Regular weakly compactness and Regular weakly connectedness in Topological spaces

M. Karpagadevi, A. Pushpalatha

Department of Mathematics, SVS College of Engineering, Coimbatore - 642 109, Tamil Nadu, India.
Department of Mathematics, Government Arts College, Udumalpet-642 107, Tamil Nadu, India.

Abstract: In this paper, we introduce and study the notions of Regular Weakly Compactness and Regular Weakly Connectedness in Topological spaces.

1. Introduction and Preliminaries

we introduce the notions of regular weakly separated sets. Also we introduce and study the concepts of regular weakly compactness and regular weakly connectedness in topological spaces.

2. Regular Weakly Separated sets in Topological Spaces

Definition 2.1 : Let (X, τ) be a topological space. Two non-empty subsets A and B are said to be weakly separated (briefly rw-separated) iff $A \cap$ rw-cl(B) = ϕ and rw-cl(A) $\cap B = \phi$.
i.e., $[A \cap$ rw-cl(B)$] \cup [$rw-cl(A) $\cap B] = \phi$.

Theorem 2.2 : Two rw-separated sets are always disjoint.

Proof : Let A and B be rw-separated sets. Then, we have $A \cap$ rw-cl(B) = ϕ and rw-cl(A) $\cap B = \phi$. Now, $A \cap B \subseteq$ rw-cl(A) $\cap B = \phi$. This implies $A \cap B = \phi$. Hence A and B are disjoint.

Theorem 2.3 : Let A and B be two rw-separated sets of (X, τ). If $C \subseteq A$ and $D \subseteq B$ then C and D are rw-separated.

Proof : Let A and B be two rw-separated sets of a topological space (X, τ). Then $A \cap$ rw-cl(B) = ϕ and rw-cl(A) $\cap B = \phi$. Let $C \subseteq A$ and $D \subseteq B$. Then, we have
$C \cap$ rw-cl(D) = ϕ and rw-cl(C) $\cap D = \phi$. Thus C and D are rw-separated.

Theorem 2.4 : Two rw-closed subsets of a topological space (X, τ) are rw-separated iff they are disjoint.

Proof : Since rw-separated sets are disjoint, rw-closed separated sets are disjoint.

Conversely, let A and B be two disjoint rw-closed sets. We have rw-cl(A) = A, rw-cl(B) = B and $A \cap B = \phi$. Consequently, $A \cap$ rw-cl(B) = ϕ and rw-cl(A) $\cap B = \phi$. Hence A and B are rw-separated.

Theorem 2.5 : If the union of two rw-separated sets in a rw-closed set then the individual sets are rw-closures are rw-closures of themselves.

Proof : Let A and B be two rw-separated sets such that $A \cup B$ is rw-closed. Then we have $A \cap$ rw-cl(B) = ϕ and rw-cl(A) $\cap B = \phi$. Also, $A \cup B$ = rw-cl($A \cup B$) = rw-cl(A) $\cup$ rw-cl(B). Therefore rw-cl(A) = rw-cl(A) $\cap$ [rw-cl(A) $\cup$ rw-cl(B)] = rw-cl(A) $\cap [A \cup B] = A \cup \phi = A$. Therefore, rw-cl(A) = A. Similarly, rw-cl(B) = B. Hence the proof.

Theorem 2.6 : Two disjoint sets A and B are rw-separated in (X, τ) iff they are both rw-open and rw-closed in $A \cup B$.

Proof : Let A and B be disjoint and rw-separated in (X, τ). Then, $A \cap$ rw-cl(B) = ϕ and rw-cl(A) $\cap B = \phi$. Let $E = A \cup B$. Then, rw-cl$_E$(A) = rw-cl(A) $\cap E$ = rw-cl(A) $\cap (A \cup B) = A$. Therefore A is rw-closed in E. Similarly, B is rw-closed in E. Now, $A \cap B = \phi$ and $A \cup B = E$. So A and B are complements of each other in E. Thus each one of A and B is rw-open in E.

Conversely, let A and B be disjoint sets which are both rw-open and rw-closed in $E = A \cup B$. Then A = rw-cl$_E$(A) = rw-cl(A) $\cap E$ = rw-cl(A) $\cap (A \cup B) = A \cup [$rw-cl(A) $\cap B]$. But $A \cap B = \phi$. This implies $A \cap [$rw-cl(A) $\cap B] = \phi$. Now, A and rw-cl(A) $\cap B$ are disjoint and their union is A. So, rw-cl(A) $\cap B = \phi$. Similarly, $A \cap$ rw-cl(B) = ϕ. Hence A and B are rw-separated.

3. Regular Weakly Compactness in Topological Spaces

Definition 3.1 : A Collection $\{A_i : i \in I\}$ of rw-open sets in a topological space X is called a rw-open cover of a subset B in X if $B \subseteq {}_{i\in I} U \{ A_i : i \in I \}$.

Definition 3.2 : A topological space X is regular weakly compact(briefly rw-compact) if every rw-open cover of X has a finite sub cover of X.

Definition 3.3 : A subset B of a topological space X is called rw-compact relative to X, if for every collection $\{A_i : i \in I\}$ of rw-open subsets of x such that $B \subseteq {}_{i\in I} U \{ A_i : i \in I \}$, there exists a finite subset I_o of I such that $B \subseteq {}_{i\in I_o} U \{ A_i : i \in I_o \}$.

Definition 3.4 : A subset B of a topological space X is called rw-compact if B is rw-compact as the subspace of X.

Theorem 3.5 : A rw-closed subset of rw-compact space is rw-compact relative to X.

Proof : Let A be a rw-closed subset of a rw-compact space X. Then A^c is rw-open in X. Let S be a rw-open cover of A in X. Then, S along with A^c forms a rw-open cover of X. Since X is rw-compact, it has a finite sub cover say $\{G_1, G_2,\ldots\ldots G_n\}$. If this subcover contains A^c, we discard it. Otherwise leave the subcover as it is. Thus we have obtained a finite subcover of A and so A is rw-compact relative to X.

Theorem 3.6 : A closed subset of rw-compact space is rw-compact relative to X.

Proof : Let A be a closed subset of a rw-compact space X. Then A is rw-closed. Therefore by Theorem 3.5, every rw-closed subset of a rw-compact space is rw-compact relative to X. Hence A is rw-compact space related to X.

4.Regular Weakly Connectedness in Topological Spaces

Definition 4.1 : A topological space (X, τ) is said to be regular weakly connected (briefly rw-connected) if X cannot be written as a union of two disjoint non-empty rw-open sets.

Theorem 4.2 : Every rw-connected space is connected .

Proof : Let (X, τ) be an rw-connected space. Suppose that X is not connected. Then $X = A \cup B$, where A and B are disjoint non-empty open sets in (X, τ). Since every open set is rw-open, A and B are rw-open. Therefore, $X = A \cup B$, where A and B are disjoint non-empty rw-open sets in (X, τ). This contradicts the fact that X is rw-connected and so X is connected.

Theorem 4.3 : A topological space (X, τ) is rw-disconnected if there exists a non-empty proper subset of X which is both rw-open and rw-closed.

Proof : Let A be a non-empty proper subset of X which is both rw-open and rw-closed. Then clearly A^c is a non-empty proper subset of X which is both rw-open and rw-closed. Thus $A \cap A^c = \phi$ and therefore $A \cap \text{rw-cl}(A^c) = \phi$ and $\text{rw-cl}(A^c) \cap A = \phi$. Also, $X = A \cup A^c$. Thus X is the union of two non-empty rw-separated sets. Hence X is rw-disconnected.

Theorem 4.4 : If the sets A and B form an rw-separation of (X, τ) and if (Y, σ) is an rw-connected subspace of (X, τ) the Y lies entirely within either A or B.

Proof : Since A and B are both rw-open in X, the sets $A \cap Y$ and $B \cap Y$ are rw-open in Y. These two sets are disjoint and their union is Y. If they were both non-empty, they would constitute an rw-separation of Y. Therefore one of them is empty. Hence Y must lie entirely in A or in B.

Theorem 4.5 : If every two points of a set A are contained in some rw-connected subset of A, then A is rw-connected.

Proof : Suppose that A is not rw-connected. Then A is the union of two non-empty disjoint rw-open sets G and H. Let $g \in G$ and $h \in H$. Then g and h are two distinct points of A. By hypothesis, there exists a rw-connected subset B of A such that $g,h \in B$. But B is an rw-connected subset of a rw-disconnected set A, we have $B \subseteq G$ or $B \subseteq H$. Since G and H are disjoint and B contains atleast one point of G and one of that of H. This is a contradiction. Hence A is rw-connected.

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