Eulerian integral associated with product of two multivariable A-functions, generalized Lauricella function and a class of polynomial and the multivariable I-function defined by Nambisan I

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ABSTRACT
The present paper is evaluated a new Eulerian integral associated with the product of two multivariable A-functions defined by Gautam et al [1] a generalized Lauricella function , a class of multivariable polynomials and multivariable I-function defined by Nambisan [2] with general arguments . We will study the case concerning the multivariable H-function defined by Srivastava et al [7]. .

Keywords: Eulerian integral, multivariable I-function, generalized Lauricella function of several variables, multivariable H-function, generalized hypergeometric function, class of polynomials,multivariable A-function.

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1. Introduction

In this paper, we evaluate a new Eulerian integral of most general characters associated with the products of two multivariable A-functions defined by Gautam et [1] , a serie expansion of multivariable I-function defined by Nambisan et al [2] and a class of polynomials with general arguments.
First time, we define the multivariable I-function by :

$$\bar{I}(z_1''',\cdots,z_v''') = \bar{I}^{0,N:M_1,N_1;\cdots;M_v,N_v}_{P,Q:P_1,Q_1;\cdots;P_v,Q_v}\left(\begin{array}{c|c} z_1''' & (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(v)};A_j)_{N+1,P}: \\ \cdot & \\ \cdot & \\ \cdot & \\ z_v''' & (b_j;\beta_j^{(1)},\cdots,\beta_j^{(v)};B_j)_{M+1,Q}: \end{array}\right.$$

$$\left.\begin{array}{c} (c_j^{(1)},\gamma_j^{(1)};1)_{1,N_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{N_1+1,P_1};\cdots;(c_j^{(v)},\gamma_j^{(v)};1)_{1,N_u},(c_j^{(v)},\gamma_j^{(v)};C_j^{(v)})_{N_v+1,P_v} \\ (d_j^{(1)},\delta_j^{(1)};1)_{1,M_1},(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{M_1+1,Q_1};\cdots;(d_j^{(v)},\delta_j^{(v)};1)_{1,M_v},(d_j^{(v)},\delta_j^{(v)};D_j^{(v)})_{M_v+1,Q_v} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^v}\int_{L_1}\cdots\int_{L_v}\phi_1(s_1,\cdots,s_v)\prod_{i=1}^{v}\xi_i'(s_i)z_i'''^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_v \tag{1.2}$$

where $\phi_1(s_1,\cdots,s_v),\xi_i'(s_i), i=1,\cdots,v$ are given by :

$$\phi_1(s_1,\cdots,s_v) = \frac{1}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{v}\alpha_j^{(i)}s_j\right)\prod_{j=M+1}^{Q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{v}\beta_j^{(i)}s_j\right)} \tag{1.3}$$

$$\xi_i'(s_i) = \frac{\prod_{j=1}^{N_i} \Gamma\left(1 - c_j^{(i)} + \gamma_j^{(i)} s_i\right) \prod_{j=1}^{M_i} \Gamma\left(d_j^{(i)} - \delta_j^{(i)} s_i\right)}{\prod_{j=N_i+1}^{P_i} \Gamma^{C_j^{(i)}}\left(c_j^{(i)} - \gamma_j^{(i)} s_i\right) \prod_{j=M_i+1}^{Q_i} \Gamma^{D_j^{(i)}}\left(1 - d_j^{(i)} - \delta_j^{(i)} s_i\right)} \tag{1.4}$$

$i = 1, \cdots, v$

Serie representation

If $z_i''' \neq 0; i = 1, \cdots, v$

$\delta_{h_i}^{(i)}(d_j^{(i)} + k_i) \neq \delta_j^{(i)}(\delta_{h_i}^{(i)} + \eta_i) for j \neq h_i, j, h_i = 1, \cdots, m_i (i = 1, \cdots, v), k_i, \eta_i = 0, 1, 2, \cdots (i = 1, \cdots, v)$, then

$$\bar{I}(z_1''', \cdots, z_v''') = \sum_{h_1=1}^{M_1} \cdots \sum_{h_v=1}^{M_v} \sum_{k_1=0}^{\infty} \cdots \sum_{k_v=0}^{\infty} \left[\phi_1\left(\frac{dh_1^{(1)} + k_1}{\delta h_1^{(1)}}, \cdots, \frac{dh_v^{(v)} + k_v}{\delta h_v^{(v)}}\right)\right] \prod_{j \neq h_i i=1}^{r} \frac{(-)^{k_i}}{\delta h_i^{(i)} k_i!} z_i'''^{\frac{dh_i + k_i}{\delta h_i}} \tag{1.5}$$

This result can be proved on computing the residues at the poles :

$$s_i = \frac{dh_i^{(i)} + k_i}{\delta h_i^{(i)}}, (h_i = 1, \cdots, m_i, k_i = 0, 1, 2, \cdots) for i = 1, \cdots, v \tag{1.6}$$

We may establish the the asymptotic expansion in the following convenient form :

$$\bar{I}(z_1''', \cdots, z_v''') = 0(|z_1'''|^{\alpha_1}, \cdots, |z_v'''|^{\alpha_v}), max(|z_1'''|, \cdots, |z_v'''|) \to 0$$

$$\bar{I}(z_1''', \cdots, z_v''') = 0(|z_1'''|^{\beta_1}, \cdots, |z_v'''|^{\beta_u}), min(|z_1'''|, \cdots, |z_v'''|) \to \infty$$

where $k = 1, \cdots, v : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will note $\eta_{h_i,k_i} = \dfrac{dh_i^{(i)} + k_i}{\delta h_i}$, $(h_i = 1, \cdots, m_i, k_i = 0, 1, 2, \cdots) for i = 1, \cdots, v$ (1.7)

The A-function is defined and represented in the following manner.

$$A(z_1, \cdots, z_r) = A_{p,q:p_1,q_1;\cdots;p_r,q_r}^{m,n:m_1,n_1;\cdots;m_r,n_r}\left(\begin{array}{c|c} z_1 & (a_j; A_j^{(1)}, \cdots, A_j^{(r)})_{1,p} : \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & (b_j; B_j^{(1)}, \cdots, B_j^{(r)})_{1,q} : \end{array}\right.$$

$$\left.\begin{array}{c} (c_j^{(1)}, C_j^{(1)})_{1,p_1}; \cdots; (c_j^{(r)}, C_j^{(r)})_{1,p_r} \\ \\ (d_j^{(1)}, D_j^{(1)})_{1,q_1}; \cdots; (d_j^{(r)}, D_j^{(r)})_{1,q_r} \end{array}\right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1, \cdots, s_r) \prod_{i=1}^{r} \theta_i(s_i) z_i^{s_i} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.9}$$

where $\phi(s_1,\cdots,s_r),\theta_i(s_i), i=1,\cdots,r$ are given by :

$$\phi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\sum_{i=1}^{r}B_j^{(i)}s_i)\prod_{j=1}^{n}\Gamma(1-a_j+\sum_{i=1}^{r}A_j^{(i)}s_j)}{\prod_{j=n+1}^{p}\Gamma(a_j-\sum_{i=1}^{r}A_j^{(i)}s_j)\prod_{j=m+1}^{q}\Gamma(1-b_j+\sum_{i=1}^{r}B_j^{(i)}s_j)} \quad (1.10)$$

$$\theta_i(s_i)=\frac{\prod_{j=1}^{n_i}\Gamma(1-c_j^{(i)}+C_j^{(i)}s_i)\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-D_j^{(i)}s_i)}{\prod_{j=n_i+1}^{p_i}\Gamma(c_j^{(i)}-C_j^{(i)}s_i)\prod_{j=m_i+1}^{q_i}\Gamma(1-d_j^{(i)}-D_j^{(i)}s_i)} \quad (1.11)$$

Here $m,n,p,m_i,n_i,p_i,c_i\in\mathbb{N}^*; i=1,\cdots,r$; $a_j,b_j,c_j^{(i)},d_j^{(i)},A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega_i)z_k|<\frac{1}{2}\eta_k\pi,\xi^*=0,\eta_i>0 \quad (1.12)$$

$$\Omega_i=\prod_{j=1}^{p}\{A_j^{(i)}\}^{A_j^{(i)}}\prod_{j=1}^{q}\{B_j^{(i)}\}^{-B_j^{(i)}}\prod_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}}\prod_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}};i=1,\cdots,r \quad (1.13)$$

$$\xi_i^*=Im(\sum_{j=1}^{p}A_j^{(i)}-\sum_{j=1}^{q}B_j^{(i)}+\sum_{j=1}^{q_i}D_j^{(i)}-\sum_{j=1}^{p_i}C_j^{(i)});i=1,\cdots,r \quad (1.14)$$

$$\eta_i=Re\left(\sum_{j=1}^{n}A_j^{(i)}-\sum_{j=n+1}^{p}A_j^{(i)}+\sum_{j=1}^{m}B_j^{(i)}-\sum_{j=m+1}^{q}B_j^{(i)}+\sum_{j=1}^{m_i}D_j^{(i)}-\sum_{j=m_i+1}^{q_i}D_j^{(i)}+\sum_{j=1}^{n_i}C_j^{(i)}-\sum_{j=n_i+1}^{p_i}C_j^{(i)}\right)$$

$i=1,\cdots,r$ (1.15)

Consider the second multivariable A-function.

$$A(z_1',\cdots,z_s')=A_{p',q':p_1',q_1';\cdots;p_r',q_r'}^{m',n':m_1',n_1';\cdots;m_r',n_r'}\left(\begin{array}{c|c}z_1 & (a'_j;A_j'^{(1)},\cdots,A_j'^{(s)})_{1,p'}:\\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & (b'_j;B_j'^{(1)},\cdots,B_j'^{(s)})_{1,q'}:\end{array}\right.$$

$$\left.\begin{array}{c}(c'^{(1)}_j,C_j'^{(1)})_{1,p_1'};\cdots;(c_j'^{(s)},C_j'^{(s)})_{1,p_s'}\\ \\ (d'^{(1)}_j,D_j'^{(1)})_{1,q_1'};\cdots;(d_j'^{(s)},D_j'^{(s)})_{1,q_s'}\end{array}\right) \quad (1.16)$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1'}\cdots\int_{L_s'}\phi'(t_1,\cdots,t_s)\prod_{i=1}^{s}\theta_i'(t_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \quad (1.17)$$

where $\phi'(t_1,\cdots,t_s),\theta_i'(t_i), i=1,\cdots,s$ are given by :

$$\phi'(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{m'}\Gamma(b'_j-\sum_{i=1}^{s}B'^{(i)}_jt_i)\prod_{j=1}^{n'}\Gamma(1-a'_j+\sum_{i=1}^{s}A'^{(i)}_jt_j)}{\prod_{j=n'+1}^{p'}\Gamma(a'_j-\sum_{i=1}^{s}A'^{(i)}_jt_j)\prod_{j=m'+1}^{q'}\Gamma\left(1-b'_j+\sum_{i=1}^{s}B'^{(i)}_jt_j\right)} \tag{1.18}$$

$$\theta'_i(t_i)=\frac{\prod_{j=1}^{n'_i}\Gamma(1-c'^{(i)}_j+C'^{(i)}_jt_i)\prod_{j=1}^{m'_i}\Gamma(d'^{(i)}_j-D'^{(i)}_jt_i)}{\prod_{j=n'_i+1}^{p'_i}\Gamma(c'^{(i)}_j-C'^{(i)}_jt_i)\prod_{j=m'_i+1}^{q'_i}\Gamma(1-d'^{(i)}_j-D'^{(i)}_jt_i)} \tag{1.19}$$

Here $m',n',p',m'_i,n'_i,p'_i,c'_i\in\mathbb{N}^*;i=1,\cdots,r\,;a'_j,b'_j,c'^{(i)}_j,d'^{(i)}_j,A'^{(i)}_j,B'^{(i)}_j,C'^{(i)}_j,D'^{(i)}_j\in\mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega'_i)z'_k|<\frac{1}{2}\eta'_k\pi,\xi'^*=0,\eta'_i>0 \tag{1.20}$$

$$\Omega'_i=\prod_{j=1}^{p'}\{A'^{(i)}_j\}^{A'^{(i)}_j}\prod_{j=1}^{q'}\{B'^{(i)}_j\}^{-B'^{(i)}_j}\prod_{j=1}^{q'_i}\{D'^{(i)}_j\}^{D'^{(i)}_j}\prod_{j=1}^{p'_i}\{C'^{(i)}_j\}^{-C'^{(i)}_j};i=1,\cdots,s \tag{1.21}$$

$$\xi'^*_i=Im(\sum_{j=1}^{p'}A'^{(i)}_j-\sum_{j=1}^{q'}B'^{(i)}_j+\sum_{j=1}^{q'_i}D'^{(i)}_j-\sum_{j=1}^{p'_i}C'^{(i)}_j);i=1,\cdots,s \tag{1.22}$$

$$\eta'_i=Re\left(\sum_{j=1}^{n'}A'^{(i)}_j-\sum_{j=n'+1}^{p'}A'^{(i)}_j+\sum_{j=1}^{m'}B'^{(i)}_j-\sum_{j=m'+1}^{q'}B'^{(i)}_j+\sum_{j=1}^{m'_i}D'^{(i)}_j-\sum_{j=m'_i+1}^{q'_i}D'^{(i)}_j+\sum_{j=1}^{n'_i}C'^{(i)}_j-\sum_{j=n'_i+1}^{p'_i}C'^{(i)}_j\right)$$

$$i=1,\cdots,s \tag{1.23}$$

Srivastava [4] introduced and defined a general class of multivariable polynomials as follows

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[y_1,\cdots,y_u]=\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!}$$

$$A[N_1,K_1;\cdots;N_u,K_u]y_1^{K_1}\cdots y_u^{K_u} \tag{1.24}$$

where $M_1,\cdots,M_u$ are arbitrary positive integers and the coefficients are $A[N_1,K_1;\cdots;N_u,K_u]$ arbitrary constants, real or complex.

2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [6 ,page 39 eq .30]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^{P}\Gamma(A_j+s_1+\cdots+s_r)}{\prod_{j=1}^{Q}\Gamma(B_j+s_1+\cdots+s_r)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j+s_1+\cdots+s_r)$

are separated from those of $\Gamma(-s_j), j=1,\cdots,r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j=1,\cdots,r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}$$

$$F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right) \tag{2.2}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i,\tau_j,h_j\in\mathbb{C},\lambda_j\in\mathbb{R}^+(i=1,\cdots,k;j=1,\cdots,l)$

$$min(Re(\alpha),Re(\beta))>0,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1,\ \max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,$$

and $F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust[5,page 454] given by :

$$F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\prod_{j=1}^{l}\Gamma(\lambda_j)\prod_{j=1}^{k}\Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}}\int_{L_1}\cdots\int_{L_{l+k}}\frac{\Gamma\left(\alpha+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}{\Gamma\left(\alpha+\beta+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}\prod_{j=1}^{l}\Gamma(\lambda_j+s_j)\prod_{j=1}^{k}\Gamma(-\sigma_j+s_{l+j})$$

$$\prod_{j=1}^{l+k}\Gamma(-s_j)z_1^{s_1}\cdots z_l^{s_l}z_{l+1}^{s_{l+1}}\cdots,z_{l+k}^{s_{l+k}}\,\mathrm{d}s_1\cdots\mathrm{d}s_{l+k} \tag{2.3}$$

Here the contour $L_j's$ are defined by $L_j=L_{w\zeta_j\infty}(Re(\zeta_j)=v_j'')$ starting at the point $v_j''-\omega\infty$ and terminating at the point $v_j''+\omega\infty$ with $v_j''\in\mathbb{R}(j=1,\cdots,l)$ and each of the remaining contour $L_{l+1},\cdots,L_{l+k}$ run from $-\omega\infty$ to $\omega\infty$

(2.2) can be easily established by expanding $\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.4}$$

integrating term by term with the help of the integral given by Saigo and Saxena [3, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [5, page 454].

3. Eulerian integral

In this section , we note :

$$\theta_i=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}},\zeta_j^{(i)}>0(i=1,\cdots,r)\,;\,\theta'_i=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'^{(i)}},\zeta_j'^{(i)}>0(i=1,\cdots,s)$$

$$\theta''_i=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j''^{(i)}},\zeta_j''^{(i)}>0(i=1,\cdots,u)$$

$$\theta'''_i=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'''^{(i)}},\zeta_j'''^{(i)}>0(i=1,\cdots,v) \tag{3.1}$$

$$X=m_1,n_1;\cdots;m_r,n_r;m'_1,n'_1;\cdots;m'_s,n'_s;1,0;\cdots;1,0;1,0;\cdots;1,0 \tag{3.2}$$

$$Y=p_1,q_1;\cdots;p_r,q_r;p'_1,q'_1;\cdots;p'_s,q'_s;0,1;\cdots;0,1;0,1;\cdots;0,1 \tag{3.3}$$

$$A=(a_j;A_j^{(1)},\cdots,A_j^{(r)},0,\cdots,0,0,\cdots,0,0,\cdots,0)_{1,p} \tag{3.4}$$

$$B=(b_j;B_j^{(1)},\cdots,B_j^{(r)},0\cdots,0,0\cdots,0,0\cdots,0)_{1,q} \tag{3.5}$$

$$A'=(a'_j;0,\cdots,0,A_j'^{(1)},\cdots,A_j'^{(s)},0,\cdots,0,0,\cdots,0)_{1,p'} \tag{3.6}$$

$$B'=(b'_j;0,\cdots,0,B_j'^{(1)},\cdots,B_j'^{(s)},0,\cdots,0,0,\cdots,0)_{1,q'} \tag{3.7}$$

$$\mathrm{C}=(\mathrm{c}_j^{(1)},C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},C_j^{(r)})_{1,p_r};(c_j'^{(1)},C_j'^{(1)})_{1,p'_1};\cdots;(c_j'^{(r)},C_j'^{(s)})_{1,p'_s}$$

$$(1,0);\cdots;(1,0);(1,0);\cdots;(1,0) \tag{3.8}$$

$$D=(\mathrm{d}_j^{(1)},D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},D_j^{(r)})_{1,q_r};(\mathrm{d}'^{(1)}_j,D_j'^{(1)})_{1,q'_1};\cdots;(d_j'^{(s)},D_j'^{(s)})_{1,q'_s};$$

$$(0,1);\cdots;(0,1);(0,1);\cdots;(0,1) \tag{3.9}$$

$$X=m_1,n_1;\cdots;m_r,n_r;m'_1,n'_1;\cdots;m'_s,n'_s;1,0;\cdots;1,0;1,0;\cdots;1,0 \quad (3.2)$$

$$Y=p_1,q_1;\cdots;p_r,q_r;p'_1,q'_1;\cdots;p'_s,q'_s;0,1;\cdots;0,1;0,1;\cdots;0,1 \quad (3.3)$$

$$A=(a_j;A_j^{(1)},\cdots,A_j^{(r)},0,\cdots,0,0,\cdots,0,0,\cdots,0)_{1,p} \quad (3.4)$$

$$B=(b_j;B_j^{(1)},\cdots,B_j^{(r)},0\cdots,0,0\cdots,0,0\cdots,0)_{1,q} \tag{3.5}$$

$$A'=(a'_j;0,\cdots,0,A'^{(1)}_j,\cdots,A'^{(s)}_j,0,\cdots,0,0,\cdots,0)_{1,p'} \tag{3.6}$$

$$B'=(b'_j;0,\cdots,0,B'^{(1)}_j,\cdots,B'^{(s)}_j,0,\cdots,0,0,\cdots,0)_{1,q'} \tag{3.7}$$

$$\mathrm{C}=(\mathrm{c}_j^{(1)},C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},C_j^{(r)})_{1,p_r};(c'^{(1)}_j,C'^{(1)}_j)_{1,p'_1};\cdots;(c'^{(r)}_j,C'^{(s)}_j)_{1,p'_s}$$

$$(1,0);\cdots;(1,0);(1,0);\cdots;(1,0) \tag{3.8}$$

$$D=(\mathrm{d}_j^{(1)},D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},D_j^{(r)})_{1,q_r};(\mathrm{d}'^{(1)}_j,D'^{(1)}_j)_{1,q'_1};\cdots;(d'^{(s)}_j,D'^{(s)}_j)_{1,q'_s};$$

$$(0,1);\cdots;(0,1);(0,1);\cdots;(0,1) \tag{3.9}$$

$$K_1=(1-\alpha-\sum_{i=1}^{u}K_ia_i-\sum_{i=1}^{v}\eta_{G_i,g_i}a'_i;\mu_1,\cdots,\mu_r,\mu'_1,\cdots,\mu'_s,h_1,\cdots,h_l,1,\cdots,1) \tag{3.10}$$

$$K_2=(1-\beta-\sum_{i=1}^{u}K_ib_i-\sum_{i=1}^{v}\eta_{G_i,g_i}b'_i;\rho_1,\cdots,\rho_r,\rho'_1,\cdots,\rho'_s,0,\cdots,0,0\cdots,0) \tag{3.11}$$

$$K_j=[1-\lambda_j-\sum_{i=1}^{u}K_i\zeta''^{(i)}_j-\sum_{i=1}^{v}\eta_{G_i,g_i}\zeta'''^{(i)}_j;\zeta_j^{(1)},\cdots,\zeta_j^{(r)},\zeta'^{(1)}_j\cdots,\zeta'^{(s)}_j,$$

$$0,\cdots,\underset{\mathrm{j}}{1},\cdots,0,0\cdots,0]_{1,l} \tag{3.12}$$

$$K'_j=[1+\sigma_j-\sum_{i=1}^{u}K_i\lambda''^{(i)}_j-\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}_j;\lambda_j^{(1)},\cdots,\lambda_j^{(r)},\lambda'^{(1)}_j\cdots,\lambda'^{(s)}_j,$$

$$0,\cdots,0,0\cdots,\underset{\mathrm{j}}{1},\cdots,0]_{1,k} \tag{3.13}$$

$$L_1=(1-\alpha-\beta-\sum_{i=1}^{u}K_i(a_i+b_i)-\sum_{i=1}^{v}(a'_i+b'_i)\eta_{G_i,g_i};\mu_1+\rho_1,\cdots,\mu_r+\rho_r,\mu'_1+\rho'_1,\cdots,\mu'_r+\rho'_r,$$

$$h_1,\cdots,h_l,1,\cdots,1) \tag{3.14}$$

$$L_j=[1-\lambda_j-\sum_{i=1}^{u}K_i\zeta''^{(i)}_j-\sum_{i=1}^{s}\zeta'''^{(i)}_j\eta_{G_i,g_i};\zeta_j^{(1)},\cdots,\zeta_j^{(r)},\zeta'^{(1)}_j\cdots,\zeta'^{(s)}_j,0,\cdots,0,0\cdots,0]_{1,l} \tag{3.15}$$

$$L'_j=[1+\sigma_j-\sum_{i=1}^{u}K_i\lambda''^{(i)}_j-\sum_{i=1}^{v}\lambda'''^{(i)}_j\eta_{G_i,g_i};\lambda_j^{(1)},\cdots,\lambda_j^{(r)},\lambda'^{(1)}_j\cdots,\lambda'^{(s)}_j,0,\cdots,0,0,\cdots,0]_{1,k} \tag{3.16}$$

$$P_1=(b-a)^{\alpha+\beta-1}\left\{\prod_{j=1}^{h}(af_j+g_j)^{\sigma_j}\right\} \quad (3.17)$$

$$B_{u,v}=(b-a)^{\sum_{i=1}^{v}(a_i'+b_i')\eta_{G_i,g_i}+\sum_{i=1}^{u}(a_i+b_i)K_i}\left\{\prod_{j=1}^{h}(af_j+g_j)^{-\sum_{i=1}^{v}\lambda_i'''\eta_{g_i,h_i}-\sum_{i=1}^{u}\lambda_i''K_i}\right\} \quad (3.18)$$

$$A_u=\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!}A[N_1,K_1;\cdots;N_u,K_u] \quad (3.19)$$

$$\mathfrak{A}=A,A';\mathfrak{B}=B,B' \quad (3.20)$$

We have the general Eulerian integral.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\begin{pmatrix} \mathrm{z}_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(u)}} \end{pmatrix}$$

$$\bar{I}\begin{pmatrix} \mathrm{z}_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{pmatrix}$$

$$A\begin{pmatrix} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{pmatrix}$$

$$A\begin{pmatrix} \mathrm{z}_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{pmatrix}\mathrm{d}t=$$

$$= P_1 \sum_{h_1=1}^{M_1} \cdots \sum_{h_v=1}^{M_v} \sum_{k_1=0}^{\infty} \cdots \sum_{k_v=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \prod_{i=1}^{v} \frac{(-)^{k_i}}{\delta h_i^{(i)} k_i!} z_i'''^{\eta_{h_i,k_i}} \prod_{k=1}^{u} z''^{R_k k} A_u B_{u,v}[\phi_1(\eta_{h_1,k_1}, \cdots, \eta_{h_r,k_r})]_{j \neq h_i}$$

$$A^{m+m',n+n'+l+k+2;X}_{p+p'+l+k+2,q+q'+l+k+1;Y} \left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^k (af_j+g_j)^{\lambda_j^{(1)}}} & \\ \cdots & \mathfrak{A}\ ,\ \mathrm{K}_1, K_2, K_j, K'_j : C \\ \cdots & \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^k (af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^k (af_j+g_j)^{\lambda'^{(1)}_j}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^k (af_j+g_j)^{\lambda'^{(s)}_j}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdots & \cdot \\ \cdots & \\ \tau_l(b-a)^{h_l} & \\ \frac{(b-a)f_1}{af_1+g_1} & \mathfrak{B}\ ,\ \mathrm{L}_1, L_j, L'_j : D \\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array} \right) \quad (3.21)$$

where $\mathfrak{A}, \mathfrak{B}, C, D, X, K_1, K_2, K_j, K'_j, L_1, L_j, L'_j, P_1, P_u, B_u$ and $\mathfrak{B}_1$ are defined above.

Provided that

(A) $a, b \in \mathbb{R}(a < b); \mu_i, \mu'_u, \rho_i, \rho'_u, \lambda_j^{(i)}, \lambda'^{(u)}_j, h_v \in \mathbb{R}^+, f_i, g_j, \tau_v, \sigma_j, \lambda_v \in \mathbb{C}\ (i = 1, \cdots, r; j = 1, \cdots; k;$

$u = 1, \cdots, s; v = 1, \cdots, l), a_i, b_i, \lambda''^{(i)}_j, \zeta''^{(i)}_j \in \mathbb{R}^+, (i = 1, \cdots, u; j = 1, \cdots, k)$

$a'_i, b'_i, \lambda'''^{(i)}_j, \zeta'''^{(i)}_j \in \mathbb{R}^+, (i = 1, \cdots, v; j = 1, \cdots, k)$

(B) $m, n, p, m_i, n_i, p_i, c_i \in \mathbb{N}^*; i = 1, \cdots, r$ **;** $a_j, b_j, c_j^{(i)}, d_j^{(i)}, A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{C}$

$m', n', p', m'_i, n'_i, p'_i, c'_i \in \mathbb{N}^*; i = 1, \cdots, r\ ; a'_j, b'_j, c'^{(i)}_j, d'^{(i)}_j, A'^{(i)}_j, B'^{(i)}_j, C'^{(i)}_j, D'^{(i)}_j \in \mathbb{C}$

(C) $\max_{1 \leqslant j \leqslant k} \left\{ \left| \frac{(b-a)f_i}{af_i + g_i} \right| \right\} < 1$

(D) $Re\left[\alpha + \sum_{j=1}^{r} \mu_j \min_{1 \leqslant k \leqslant m_i} \frac{d_k^{(j)}}{D_k^{(j)}} + \sum_{j=1}^{s} \mu'_i \min_{1 \leqslant k \leqslant m'_i} \frac{d'^{(j)}_k}{D'^{(j)}_k}\right] > 0$

$$Re[\beta+\sum_{j=1}^{r}\rho_j\min_{1\leqslant k\leqslant m_i}\frac{d_k^{(j)}}{D_k^{(j)}}+\sum_{j=1}^{s}\rho'_j\min_{1\leqslant k\leqslant m'_i}\frac{d_k'^{(j)}}{D_k'^{(j)}}]>0$$

(E) $Re\left(\alpha+\sum_{i=1}^{v}\eta_{G_i,g_i}a'_i+\sum_{i=1}^{u}K_ia_i+\sum_{i=1}^{r}\mu_is_i+\sum_{i=1}^{s}t_i\mu'_i\right)>0$

$$Re\left(\beta+\sum_{i=1}^{v}\eta_{G_i,g_i}b'_i+\sum_{i=1}^{u}K_ib_i+\sum_{i=1}^{r}v_is_i+\sum_{i=1}^{s}t_i\rho'_i\right)>0$$

$$Re\left(\lambda_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda_j'''^{(i)}+\sum_{i=1}^{u}K_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\zeta_j^{(i)}+\sum_{i=1}^{s}t_i\zeta_j'^{(i)}\right)>0(j=1,\cdots,l);$$

$$Re\left(-\sigma_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}+\sum_{i=1}^{u}K_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\lambda_j^{(i)}+\sum_{i=1}^{s}t_i\lambda_j'^{(i)}\right)>0(j=1,\cdots,k);$$

(F) $|arg(\Omega_i)z_k|<\frac{1}{2}\eta_i\pi,\xi^*=0,\eta_i>0$

$$\Omega_i=\prod_{j=1}^{p}\{A_j^{(i)}\}^{A_j^{(i)}}\prod_{j=1}^{q}\{B_j^{(i)}\}^{-B_j^{(i)}}\prod_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}}\prod_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}};i=1,\cdots,r$$

$$\xi_i^*=Im(\sum_{j=1}^{p}A_j^{(i)}-\sum_{j=1}^{q}B_j^{(i)}+\sum_{j=1}^{q_i}D_j^{(i)}-\sum_{j=1}^{p_i}C_j^{(i)});i=1,\cdots,r$$

$$\eta_i=Re\left(\sum_{j=1}^{n}A_j^{(i)}-\sum_{j=n+1}^{p}A_j^{(i)}+\sum_{j=1}^{m}B_j^{(i)}-\sum_{j=m+1}^{q}B_j^{(i)}+\sum_{j=1}^{m_i}D_j^{(i)}-\sum_{j=m_i+1}^{q_i}D_j^{(i)}+\sum_{j=1}^{n_i}C_j^{(i)}-\sum_{j=n_i+1}^{p_i}C_j^{(i)}\right)$$

$$-\mu_i-\rho_i-\sum_{l=1}^{k}\lambda_j^{(i)}-\sum_{l=1}^{l}\zeta_j^{(i)}>0;\ i=1,\cdots,r$$

$$|arg(\Omega'_i)z'_k|<\frac{1}{2}\eta'_i\pi,\xi'^*=0,\eta'_i>0$$

$$\Omega'_i=\prod_{j=1}^{p'}\{A_j'^{(i)}\}^{A_j'^{(i)}}\prod_{j=1}^{q'}\{B_j'^{(i)}\}^{-B_j'^{(i)}}\prod_{j=1}^{q'_i}\{D_j'^{(i)}\}^{D_j'^{(i)}}\prod_{j=1}^{p'_i}\{C_j'^{(i)}\}^{-C_j'^{(i)}};i=1,\cdots,s$$

$$\xi_i'^*=Im(\sum_{j=1}^{p'}A_j'^{(i)}-\sum_{j=1}^{q'}B_j'^{(i)}+\sum_{j=1}^{q'_i}D_j'^{(i)}-\sum_{j=1}^{p'_i}C_j'^{(i)});i=1,\cdots,s$$

$$\eta'_i=Re\left(\sum_{j=1}^{n'}A_j'^{(i)}-\sum_{j=n'+1}^{p'}A_j'^{(i)}+\sum_{j=1}^{m'}B_j'^{(i)}-\sum_{j=m'+1}^{q'}B_j'^{(i)}+\sum_{j=1}^{m'_i}D_j'^{(i)}-\sum_{j=m'_i+1}^{q'_i}D_j'^{(i)}+\sum_{j=1}^{n'_i}C_j'^{(i)}-\sum_{j=n'_i+1}^{p'_i}C_j'^{(i)}\right)$$

$$-\mu'_i-\rho'_i-\sum_{l=1}^{k}\lambda_j'^{(i)}-\sum_{l=1}^{l}\zeta_j'^{(i)}>0;i=1,\cdots,s$$

(H) $\left|arg\left(z_i\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(i)}}\right)\right|<\frac{1}{2}\eta_i\pi\ \ (a\leqslant t\leqslant b;i=1,\cdots,r)$

$$\left|arg\left(z_i'\prod_{j=1}^{l}\left[1-\tau_j'(t-a)^{h_i'}\right]^{-\zeta_j'^{(i)}}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(i)}}\right)\right|<\frac{1}{2}\eta_i'\pi\ (a\leqslant t\leqslant b;i=1,\cdots,s)$$

(I) The multiple series occuring on the right-hand side of (3.21) is absolutely and uniformly convergent.

Proof

To prove (3.21), first, we express in serie the multivariable I-function defined by Nambisan et al [2] with the help of (1.5), a class of multivariable polynomials defined by Srivastava $S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[.]$ in serie with the help of (1.24), the A-functions of r-variables and s-variables defined by Gautam et al [1] in terms of Mellin-Barnes type contour integral with the help of (1.9) and (1.17) respectively. Now collect the power of $\left[1-\tau_j(t-a)^{h_i}\right]$ with $(i=1,\cdots,r;j=1,\cdots,l)$ and collect the power of (f_jt+g_j) with $j=1,\cdots,k$. Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the $(r+s+k+l)$ dimensional Mellin-Barnes integral in multivariable A-function defined by Gautam et al [1], we obtain the equation (3.20).

Remarks

If a) $\rho_1=\cdots,\rho_r=\rho_1'=\cdots,\rho_s'=0$; b) $\mu_1=\cdots,\mu_r=\mu_1'=\cdots,\mu_s'=0$, we obtain the similar formulas that (3.21) with the corresponding simplifications.

4. Particular cases

If $A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{R}$,$m=0$ and $A_j'^{(i)},B_j'^{(i)},C_j'^{(i)},D_j'^{(i)}\in\mathbb{R}$ and $m'=0$, the multivariable A-functions reduces to multivariable H-functions defined by Srivastava et al [7], we obtain the following result.

$$\int_a^b(t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\left(\begin{array}{c}z_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j''^{(1)}}\\ .\\ .\\ .\\ z_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j''^{(u)}}\end{array}\right)$$

$$\bar{I}\left(\begin{array}{c}z_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'''^{(1)}}\\ .\\ .\\ .\\ z_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'''^{(v)}}\end{array}\right)$$

$$H\left(\begin{array}{c}z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(1)}}\\ .\\ .\\ .\\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(r)}}\end{array}\right)$$

$$H\left(\begin{array}{c} z'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ . \\ . \\ . \\ z'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t =$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{k}A_uB_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$H^{0,n+n'+l+k+2;X}_{p+p'+l+k+2,q+q'+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \\ \cdots & \\ \cdots & \mathfrak{A}\ ,\mathrm{K}_1,K_2,K_j,K'_j:C \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & . \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & . \\ \tau_1(b-a)^{h_1} & . \\ \cdots & . \\ \cdots & \\ \tau_l(b-a)^{h_l} & \mathfrak{B}\ ,\mathrm{L}_1,L_j,L'_j:D \\ \frac{(b-a)f_1}{af_1+g_1} & \\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (4.1)$$

under the same notations and validity conditions that (3.21) with $A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{R}$,$m=0$ and $A_j'^{(i)},B_j'^{(i)},C_j'^{(i)},D_j'^{(i)}\in\mathbb{R}$ and $m'=0$

Remark:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable A-functions and a class of multivariable polynomials defined by Srivastava et al [4].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable A-functions defined by Gautam et al [1],a expansion of multivariable I-function defined by Nambisan et al [2] and a class of multivariable polynomials defined by Srivastava [4] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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