Selberg integral involving the incomplete generalized hypergeometric function,a class of polynomials and multivariable Aleph-functions

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ABSTRACT
In the present paper we evaluate the modified Selberg integral involving the product of a multivariable Aleph-functions, a extension of the incomplete generalized hypergeometric function and general class of polynomials of several variables. The importance of the result established in this paper lies in the fact they involve the Aleph-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializating the parameters their in.We will study two particular cases.

Keywords:Multivariable Aleph-function, general class of polynomials, modified Selberg integral, incomplete generalized hypergeometric function , multivariable I-function

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1.Introduction and preliminaries.

In this paper, we evaluate the modified Selberg integral involving the product of a multivariable Aleph-functions, a extension of the incomplete generalized hypergeometric function and general class of polynomials of several variables.

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [2] , itself is an a generalisation of G and H-functions of multiple variables defined by Srivastava et al [6]. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define : $\aleph(z_1,\cdots,z_r) = \aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}} \left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array} \right|$

$[(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}] \ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\qquad\cdots\cdots\cdots \ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$\left. \begin{array}{c} [(c_j^{(1)});\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(c_j^{(r)});\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}] \\ [(d_j^{(1)});\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(d_j^{(r)});\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}] \end{array} \right)$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1}\cdots\int_{L_r} \psi(s_1,\cdots,s_r) \prod_{k=1}^{r} \theta_k(s_k) y_k^{s_k} \mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega = \sqrt{-1}$

$$\psi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k) = \frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \quad (1.3)$$

Suppose , as usual , that the parameters

$a_j, j=1,\cdots,p; b_j, j=1,\cdots,q;$

$c_j^{(k)}, j=1,\cdots,n_k; c_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,p_{i^{(k)}};$

$d_j^{(k)}, j=1,\cdots,m_k; d_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,q_{i^{(k)}};$

with $k=1\cdots,r, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)} + \tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} + \tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)} - \tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)} - \sum_{j=1}^{m_k}\delta_j^{(k)}$$

$$-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} \leqslant 0 \quad (1.4)$$

The reals numbers τ_i are positives for $i=1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)$ with $j=1$ to m_k are separated from those of $\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(k)}s_k)$ with $j=1$ to n and $\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)$ with $j=1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)} - \tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)} - \tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} - \tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)} - \tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k=1,\cdots,r, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)} \quad (1.5)$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_r) = 0(\ |z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\ ), max(\ |z_1|,\cdots,|z_r|\ ) \rightarrow 0$

$\aleph(z_1,\cdots,z_r) = 0(\ |z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\ ), min(\ |z_1|,\cdots,|z_r|\ ) \rightarrow \infty$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

Serie representation of Aleph-function of several variables is given by

$$\aleph(y_1, \cdots, y_r) = \sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} \psi(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r})$$

$$\times\ \theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r}) y_1^{-\eta_{G_1,g_1}} \cdots y_r^{-\eta_{G_r,g_r}} \tag{1.6}$$

Where $\psi(., \cdots, .), \theta_i(.), i = 1, \cdots, r$ are given respectively in (1.2), (1.3) and

$$\eta_{G_1,g_1} = \frac{d_{g_1}^{(1)} + G_1}{\delta_{g_1}^{(1)}}, \cdots, \ \eta_{G_r,g_r} = \frac{d_{g_r}^{(r)} + G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[d_j^i + p_i] \neq \delta_j^{(i)}[d_{g_i}^i + G_i]$ (1.7)

for $j \neq m_i, m_i = 1, \cdots \eta_{G_i,g_i}; p_i, n_i = 0, 1, 2, \cdots, ; y_i \neq 0, i = 1, \cdots, r$ (1.8)

Consider the Aleph-function of s variables

$$\aleph(z_1, \cdots, z_s) = \aleph^{0,N:M_1,N_1,\cdots,M_s,N_s}_{P_i,Q_i,\iota_i;r':P_{i(1)},Q_{i(1)},\iota_{i(1)};r^{(1)};\cdots;P_{i(s)},Q_{i(s)};\iota_{i(s)};r^{(s)}} \left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_s \end{array} \right|$$

$$\begin{array}{l} [(u_j; \mu_j^{(1)}, \cdots, \mu_j^{(r')})_{1,N}] \ ,[\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots, \mu_{ji}^{(r')})_{N+1,P_i}] : \\ \quad \ldots\ldots\ldots\ldots\ldots\ldots \ ,[\iota_i(v_{ji}; \upsilon_{ji}^{(1)}, \cdots, \upsilon_{ji}^{(r')})_{M+1,Q_i}] : \end{array}$$

$$\left. \begin{array}{l} [(a_j^{(1)}); \alpha_j^{(1)})_{1,N_1}], [\iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_i^{(1)}}]; \cdots; [(a_j^{(s)}); \alpha_j^{(s)})_{1,N_s}], [\iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)}; \alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_i^{(s)}}] \\ [(b_j^{(1)}); \beta_j^{(1)})_{1,M_1}], [\iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)}; \beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_i^{(1)}}]; \cdots; [(b_j^{(s)}); \beta_j^{(s)})_{1,M_s}], [\iota_{i^{(s)}}(b_{ji^{(s)}}^{(s)}; \beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_i^{(s)}}] \end{array} \right)$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L'_1} \cdots \int_{L'_s} \zeta(t_1, \cdots, t_s) \prod_{k=1}^{s} \phi_k(t_k) z_k^{t_k} \,\mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.9}$$

with $\omega = \sqrt{-1}$

$$\zeta(t_1, \cdots, t_s) = \frac{\prod_{j=1}^{N} \Gamma(1 - u_j + \sum_{k=1}^{s} \mu_j^{(k)} t_k)}{\sum_{i=1}^{r'} [\iota_i \prod_{j=N+1}^{P_i} \Gamma(u_{ji} - \sum_{k=1}^{s} \mu_{ji}^{(k)} t_k) \prod_{j=1}^{Q_i} \Gamma(1 - \upsilon_{ji} + \sum_{k=1}^{s} \upsilon_{ji}^{(k)} t_k)]} \tag{1.10}$$

and $$\phi_k(t_k) = \frac{\prod_{j=1}^{M_k}\Gamma(b_j^{(k)}-\beta_j^{(k)}t_k)\prod_{j=1}^{N_k}\Gamma(1-a_j^{(k)}+\alpha_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{r^{(k)}}[\iota_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-b_{ji^{(k)}}^{(k)}+\beta_{ji^{(k)}}^{(k)}t_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(a_{ji^{(k)}}^{(k)}-\alpha_{ji^{(k)}}^{(k)}s_k)]} \quad (1.11)$$

Suppose , as usual , that the parameters

$u_j, j=1,\cdots,P; v_j, j=1,\cdots,Q;$

$a_j^{(k)}, j=1,\cdots,N_k; a_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,P_{i^{(k)}};$

$b_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,Q_{i^{(k)}}; b_j^{(k)}, j=1,\cdots,M_k;$

with $k=1\cdots,s, i=1,\cdots,r', i^{(k)}=1,\cdots,r^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{N}\mu_j^{(k)} + \iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)} + \sum_{j=1}^{N_k}\alpha_j^{(k)} + \iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)} - \iota_i\sum_{j=1}^{Q_i}\upsilon_{ji}^{(k)} - \sum_{j=1}^{M_k}\beta_j^{(k)}$$

$$-\iota_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)} \leqslant 0 \quad (1.12)$$

The reals numbers τ_i are positives for $i=1,\cdots,r$, $\iota_{i^{(k)}}$ are positives for $i^{(k)}=1\cdots r^{(k)}$

The contour L_k is in the t_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(b_j^{(k)}-\beta_j^{(k)}t_k)$ with $j=1$ to M_k are separated from those of $\Gamma(1-u_j+\sum_{i=1}^{s}\mu_j^{(k)}t_k)$with $j=1$ to N and $\Gamma(1-a_j^{(k)}+\alpha_j^{(k)}t_k)$ with $j=1$ to N_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}B_i^{(k)}\pi$, where

$$B_i^{(k)} = \sum_{j=1}^{N}\mu_j^{(k)} - \iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)} - \iota_i\sum_{j=1}^{Q_i}\upsilon_{ji}^{(k)} + \sum_{j=1}^{N_k}\alpha_j^{(k)} - \iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{M_k}\beta_j^{(k)} - \iota_{i^{(k)}}\sum_{j=M_k+1}^{q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)} > 0, \text{ with } k=1,\cdots,s, i=1,\cdots,r, i^{(k)}=1,\cdots,r^{(k)} \quad (1.13)$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_s) = 0(|z_1|^{\alpha'_1},\cdots,|z_s|^{\alpha'_s}), max(|z_1|,\cdots,|z_s|) \to 0$

$\aleph(z_1,\cdots,z_s) = 0(|z_1|^{\beta'_1},\cdots,|z_s|^{\beta'_s}), min(|z_1|,\cdots,|z_s|) \to \infty$

where, with $k = 1, \cdots, z : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, M_k$ and

$$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, N_k$$

We will use these following notations in this paper

$$U = P_i, Q_i, \iota_i; r'; \; V = M_1, N_1; \cdots; M_s, N_s \tag{1.15}$$

$$\mathrm{W} = P_{i^{(1)}}, Q_{i^{(1)}}, \iota_{i^{(1)}}; r^{(1)}, \cdots, P_{i^{(r)}}, Q_{i^{(r)}}, \iota_{i^{(s)}}; r^{(s)} \tag{1.16}$$

$$A = \{(u_j; \mu_j^{(1)}, \cdots, \mu_j^{(s)})_{1,N}\}, \{\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots, \mu_{ji}^{(s)})_{N+1,P_i}\} \tag{1.17}$$

$$B = \{\iota_i(v_{ji}; v_{ji}^{(1)}, \cdots, v_{ji}^{(s)})_{M+1,Q_i}\} \tag{1.18}$$

$$C = (a_j^{(1)}; \alpha_j^{(1)})_{1,N_1}, \iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_{i^{(1)}}}, \cdots, (a_j^{(s)}; \alpha_j^{(s)})_{1,N_s}, \iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)}; \alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_{i^{(s)}}} \tag{1.19}$$

$$D = (b_j^{(1)}; \beta_j^{(1)})_{1,M_1}, \iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)}; \beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_{i^{(1)}}}, \cdots, (b_j^{(s)}; \beta_j^{(s)})_{1,M_s}, \iota_{i^{(s)}}(\beta_{ji^{(s)}}^{(s)}; \beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_{i^{(s)}}} \tag{1.20}$$

The contracted form is :

$$\aleph(z_1, \cdots, z_s) = \aleph_{U:W}^{0,N:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A} : \mathrm{C} \\ \cdot & \cdot\;\cdot\;\cdot \\ \cdot & \mathrm{B} : \mathrm{D} \\ \mathrm{z}_s & \end{array}\right) \tag{1.21}$$

Srivastava and Garg [5] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_t}[z_1,\cdots,z_t] = \sum_{R_1,\cdots,R_t=0}^{h_1R_1+\cdots h_tR_t \leqslant L} (-L)_{h_1R_1+\cdots+h_tR_t} B(E; R_1, \cdots, R_t)\frac{z_1^{R_1}\cdots z_t^{R_t}}{R_1!\cdots R_t!} \tag{1.22}$$

the coefficients $B(E; R_1, \cdots, R_t)$ are arbitrary constants, real or complex.

2. Generalized incomplete hypergeometric function

The generalized incomplete hypergeometric function introduced by Srivastava et al [3 page 675, Eq.(4.1) is represented in the following manner.

$${}_p\gamma_q\left[\begin{array}{c} (\mathrm{e}_1;\sigma), (e_2), \cdots, (e_p) \\ \cdot\;\cdot\;\cdot \\ (\mathrm{f}_1), \cdots, (f_q) \end{array} ; z\right] = \sum_{n=0}^{\infty} \frac{(e_1;\sigma)_n (e_2)_n \cdots (e_p)_n}{(f_1)_n \cdots (f_q)_n} \frac{z^n}{n!} \tag{2.1}$$

where the incomplete Pochhammer symbols are defined as follows :

$$(a;\sigma)_n = \frac{\gamma(a+n;\sigma)}{\Gamma(a)} \quad (a, n \in \mathbb{C}; x \geqslant 0) \tag{2.2}$$

$$\gamma(s,x) = \int_0^x t^{s-1}e^{-t}\mathrm{d}t \quad (Re(s) > 0, x \geqslant 0) \tag{2.3}$$

3. Required integral

We note $S(a,b,c)$, the Selberg integral , see Askey et al ([1], page 402) by :

$$S(a,b,c)=\int_0^1\cdots\int_0^1\prod_{i=1}^{n}x_i^{a-1}(1-x_i)^{b-1}\prod_{1\leqslant j<k\leqslant n}|x_j-x_k|^{2c}\,\mathrm{d}x_1\cdots\mathrm{d}x_n=$$

$$=\prod_{j=0}^{n-1}\frac{\Gamma(a+jc)\Gamma(b+jc)\Gamma(1+(j+1)c)}{\Gamma(a+b+(n-1+j)c)\Gamma(1+c)} \tag{3.1}$$

with $Re(a)>0, Re(b)>0, Re(c)>Max\left\{-\frac{1}{n},-\frac{Re(a)}{n-1},-\frac{Re(b)}{n-1}\right\}$

We consider the new integral , see Askey et al ([1], page 402) defined by :

Lemme

$$\int_0^1\cdots\int_0^1\prod_{i=1}^{k}x_i\prod_{i=1}^{n}x_i^{a-1}(1-x_i)^{b-1}\prod_{1\leqslant j<k\leqslant n}|x_j-x_k|^{2c}\,\mathrm{d}x_1\cdots\mathrm{d}x_n=$$

$$=\prod_{i=1}^{k}\frac{(a+(n-i)c)}{(a+b+(2n-i-1)c)}\;S(a,b,c) \tag{3.2}$$

with $Re(a)>0, Re(b)>0, Re(c)>Max\left\{-\frac{1}{n},-\frac{Re(a)}{n-1},-\frac{Re(b)}{n-1}\right\}$ and $k\leqslant n$

where $S(a,b,c)$ is defined by (3.1). In this paper, we will denote the modified Selberg integral

4. Main integral

Let $X_{u,v,w}=\prod_{i=1}^{n}x_i^{u}(1-x_i)^{v}\prod_{1\leqslant j<k\leqslant n}|x_j-x_k|^{2w}$ $A_{n'}=\frac{(e_1;\sigma)_n(e_2)_n\cdots(e_p)_n}{(f_1)_n\cdots(f_q)_n}$ and

$$B_t=\frac{(-L)_{h_1R_1+\cdots+h_tR_t}B(E;R_1,\cdots,R_t)}{R_1!\cdots R_t!}$$

we have the following formula

Theorem

$$\int_0^1\cdots\int_0^1\prod_{i=1}^{k}x_i\prod_{i=1}^{n}x_i^{a-1}(1-x_i)^{b-1}\prod_{1\leqslant j<k\leqslant n}|x_j-x_k|^{2c}\;{}_p\gamma_q\left[\begin{matrix}(\mathrm{e}_1;\sigma),(e_2),\cdots,(e_p)\\ \cdots\\ (\mathrm{f}_1),\cdots,(f_q)\end{matrix};\;yX_{\alpha,\beta,\gamma}\right]$$

$$S_L^{h_1,\cdots,h_t}\left(\begin{array}{c} \mathrm{y}_1 X_{\alpha_1,\beta_1,\gamma_1} \\ \cdot\ \cdot\ \cdot \\ \mathrm{y}_t X_{\alpha_t,\beta_t,\gamma_t} \end{array}\right) \aleph_{u:w}^{0,\mathfrak{n}:v}\left(\begin{array}{c} \mathrm{z}_1 X_{\delta_1,\psi_1,\phi_1} \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_r X_{\delta_r,\psi_r,\phi_r} \end{array}\right) \aleph_{U:W}^{0,N:V}\left(\begin{array}{c} \mathrm{Z}_1 X_{\epsilon_1,\eta_1,\zeta_1} \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z}_s X_{\epsilon_s,\eta_s,\zeta_s} \end{array}\right) \mathrm{d}x_1 \cdots \mathrm{d}x_n =$$

$$\sum_{R_1,\cdots,R_t=0}^{h_1R_1+\cdots h_tR_t\leqslant L} \sum_{n'=0}^{\infty} \sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} G(\eta_{G_1,g_1},\cdots \eta_{G_r,g_r}) B_t \frac{A_{n'} z^{n'}}{n'!}$$

$$z_1^{\eta_{G_1,g_1}} \cdots z_r^{\eta_{G_r,g_r}} y_1^{R_1} \cdots y_t^{R_t} \aleph_{U_{3n+2k,2n+2k}:W}^{0,N+3n+2k:V}\left(\begin{array}{c|l} \mathrm{Z}_1 & \mathrm{A}\ , \\ \cdot\ \cdot\ \cdot & \\ \cdot\ \cdot\ \cdot & \\ \mathrm{Z}_s & \mathrm{B}, \end{array}\right.$$

$$[\text{1-a-n'}\alpha - \textstyle\sum_{i=1}^t R_i\alpha_i - \sum_{i=1}^r \eta_{G_i,g_i}\delta_i - j(c+\gamma' R + \sum_{i=1}^t \gamma_i K_i + \sum_{i=1}^r \phi_i \eta_{G_i,g_i}); \epsilon_1 + j\zeta_1, \cdots, \epsilon_s + j\zeta_s]_{1,n}$$

$$\cdot\ \cdot\ \cdot$$

$$(\text{-c-n'}\gamma - \textstyle\sum_{i=1}^t R_i\gamma_i - \sum_{i=1}^r \phi_i\eta_{G_i,g_i}; \zeta_1, \cdots, \zeta_s), \cdots,$$

$$[\text{1-b-n'}\beta - \textstyle\sum_{i=1}^t R_i\beta_i - \sum_{i=1}^r \eta_{G_i,g_i}\psi_i - j(c+\gamma' R + \sum_{i=1}^t \gamma_i K_i + \sum_{i=1}^r \phi_i \eta_{G_i,g_i}); \eta_1 + j\zeta_1, \cdots, \eta_s + j\zeta_s]_{1,n}$$

$$\cdot\ \cdot\ \cdot$$

$$(\text{-c-n'}\gamma - \textstyle\sum_{i=1}^t \gamma_i R_i - \sum_{i=1}^r \phi_i\eta_{G_i,g_i}; \zeta_1, \cdots, \zeta_s), B_1, B_2, B_3,$$

$$\left.\begin{array}{c} [\ \text{-(j+1)(c+n'}\gamma + \sum_{i=1}^t \gamma_i R_i + \sum_{i=1}^r \phi_i\eta_{G_i,g_i}); (j+1)\zeta_1, \cdots, (j+1)\zeta_s)]_{0,n-1}, A_2, A_3 : C \\ \cdot\ \cdot\ \cdot \\ \mathrm{D} \end{array}\right) \tag{4.1}$$

where $B_1 = [1 - a - b - (\alpha+\beta)n' - \sum_{i=1}^t R_i(\alpha_i+\beta_i) - \sum_{i=1}^r (\delta_i+\phi_i)\eta_{G_i,g_i} - (n-1+j)\times$

$$(c + n'\gamma + \sum_{i=1}^t R_i\gamma_i + \sum_{i=1}^r \phi_i\eta_{G_i,g_i}); \epsilon_1 + \eta_1 + j\zeta_1, \cdots, \epsilon_s + \eta_s + j\zeta_s]_{0,n-1} \tag{4.2}$$

$$A_2 = \left[-a - n'\alpha - \sum_{i=1}^t R_i\alpha_i - \sum_{i=1}^r \eta_{G_i,g_i}\delta_i - (n-j)(c + m\gamma + \sum_{i=1}^t \gamma_i R_i + \sum_{i=1}^r \phi_i\eta_{G_i,g_i}); \right.$$

$$\left. \epsilon_1 + (n-j)\zeta_1, \cdots, \epsilon_s + (n-j)\zeta_s \right]_{1,k} \tag{4.3}$$

$$B_2 = \left[1 - a - n'\alpha - \sum_{i=1}^t R_i\alpha_i - \sum_{i=1}^r \eta_{G_i,g_i}\delta_i - (n-j)(c + m\gamma + \sum_{i=1}^t \gamma_i K_i + \sum_{i=1}^r \phi_i\eta_{G_i,g_i}); \right.$$

$$\epsilon_1+(n-j)\zeta_1,\cdots,\epsilon_s+(n-j)\zeta_s]_{1,k} \tag{4.4}$$

$$B_3=[-a-n'\alpha-\sum_{i=1}^{t}R_i\alpha_i-\sum_{i=1}^{r}\eta_{G_i,g_i}\delta_i-b-m\beta-\sum_{i=1}^{t}R_i\beta_i-\sum_{i=1}^{r}\eta_{G_i,g_i}\psi_i$$

$$\epsilon_1+\eta_1+(2n-j-1)\zeta_1,\cdots-(2n-j-1)(c+m\gamma+\sum_{i=1}^{t}\gamma_iR_i+\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});$$

$$\epsilon_s+\eta_s+(2n-j-1)\zeta_s]_{1,k} \tag{4.5}$$

$$A_3=[1-a-n'\alpha-\sum_{i=1}^{t}R_i\alpha_i-\sum_{i=1}^{r}\eta_{G_i,g_i}\delta_i-b-m\beta-\sum_{i=1}^{t}R_i\beta_i-\sum_{i=1}^{r}\eta_{G_i,g_i}\psi_i$$

$$\epsilon_1+\eta_1+(2n-j-1)\zeta_1,\cdots-(2n-j-1)(c+m\gamma+\sum_{i=1}^{t}\gamma_iR_i+\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});$$

$$\epsilon_s+\eta_s+(2n-j-1)\zeta_s]_{1,k} \tag{4.6}$$

A,B,C and D are defined by (1.17), (1.18), (1.19) and (1.20) respectively.

where $U_{3n+2k,2n+2k}=P_i+3n+2k,Q_i+2n+2k,\iota_i;r'$

Provided that

a) $min\{\alpha,\beta,\gamma,\alpha_i,\beta_i,\gamma_i,\delta_j,\psi_j,\phi_j,\epsilon_l,\eta_l,\zeta_l\}>0, i=1,\cdots,t, j=1,\cdots,r, l=1,\cdots,s$,

b) $A=Re[a+n'\alpha+\sum_{i=1}^{r}\delta_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^{s}\epsilon_i\min_{1\leqslant j\leqslant M_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>0$

c) $B=Re[b+n'\beta+\sum_{i=1}^{r}\psi_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^{s}\eta_i\min_{1\leqslant j\leqslant M_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>0$

d) $C=Re[c+n'\gamma+\sum_{i=1}^{r}\phi_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^{s}\zeta_i\min_{1\leqslant j\leqslant M_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>Max\left\{-\frac{1}{n},-\frac{A}{n-1},-\frac{B}{n-1}\right\}$

e) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.5) ; $i=1,\cdots,r$

f) The conditions (f) are satisfied and $k\leqslant n$

g) $|argZ_k|<\frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (1.13) ; $i=1,\cdots,s$

h) The series occuring on the right-hand side of (3.1) are absolutely and uniformly convergent.

Proof

first, expressing the generalized the generalized incomplete hypergeometric function ${}_p\gamma_q(.)$ in serie with the help of equation (2.1), the Aleph-function of r-variables in series with the help of equation (1.6), the general class of polynomial of several variables $S_L^{h_1,\cdots,h_t}[.]$ with the help of equation (1.22) and the Aleph-function of s variables in Mellin-Barnes contour integral with the help of equation (1.9), changing the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process) Now evaluating the resulting modified Selberg integral with the help of equation (3.2). Use the following relations $\Gamma(a)(a)_n = \Gamma(a+n)$ and $a = \frac{\Gamma(a+1)}{\Gamma(a)}$ several times with $Re(a) > 0$. Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

5. Particular cases

1) If $\iota, \iota_{i^{(1)}}, \cdots, \iota_{i^{(s)}} \rightarrow 1$, the Aleph-function of s-variables reduces to I-function of s-variables defined by Sharma et al [2] and we obtain.

Corollary 1

$$\int_0^1 \cdots \int_0^1 \prod_{i=1}^k x_i \prod_{i=1}^n x_i^{a-1}(1-x_i)^{b-1} \prod_{1 \leqslant j<k \leqslant n} |x_j - x_k|^{2c} \; {}_p\gamma_q \left[\begin{array}{c} (\mathrm{e}_1;\sigma),(e_2),\cdots,(e_p) \\ \cdot\cdot\cdot \\ (\mathrm{f}_1),\cdots,(f_q) \end{array} ; \; yX_{\alpha,\beta,\gamma} \right]$$

$$S_L^{h_1,\cdots,h_t}\left(\begin{array}{c} \mathrm{y}_1 X_{\alpha_1,\beta_1,\gamma_1} \\ \cdot\cdot\cdot \\ \mathrm{y}_t X_{\alpha_t,\beta_t,\gamma_t} \end{array} \right) \aleph_{u:w}^{0,\mathrm{n}:v} \left(\begin{array}{c} \mathrm{z}_1 X_{\delta_1,\psi_1,\phi_1} \\ \cdot\cdot\cdot \\ \mathrm{z}_r X_{\delta_r,\psi_r,\phi_r} \end{array} \right) I_{U:W}^{0,N:V} \left(\begin{array}{c} \mathrm{Z}_1 X_{\epsilon_1,\eta_1,\zeta_1} \\ \cdot\cdot\cdot \\ \mathrm{Z}_s X_{\epsilon_s,\eta_s,\zeta_s} \end{array} \right) \mathrm{d}x_1 \cdots \mathrm{d}x_n =$$

$$\sum_{R_1,\cdots,R_t=0}^{h_1R_1+\cdots h_tR_t \leqslant L} \sum_{n'=0}^{\infty} \sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} G(\eta_{G_1,g_1}, \cdots \eta_{G_r,g_r}) B_t \frac{A_{n'} z^{n'}}{n'!}$$

$$z_1^{\eta_{G_1,g_1}} \cdots z_r^{\eta_{G_r,g_r}} y_1^{R_1} \cdots y_t^{R_t} I_{U_{3n+2k,2n+2k}:W}^{0,N+3n+2k:V} \left(\begin{array}{c} \mathrm{Z}_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{Z}_s \end{array} \middle| \begin{array}{l} \mathrm{A}\;, \\ \\ \\ \mathrm{B}, \end{array} \right.$$

$$[\text{1-a-n'}\alpha - \sum_{i=1}^t R_i\alpha_i - \sum_{i=1}^r \eta_{G_i,g_i}\delta_i - j(c+\gamma' R + \sum_{i=1}^t \gamma_i K_i + \sum_{i=1}^r \phi_i \eta_{G_i,g_i}); \epsilon_1 + j\zeta_1, \cdots, \epsilon_s + j\zeta_s]_{1,n}$$
$$\cdot\cdot\cdot$$
$$(\text{-c-n'}\gamma - \sum_{i=1}^t R_i\gamma_i - \sum_{i=1}^r \phi_i \eta_{G_i,g_i}; \zeta_1, \cdots, \zeta_s), \cdots,$$

$$[\text{1-b-n'}\beta - \sum_{i=1}^t R_i\beta_i - \sum_{i=1}^r \eta_{G_i,g_i}\psi_i - j(c+\gamma' R + \sum_{i=1}^t \gamma_i K_i + \sum_{i=1}^r \phi_i \eta_{G_i,g_i}); \eta_1 + j\zeta_1, \cdots, \eta_s + j\zeta_s]_{1,n}$$
$$\cdot\cdot\cdot$$
$$(\text{-c-n'}\gamma - \sum_{i=1}^t \gamma_i R_i - \sum_{i=1}^r \phi_i \eta_{G_i,g_i}; \zeta_1, \cdots, \zeta_s), B_1, B_2, B_3,$$

$$\left[\text{-(j+1)(c+n'}\gamma + \sum_{i=1}^{t}\gamma_i R_i + \sum_{i=1}^{r}\phi_i\eta_{G_i,g_i}); (j+1)\zeta_1,\cdots,(j+1)\zeta_s)]_{0,n-1}, A_2, A_3 : C \atop \cdots \atop \text{D}\right) \quad (5.1)$$

under the same conditions and notations that (4.1) with $\iota, \iota_{i^{(1)}}, \cdots, \iota_{i^{(s)}} \rightarrow 1$

2) **If** $B(L; R_1, \cdots, R_t) = \dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_t\theta_j^{(t)}} \prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j} \cdots \prod_{j=1}^{B^{(t)}}(b_j^{(t)})_{R_t\phi_j^{(t)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi'_j+\cdots+m_t\psi_j^{(t)}} \prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j} \cdots \prod_{j=1}^{D^{(t)}}(d_j^{(t)})_{R_t\delta_j^{(t)}}}$ **(5.2)**

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_t}[z_1,\cdots,z_t]$ reduces to generalized Lauricella function defined by Srivastava et al [4].

$$F_{\bar{C}:D';\cdots;D^{(t)}}^{1+\bar{A}:B';\cdots;B^{(t)}}\left(\begin{array}{c} \text{z}_1 \\ \cdots \\ \cdots \\ \text{z}_t \end{array}\middle| \begin{array}{c} [\text{(-L);R}_1,\cdots,R_t][(a);\theta',\cdots,\theta^{(t)}] : [(b');\phi'];\cdots;[(b^{(t)});\phi^{(t)}] \\ [\text{(c)};\psi',\cdots,\psi^{(t)}] : [(d');\delta'];\cdots;[(d^{(t)});\delta^{(t)}] \end{array}\right) \quad (5.3)$$

and we have the following formula

Corollary 2

$$\int_0^1 \cdots \int_0^1 \prod_{i=1}^{k} x_i \prod_{i=1}^{n} x_i^{a-1}(1-x_i)^{b-1} \prod_{1\leqslant j<k\leqslant n} |x_j - x_k|^{2c} \; {}_p\gamma_q\left[\begin{array}{c} (\text{e}_1;\sigma),(e_2),\cdots,(e_p) \\ \cdots \\ (\text{f}_1),\cdots,(f_q) \end{array} ; \; yX_{\alpha,\beta,\gamma}\right]$$

$$F_{\bar{C}:D';\cdots;D^{(t)}}^{1+\bar{A}:B';\cdots;B^{(t)}}\left(\begin{array}{c} \text{y}_1 X_{\alpha_1,\beta_1,\gamma_1} \\ \cdots \\ \text{y}_t X_{\alpha_t,\beta_t,\gamma_t} \end{array}\middle| \begin{array}{c} [\text{(-L);R}_1,\cdots,R_t][(a);\theta',\cdots,\theta^{(t)}] : [(b');\phi'];\cdots;[(b^{(t)});\phi^{(t)}] \\ [\text{(c)};\psi',\cdots,\psi^{(t)}] : [(d');\delta'];\cdots;[(d^{(t)});\delta^{(t)}] \end{array}\right)$$

$$\aleph_{u:w}^{0,\mathfrak{n}:v}\left(\begin{array}{c} \text{z}_1 X_{\delta_1,\psi_1,\phi_1} \\ \cdots \\ \text{z}_r X_{\delta_r,\psi_r,\phi_r} \end{array}\right) \aleph_{U:W}^{0,N:V}\left(\begin{array}{c} \text{Z}_1 X_{\epsilon_1,\eta_1,\zeta_1} \\ \cdots \\ \text{Z}_s X_{\epsilon_s,\eta_s,\zeta_s} \end{array}\right) \mathrm{d}x_1 \cdots \mathrm{d}x_n = \sum_{R_1,\cdots,R_t=0}^{h_1R_1+\cdots h_tR_t\leqslant L}$$

$$\sum_{n'=0}^{\infty} \sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!} G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r}) B'_t \frac{A_{n'}z^{n'}}{n'!}$$

$$z_1^{\eta_{G_1,g_1}} \cdots z_r^{\eta_{G_r,g_r}} y_1^{R_1} \cdots y_t^{R_t} \aleph^{0,N+3n+2k:V}_{U_{3n+2k,2n+2k}:W} \left(\begin{array}{c|l} \mathrm{Z}_1 & \mathrm{A} \ , \\ \cdot\cdot\cdot & \\ \cdot\cdot\cdot & \\ \mathrm{Z}_s & \mathrm{B}, \end{array} \right.$$

$$[\text{1-a-n'}\alpha - \sum_{i=1}^t R_i\alpha_i - \sum_{i=1}^r \eta_{G_i,g_i}\delta_i - j(c+\gamma' R + \sum_{i=1}^t \gamma_i K_i + \sum_{i=1}^r \phi_i\eta_{G_i,g_i}); \epsilon_1 + j\zeta_1, \cdots, \epsilon_s + j\zeta_s]_{1,n}$$

$$\cdot\cdot\cdot$$

$$(\text{-c-n'}\gamma - \sum_{i=1}^t R_i\gamma_i - \sum_{i=1}^r \phi_i\eta_{G_i,g_i}; \zeta_1, \cdots, \zeta_s), \cdots,$$

$$[\text{1-b-n'}\beta - \sum_{i=1}^t R_i\beta_i - \sum_{i=1}^r \eta_{G_i,g_i}\psi_i - j(c+\gamma' R + \sum_{i=1}^t \gamma_i K_i + \sum_{i=1}^r \phi_i\eta_{G_i,g_i}); \eta_1 + j\zeta_1, \cdots, \eta_s + j\zeta_s]_{1,n}$$

$$\cdot\cdot\cdot$$

$$(\text{-c-n'}\gamma - \sum_{i=1}^t \gamma_i R_i - \sum_{i=1}^r \phi_i\eta_{G_i,g_i}; \zeta_1, \cdots, \zeta_s), B_1, B_2, B_3,$$

$$\left. \begin{array}{c} [\ \text{-(j+1)(c+n'}\gamma + \sum_{i=1}^t \gamma_i R_i + \sum_{i=1}^r \phi_i\eta_{G_i,g_i}); (j+1)\zeta_1, \cdots, (j+1)\zeta_s)]_{0,n-1}, A_2, A_3 : C \\ \cdot\cdot\cdot \\ \mathrm{D} \end{array} \right) \quad (5.4)$$

under the same conditions and notations that (4.1)

and $B_t' = \dfrac{(-L)_{h_1R_1+\cdots+h_tR_t} B(E; R_1, \cdots, R_t)}{R_1! \cdots R_t!}$; $B(L; R_1, \cdots, R_t)$ **is defined by (5.2)**

6. Conclusion

In this paper we have evaluated a modified Selberg integral involving the product of two multivariable Aleph-functions, a class of polynomials of several variables and the generalized incomplete hypergeometric functions.The integral established in this paper is of very general nature as it contains multivariable Aleph-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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