cdots,z_v''')=\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\left[\phi_1\left(\frac{dh_1^{(1)}+k_1}{\delta h_1^{(1)}},\cdots,\frac{dh_v^{(v)}+k_v}{\delta h_v^{(v)}}\right)\right]\prod_{j\neq h_i i=1}^{r}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\frac{dh_i+k_i}{\delta h_i}} \tag{1.5}$$

This result can be proved on computing the residues at the poles :

$$s_i=\frac{dh_i^{(i)}+k_i}{\delta h_i^{(i)}},(h_i=1,\cdots,m_i,k_i=0,1,2,\cdots) for i=1,\cdots,v \tag{1.6}$$

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1''',\cdots,z_v''')=0(|z_1'''|^{\alpha_1},\cdots,|z_v'''|^{\alpha_v}), max(|z_1'''|,\cdots,|z_v'''|)\rightarrow 0$$

$$\bar{I}(z_1''',\cdots,z_v''')=0(|z_1'''|^{\beta_1},\cdots,|z_v'''|^{\beta_u}), min(|z_1'''|,\cdots,|z_v'''|)\rightarrow \infty$$

where $k=1,\cdots,v:\alpha_k=min[Re(\bar{d}_j^{(k)}/\bar{\delta}_j^{(k)})], j=1,\cdots,m_k$ and

$$\beta_k=max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,n_k$$

We will note $\eta_{h_i,k_i}=\dfrac{dh_i^{(i)}+k_i}{\delta h_i}$ $,(h_i=1,\cdots,m_i,k_i=0,1,2,\cdots) for i=1,\cdots,v$ (1.7)

The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r)=I_{p,q:p_1,q_1;\cdots;p_r,q_r}^{0,n:m_1,n_1;\cdots;m_r,n_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,p}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,q}: \end{array}\right.$$

$$\left.\begin{array}{l}(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r} \\ \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r}\end{array}\right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1, \cdots, s_r) \prod_{i=1}^{r} \theta_i(s_i) z_i^{s_i} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.9}$$

where $\phi(s_1, \cdots, s_r), \theta_i(s_i), i = 1, \cdots, r$ are given by :

$$\phi(s_1, \cdots, s_r) = \frac{\prod_{j=1}^{n} \Gamma^{A_j}\left(1 - aj + \sum_{i=1}^{r} \alpha_j^{(i)} s_j\right)}{\prod_{j=n+1}^{p} \Gamma^{A_j}\left(a_j - \sum_{i=1}^{r} \alpha_j^{(i)} s_j\right) \prod_{j=1}^{q} \Gamma^{B_j}\left(1 - bj + \sum_{i=1}^{r} \beta_j^{(i)} s_j\right)} \tag{1.10}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma^{C_j^{(i)}}\left(1 - c_j^{(i)} + \gamma_j^{(i)} s_i\right) \prod_{j=1}^{m_i} \Gamma^{D_j^{(i)}}\left(d_j^{(i)} - \delta_j^{(i)} s_i\right)}{\prod_{j=n_i+1}^{p_i} \Gamma^{C_j^{(i)}}\left(c_j^{(i)} - \gamma_j^{(i)} s_i\right) \prod_{j=m_i+1}^{q_i} \Gamma^{D_j^{(i)}}\left(1 - d_j^{(i)} - \delta_j^{(i)} s_i\right)} \tag{1.11}$$

For more details, see Nambisan et al [2].

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U_i = \sum_{j=1}^{p} A_j \alpha_j^{(i)} - \sum_{j=1}^{q} B_j \beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=1}^{q_i} D_j^{(i)} \delta_j^{(i)} \leqslant 0, i = 1, \cdots, r \tag{1.12}$$

The integral (2.1) converges absolutely if

$|arg(z_k)| < \frac{1}{2}\Delta_k \pi, k = 1, \cdots, r$ where

$$\Delta_k = -\sum_{j=n+1}^{p} A_j \alpha_j^{(k)} - \sum_{j=1}^{q} B_j \beta_j^{(k)} + \sum_{j=1}^{m_k} D_j^{(k)} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n_k} C_j^{(k)} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)} > 0 \tag{1.13}$$

Consider the second multivariable I-function.

$$I(z_1', \cdots, z_s') = I_{p',q':p_1',q_1';\cdots;p_s',q_s'}^{0,n':m_1',n_1';\cdots;m_s',n_s'} \left(\begin{array}{c|c} z'_1 & (a'_j; \alpha_j'^{(1)}, \cdots, \alpha_j'^{(s)}; A_j')_{1,p'} : \\ \cdot & \\ \cdot & \\ \cdot & \\ z'_s & (b'_j; \beta_j'^{(1)}, \cdots, \beta_j'^{(s)}; B_j')_{1,q'} : \end{array} \right.$$

$$\left. \begin{array}{l} (c'^{(1)}_j, \gamma_j'^{(1)}; C_j'^{(1)})_{1,p_1'}; \cdots; (c_j'^{(s)}, \gamma_j'^{(s)}; C_j'^{(s)})_{1,p_s'} \\ \\ (d'^{(1)}_j, \delta_j'^{(1)}; D_j'^{(1)})_{1,q_1'}; \cdots; (d_j'^{(s)}, \delta_j'^{(s)}; D_j'^{(s)})_{1,q_s'} \end{array} \right) \tag{1.14}$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L_1'} \cdots \int_{L_s'} \psi(t_1, \cdots, t_s) \prod_{i=1}^{s} \xi_i(t_i) z_i^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.15}$$

where $\psi(t_1, \cdots, t_s), \xi_i(s_i), i = 1, \cdots, s$ are given by :

$$\psi(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{n'}\Gamma^{A'_j}\left(1-a'_j+\sum_{i=1}^{s}\alpha_j'^{(i)}t_j\right)}{\prod_{j=n'+1}^{p'}\Gamma^{A'_j}\left(a'_j-\sum_{i=1}^{s}\alpha_j'^{(i)}t_j\right)\prod_{j=1}^{q'}\Gamma^{B'_j}\left(1-b'_j+\sum_{i=1}^{s}\beta_j'^{(i)}t_j\right)} \tag{1.16}$$

$$\xi_i(s_i)=\frac{\prod_{j=1}^{n'_i}\Gamma^{C_j'^{(i)}}\left(1-c_j'^{(i)}+\gamma_j'^{(i)}t_i\right)\prod_{j=1}^{m'_i}\Gamma^{D_j'^{(i)}}\left(d_j'^{(i)}-\delta_j'^{(i)}t_i\right)}{\prod_{j=n'_i+1}^{p'_i}\Gamma^{C_j'^{(i)}}\left(c_j'^{(i)}-\gamma_j'^{(i)}t_i\right)\prod_{j=m'_i+1}^{q'_i}\Gamma^{D_j'^{(i)}}\left(1-d_j'^{(i)}-\delta_j'^{(i)}t_i\right)} \tag{1.17}$$

For more details, see Nambisan et al [2].

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U'_i=\sum_{j=1}^{p'}A'_j\alpha_j'^{(i)}-\sum_{j=1}^{q'}B'_j\beta_j'^{(i)}+\sum_{j=1}^{p'_i}C_j'^{(i)}\gamma_j'^{(i)}-\sum_{j=1}^{q'_i}D_j'^{(i)}\delta_j'^{(i)}\leqslant 0, i=1,\cdots,s \tag{1.18}$$

The integral (2.1) converges absolutely if

where $|arg(z'_k)|<\frac{1}{2}\Delta'_k\pi, k=1,\cdots,s$

$$\Delta'_k=-\sum_{j=n'+1}^{p'}A'_j\alpha_j'^{(k)}-\sum_{j=1}^{q'}B'_j\beta_j'^{(k)}+\sum_{j=1}^{m'_k}D_j'^{(k)}\delta_j'^{(k)}-\sum_{j=m'_k+1}^{q'_k}D_j'^{(k)}\delta_j'^{(k)}+\sum_{j=1}^{n'_k}C_j'^{(k)}\gamma_j'^{(k)}-\sum_{j=n'_k+1}^{p'_k}C_j'^{(k)}\gamma_j'^{(k)}>0 \tag{1.19}$$

Srivastava [4] introduced and defined a general class of multivariable polynomials as follows

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[y_1,\cdots,y_u]=\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!}$$

$$A[N_1,K_1;\cdots;N_u,K_u]y_1^{K_1}\cdots y_u^{K_u} \tag{1.20}$$

where $M_1,\cdots,M_u$ are arbitrary positive integers and the coefficients are $A[N_1,K_1;\cdots;N_u,K_u]$ arbitrary constants, real or complex.

2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [6 ,page 39 eq .30]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^{P}\Gamma(A_j+s_1+\cdots+s_r)}{\prod_{j=1}^{Q}\Gamma(B_j+s_1+\cdots+s_r)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j+s_1+\cdots+s_r)$ are separated from those of $\Gamma(-s_j), j=1,\cdots,r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j=1,\cdots,r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}\mathrm{d}t \;=(b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}$$

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right) \tag{2.2}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i,\tau_j,h_j\in\mathbb{C},\lambda_j\in\mathbb{R}^+(i=1,\cdots,k;j=1,\cdots,l)$

$$min(Re(\alpha),Re(\beta))>0,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1,\ \max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,$$

and $F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust[3,page 454] given by :

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\prod_{j=1}^{l}\Gamma(\lambda_j)\prod_{j=1}^{k}\Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}}\int_{L_1}\cdots\int_{L_{l+k}}\frac{\Gamma\left(\alpha+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}{\Gamma\left(\alpha+\beta+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}\prod_{j=1}^{l}\Gamma(\lambda_j+s_j)\prod_{j=1}^{k}\Gamma(-\sigma_j+s_{l+j})$$

$$\prod_{j=1}^{l+k}\Gamma(-s_j)z_1^{s_1}\cdots z_l^{s_l}z_{l+1}^{s_{l+1}}\cdots,z_{l+k}^{s_{l+k}}\,\mathrm{d}s_1\cdots\mathrm{d}s_{l+k} \tag{2.3}$$

Here the contour $L_j's$ are defined by $L_j=L_{w\zeta_j\infty}(Re(\zeta_j)=v_j'')$ starting at the point $v_j''-\omega\infty$ and terminating at the point $v_j''+\omega\infty$ with $v_j''\in\mathbb{R}(j=1,\cdots,l)$ and each of the remaining contour $L_{l+1},\cdots,L_{l+k}$ run from $-\omega\infty$ to $\omega\infty$

(2.2) can be easily established by expanding $\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.4}$$

integrating term by term with the help of the integral given by Saigo and Saxena [3, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [5, page 454].

3. Eulerian integral

In this section , we note :

$$\theta_i = \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}}, \zeta_j^{(i)} > 0 (i=1,\cdots,r) ; \theta'_i = \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'^{(i)}}, \zeta_j'^{(i)} > 0 (i=1,\cdots,s)$$

$$\theta''_i = \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j''^{(i)}}, \zeta_j''^{(i)} > 0 (i=1,\cdots,u)$$

$$\theta'''_i = \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'''^{(i)}}, \zeta_j'''^{(i)} > 0 (i=1,\cdots,v) \tag{3.1}$$

$$X = m_1,n_1;\cdots;m_r,n_r;m'_1,n'_1;\cdots;m'_s,n'_s;1,0;\cdots;1,0;1,0;\cdots;1,0 \tag{3.2}$$

$$Y = p_1,q_1;\cdots;p_r,q_r;p'_1,q'_1;\cdots;p'_s,q'_s;0,1;\cdots;0,1;0,1;\cdots;0,1 \tag{3.3}$$

$$A = (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)},0,\cdots,0,0,\cdots,0,0,\cdots,0;A_j)_{1,p} \tag{3.4}$$

$$B = (b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)},0\cdots,0,0\cdots,0,0\cdots,0;B_j)_{1,q} \tag{3.5}$$

$$A' = (a'_j;0,\cdots,0,\alpha_j'^{(1)},\cdots,\alpha_j'^{(s)},0,\cdots,0,0,\cdots,0;A'_j)_{1,p'} \tag{3.6}$$

$$B' = (b'_j;0,\cdots,0,\beta_j'^{(1)},\cdots,\beta_j'^{(s)},0,\cdots,0,0,\cdots,0;B'_j)_{1,q'} \tag{3.7}$$

$$\mathrm{C} = (c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r};(c_j'^{(1)},\gamma_j'^{(1)};C_j'^{(1)})_{1,p'_1};\cdots;(c_j'^{(r)},\gamma_j^{(s)};C_j'^{(s)})_{1,p'_s}$$

$$(1,0;1);\cdots;(1,0;1);(1,0;1);\cdots;(1,0;1) \tag{3.8}$$

$$D = (d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r};(d_j'^{(1)},\delta_j'^{(1)};D_j'^{(1)})_{1,q'_1};\cdots;(d_j'^{(s)},\delta_j'^{(s)};D_j'^{(s)})_{1,q'_s};$$

$$(0,1;1);\cdots;(0,1;1);(0,1;1);\cdots;(0,1;1) \tag{3.9}$$

$$K_1 = (1-\alpha-\sum_{i=1}^{u} K_i a_i - \sum_{i=1}^{v} \eta_{G_i,g_i} a'_i;\mu_1,\cdots,\mu_r,\mu'_1,\cdots,\mu'_s,h_1,\cdots,h_l,1,\cdots,1;1) \tag{3.10}$$

$$K_2 = (1-\beta-\sum_{i=1}^{u} K_i b_i - \sum_{i=1}^{v} \eta_{G_i,g_i} b'_i;\rho_1,\cdots,\rho_r,\rho'_1,\cdots,\rho'_s,0,\cdots,0,0\cdots,0;1) \tag{3.11}$$

$$K_j = [1-\lambda_j-\sum_{i=1}^{u} K_i \zeta_j''^{(i)} - \sum_{i=1}^{v} \eta_{G_i,g_i} \zeta_j'''^{(i)};\zeta_j^{(1)},\cdots,\zeta_j^{(r)},\zeta_j'^{(1)}\cdots,\zeta_j'^{(s)},$$

$$0,\cdots,\underset{\mathrm{j}}{1},\cdots,0,0\cdots,0;1]_{1,l} \tag{3.12}$$

$$K'_j = [1 + \sigma_j - \sum_{i=1}^{u} K_i \lambda_j''^{(i)} - \sum_{i=1}^{v} \eta_{G_i,g_i} \lambda_j'''^{(i)}; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)},$$

$$0, \cdots, 0, 0 \cdots, \underset{\text{j}}{1}, \cdots, 0; 1]_{1,k} \tag{3.13}$$

$$L_1 = (1 - \alpha - \beta - \sum_{i=1}^{u} K_i(a_i + b_i) - \sum_{i=1}^{v} (a'_i + b'_i)\eta_{G_i,g_i}; \mu_1 + \rho_1, \cdots, \mu_r + \rho_r, \mu'_1 + \rho'_1, \cdots, \mu'_r + \rho'_r,$$

$$h_1, \cdots, h_l, 1, \cdots, 1; 1) \tag{3.14}$$

$$L_j = [1 - \lambda_j - \sum_{i=1}^{u} K_i \zeta_j''^{(i)} - \sum_{i=1}^{s} \zeta_j'''^{(i)} \eta_{G_i,g_i}; \zeta_j^{(1)}, \cdots, \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots, \zeta_j'^{(s)}, 0, \cdots, 0, 0 \cdots, 0; 1]_{1,l} \tag{3.15}$$

$$L'_j = [1 + \sigma_j - \sum_{i=1}^{u} K_i \lambda_j''^{(i)} - \sum_{i=1}^{v} \lambda_j'''^{(i)} \eta_{G_i,g_i}; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)}, 0, \cdots, 0, 0, \cdots, 0; 1]_{1,k} \tag{3.16}$$

$$P_1 = (b-a)^{\alpha+\beta-1} \left\{ \prod_{j=1}^{h} (af_j + g_j)^{\sigma_j} \right\} \tag{3.17}$$

$$B_{u,v} = (b-a)^{\sum_{i=1}^{v} (a'_i + b'_i)\eta_{G_i,g_i} + \sum_{i=1}^{u} (a_i + b_i) K_i} \left\{ \prod_{j=1}^{h} (af_j + g_j)^{-\sum_{i=1}^{v} \lambda_i''' \eta_{g_i,h_i} - \sum_{i=1}^{u} \lambda_i'' K_i} \right\} \tag{3.18}$$

$$A_u = \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_u)_{M_uK_u}}{K_u!} A[N_1, K_1; \cdots; N_u, K_u] \tag{3.19}$$

$$\mathfrak{A} = A, A'; \mathfrak{B} = B, B' \tag{3.20}$$

We have the general Eulerian integral.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1} \prod_{j=1}^{l} \left[1 - \tau_j (t-a)^{h_i}\right]^{-\lambda_j} \prod_{j=1}^{k} (f_j t + g_j)^{\sigma_j}$$

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u} \left(\begin{array}{c} \text{z}_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \text{z}_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j''^{(u)}} \end{array} \right)$$

$$\bar{I} \left(\begin{array}{c} \text{z}_1'''\theta_1'''(t-a)^{a'_1}(b-t)^{b'_1} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \text{z}_v'''\theta_v'''(t-a)^{a'_v}(b-t)^{b'_v} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'''^{(v)}} \end{array} \right)$$

$$I\begin{pmatrix} z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{pmatrix}$$

$$I\begin{pmatrix} z'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{pmatrix}\mathrm{d}t =$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{K_k k}A_uB_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$I^{0,n+n'+l+k+2;X}_{p+p'+l+k+2,q+q'+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \\ \cdots & \mathfrak{A}\ ,\ \mathrm{K}_1, K_2, K_j, K'_j : C \\ \cdots & \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdots & \cdot \\ \cdots & \\ \tau_l(b-a)^{h_l} & \\ \frac{(b-a)f_1}{af_1+g_1} & \mathfrak{B}\ ,\ \mathrm{L}_1, L_j, L'_j : D \\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (3.21)$$

where $\mathfrak{A}, \mathfrak{B}, C, D, X, K_1, K_2, K_j, K'_j, L_1, L_j, L'_j, \mathfrak{B}, \mathfrak{B}', P_1, P_u, B_u$ and $\mathfrak{B}_1$ are defined above.

Provided that

(A) $a, b \in \mathbb{R}(a<b); \mu_i, \mu'_u, \rho_i, \rho'_u, \lambda_j^{(i)}, \lambda_j'^{(u)}, h_v \in \mathbb{R}^+, f_i, g_j, \tau_v, \sigma_j, \lambda_v \in \mathbb{C}\ (i=1,\cdots,r; j=1,\cdots;k;$

$u=1,\cdots,s; v=1,\cdots,l), a_i, b_i, \lambda_j''^{(i)}, \zeta_j''^{(i)} \in \mathbb{R}^+, (i=1,\cdots,u; j=1,\cdots,k)$

$a'_i, b'_i, \lambda_j'''^{(i)}, \zeta_j'''^{(i)} \in \mathbb{R}^+, (i=1,\cdots,v; j=1,\cdots,k)$

(B) $m_j,n_j,p_j,q_j(j=1,\cdots,r),n,p,q\in\mathbb{N}^*$ **;** $\delta_j^{(i)}\in\mathbb{R}_+(j=1,\cdots,q_i;i=1,\cdots,r)$

$\alpha_j^{(i)}\in\mathbb{R}_+(j=1,\cdots,p;i=1,\cdots,r),\beta_j^{(i)}\in\mathbb{R}_+(j=1,\cdots,q;i=1,\cdots,r),\gamma_j^{(i)}\in\mathbb{R}_+(j=1,\cdots,p_i;i=1,\cdots,r)$

$a_j(j=1,\cdots,p),b_j(j=1,\cdots,q),c_j^{(i)}(j=1,\cdots,p_i,i=1,\cdots,r),d_j^{(i)}(j=1,\cdots,q_i,i=1,\cdots,r)\in\mathbb{C}$

The exposants $A_j(j=1,\cdots,p),B_j(j=1,\cdots,q),C_j^{(i)}(j=1,\cdots,p_i;i=1,\cdots,r),D_j^{(i)}(j=1,\cdots,q_i;i=1,\cdots,r)$

of various gamma function involved in (1.3) and (1.4) may take non integer values.

$m'_j,n'_j,p'_j,q'_j(j=1,\cdots,s),n',p',q'\in\mathbb{N}^*$ **;** $\delta_j'^{(i)}\in\mathbb{R}_+(j=1,\cdots,q'_i;i=1,\cdots,s)$

$\alpha_j'^{(i)}\in\mathbb{R}_+(j=1,\cdots,p';i=1,\cdots,s),\beta_j'^{(i)}\in\mathbb{R}_+(j=1,\cdots,q';i=1,\cdots,r),\gamma_j'^{(i)}\in\mathbb{R}_+(j=1,\cdots,p'_i;i=1,\cdots,s)$

$a'_j(j=1,\cdots,p'),b'_j(j=1,\cdots,q'),c_j'^{(i)}(j=1,\cdots,p'_i,i=1,\cdots,s),d_j'^{(i)}(j=1,\cdots,q'_i,i=1,\cdots,s)\in\mathbb{C}$

The exposants

$A'_j(j=1,\cdots,p'),B'_j(j=1,\cdots,q'),C_j'^{(i)}(j=1,\cdots,p'_i;i=1,\cdots,s),D_j'^{(i)}(j=1,\cdots,q'_i;i=1,\cdots,s)$

of various gamma function involved in (1.9) and (1.10) may take non integer values.

(C) $\max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1$

(D) $Re\big[\alpha+\sum_{j=1}^{r}\mu_j\min_{1\leqslant k\leqslant m_i}\frac{d_k^{(j)}}{\delta_k^{(j)}}+\sum_{j=1}^{s}\mu'_i\min_{1\leqslant k\leqslant m'_i}\frac{d_k'^{(j)}}{\delta_k'^{(j)}}\big]>0$

$$Re\big[\beta+\sum_{j=1}^{r}\rho_j\min_{1\leqslant k\leqslant m_i}\frac{d_k^{(j)}}{\delta_k^{(j)}}+\sum_{j=1}^{s}\rho'_j\min_{1\leqslant k\leqslant m'_i}\frac{d_k'^{(j)}}{\delta_k'^{(j)}}\big]>0$$

(E) $Re\left(\alpha+\sum_{i=1}^{v}\eta_{G_i,g_i}a'_i+\sum_{i=1}^{u}K_ia_i+\sum_{i=1}^{r}\mu_is_i+\sum_{i=1}^{s}t_i\mu'_i\right)>0$

$$Re\left(\beta+\sum_{i=1}^{v}\eta_{G_i,g_i}b'_i+\sum_{i=1}^{u}K_ib_i+\sum_{i=1}^{r}v_is_i+\sum_{i=1}^{s}t_i\rho'_i\right)>0$$

$$Re\left(\lambda_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda_j'''^{(i)}+\sum_{i=1}^{u}K_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\zeta_j^{(i)}+\sum_{i=1}^{s}t_i\zeta_j'^{(i)}\right)>0(j=1,\cdots,l);$$

$$Re\left(-\sigma_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}+\sum_{i=1}^{u}K_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\lambda_j^{(i)}+\sum_{i=1}^{s}t_i\lambda_j'^{(i)}\right)>0(j=1,\cdots,k);$$

(F) $U_i=\sum_{j=1}^{p}A_j\alpha_j^{(i)}-\sum_{j=1}^{q}B_j\beta_j^{(i)}+\sum_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=1}^{q_i}D_j^{(i)}\delta_j^{(i)}\leqslant 0,i=1,\cdots,r$

$$U'_i=\sum_{j=1}^{p'}A'_j\alpha_j'^{(i)}-\sum_{j=1}^{q'}B'_j\beta_j'^{(i)}+\sum_{j=1}^{p'_i}C_j'^{(i)}\gamma_j'^{(i)}-\sum_{j=1}^{q'_i}D_j'^{(i)}\delta_j'^{(i)}\leqslant 0,i=1,\cdots,s$$

(G) $$\Delta_k=-\sum_{j=n+1}^{p}A_j\alpha_j^{(k)}-\sum_{j=1}^{q}B_j\beta_j^{(k)}+\sum_{j=1}^{m_k}D_j^{(k)}\delta_j^{(k)}-\sum_{j=m_k+1}^{q_k}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n_k}C_j^{(k)}\gamma_j^{(k)}-\sum_{j=n_k+1}^{p_k}C_j^{(k)}\gamma_j^{(k)}$$

$$-\mu_i-\rho_i-\sum_{l=1}^{k}\lambda_j^{(i)}-\sum_{l=1}^{l}\zeta_j^{(i)}>0\quad(i=1,\cdots,r)$$

$$\Delta'_k=-\sum_{j=n'+1}^{p'}A'_j\alpha_j'^{(k)}-\sum_{j=1}^{q'}B'_j\beta_j'^{(k)}+\sum_{j=1}^{m'_k}D_j'^{(k)}\delta_j'^{(k)}-\sum_{j=m'_k+1}^{q'_k}D_j'^{(k)}\delta_j'^{(k)}+\sum_{j=1}^{n'_k}C_j'^{(k)}\gamma_j'^{(k)}-\sum_{j=n'_k+1}^{p'_k}C_j'^{(k)}\gamma_j'^{(k)}$$

$$-\mu'_i-\rho'_i-\sum_{l=1}^{k}\lambda_j'^{(i)}-\sum_{l=1}^{l}\zeta_j'^{(i)}>0\quad(i=1,\cdots,s)$$

(H) $$\left|arg\left(z_i\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(i)}}\right)\right|<\frac{1}{2}\Delta_i\pi\ \ (a\leqslant t\leqslant b;i=1,\cdots,r)$$

$$\left|arg\left(z'_i\prod_{j=1}^{l}\left[1-\tau'_j(t-a)^{h'_i}\right]^{-\zeta_j'^{(i)}}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(i)}}\right)\right|<\frac{1}{2}\Delta'_i\pi\ (a\leqslant t\leqslant b;i=1,\cdots,s)$$

(I) The multiple series occuring on the right-hand side of (3.21) is absolutely and uniformly convergent.

Proof

To prove (3.21), first, we express in serie the multivariable I-function defined by Nambisan et al [2] with the help of (1.5), a class of multivariable polynomials defined by Srivastava $S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[.]$ in serie with the help of (1.20), the I-functions of r-variables and s-variables defined by Nambisan et al [2] in terms of Mellin-Barnes type contour integral with the help of (1.8) and (1.15) respectively. Now collect the power of $\left[1-\tau_j(t-a)^{h_i}\right]$ with $(i=1,\cdots,r;j=1,\cdots,l)$ and collect the power of (f_jt+g_j) with $j=1,\cdots,k$. Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the $(r+s+k+l)$ dimensional Mellin-Barnes integral in multivariable I-function defined by Nambisan et [2], we obtain the equation (3.20).

Remarks

If a) $\rho_1=\cdots,\rho_r=\rho'_1=\cdots,\rho'_s=0$; b) $\mu_1=\cdots,\mu_r=\mu'_1=\cdots,\mu'_s=0$, we obtain the similar formulas that (3.21) with the corresponding simplifications.

4. Particular cases

If $A_j=B_j=C_j^{(i)}=D_j^{(i)}=A'_j=B'_j=C_j'^{(i)}=D_j'^{(i)}=1$, the multivariable I-function defined by Nambisan et al [2] reduces to multivariable H-function defined by Srivastava et al [7].
We the following generalized Eulerian integral concerning the multivariable H-function :

$$\int_a^b(t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\begin{pmatrix}z''_1\theta''_1(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ z''_u\theta''_u(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(u)}}\end{pmatrix}$$

$$\bar{I}\left(\begin{array}{c} \mathrm{z}_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{array}\right)$$

$$H\left(\begin{array}{c} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array}\right)$$

$$H\left(\begin{array}{c} \mathrm{z}_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ . \\ . \\ . \\ \mathrm{z}_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t =$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{K_k k}A_uB_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$H^{0,n+n'+l+k+2;X}_{p+p'+l+k+2,q+q'+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathfrak{A}\;,\mathrm{K}_1,K_2,K_j,K_j':C \\ \cdots & . \\ \cdots & . \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & . \\ \frac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z_s'(b-a)^{\mu_s'+\rho_s'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & . \\ \tau_1(b-a)^{h_1} & \\ \cdots & \\ \cdots & \\ \tau_l(b-a)^{h_l} & \\ \frac{(b-a)f_1}{af_1+g_1} & \mathfrak{B}\;,\mathrm{L}_1,L_j,L_j':D \\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (4.1)$$

under the same notations and conditions that (3.21) with $A_j = B_j = C_j^{(i)} = D_j^{(i)} = A'_j = B'_j = C'^{(i)}_j = D'^{(i)}_j = 1$

Remark:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable I-functions of Nambisan et al [2] and a class of multivariable polynomials defined by Srivastava [4].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable I-functions defined by Nambisan et al [2], a expansion of multivariable I-function defined by Nambisan et al [2] and a class of multivariable polynomials defined by Srivastava [4] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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