Casson fluid flow toward a vertical plate embedded in porous medium in presence of heat source/sink

Manish Raj¹, Abhay Jha² and Anil Sharma³

^{1,3}Department of Mathematics, University of RajasthanJaipur-302004, India ²Department of Mathematics, JECRC University, Jaipur¹ Abhay Jha Corresponding Author

Abstract

Casson fluid flow through a porous medium towards a vertical plate in presence of heat generation or absorption is considered in this analysis. The governing boundary layer equations are formulated and transformed into set of ordinary differential equation using similarity transformation. Ordinary differential equations solved numerically by shooting technique with fourth order Runge-Kutta method. Solutions for the velocity and temperature were obtained for some special cases. The effect of Casson parameter, thermal Grashof number, Prandtl number on the velocity boundary layer and on the thermal boundary layer are studied and plotted.

Key words: Casson fluid, Vertical plate, Porous medium, Stretching sheet, Heat Generation/absorption, Fluid viscosity.

1. Introduction

Casson fluid flows past a surface embedded in a saturated porous medium have received considerable attention because of numerous applications in engineering and geophysics. Crane [1] study the flow caused by stretching of a sheet. Many researchers such a [2-4] extended the mark of Crane [1] by including the effect of heat and mass transfer analysis under different physical situations. Several author have considered various aspects of this problem and obtained similarity solutions [5-13].All the above mentioned studies continued their discussions by assuming the uniform fluid viscosity. However, it is known that the physical properties of fluid may change significantly with temperature [14-18] the variations of properties with temperature has several practical applications in the field of metallurgy and chemical engineering in the extrusion process, the heat treated materials traveling between a feed roll and wind-up roll or on conveyor belt possess the feature of moving continuous surface. The increase of temperature leads to a local increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and so rate of heat transfer at the wall is also affected. Therefore, to predict the flow

behaviour accurately it is necessary to take into account the viscosity variation for incompressible fluids. Gary et al. [19] and Mehta and Sood [20] showed that, when this effect is included the flow characteristics may change substantially compared to constant viscosity assumption. For lubricating fluids heat generated by internal friction and the corresponding rise in the temperature affects the viscosity of the fluid and so that the fluid viscosity no longer be assumed constant. Mukhapadhyay et al. [21] investigated the MHD boundary layer flow with variable fluid viscosity over a heated stretching sheet. The effects of temperature dependent viscosity and thermal conductivity on flow and heat transfer over a stretching surface in different flow situations and for different fluids were considered by El-Aziz [22], Dandupat et al. [23], Salem [24], Mukhapadhyay and Layek [25], Prasad et al. [26] etc. The increasing of temperature leads to the increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and due to which the heat transfer rate at the wall is also affected. A new dimension is added to the above mentioned study of Mukhapadhyay et al. [21] by considering the effects of porous media.

Flows through porous media are of principal interest because these are quite prevalent in nature such type of flow finds its applications in a broad spectrum of disciplinary covering chemical engineering to geophysics. Flow through fluidsaturated porous medium is important in many technological application and it has increasing importance with the growth of geo-thermal energy usage and in astrophysical problems. In certain porous media applications, working fluid heat generation or absorption effects are important. Representative studied dealing with these effects has been reported by Authors such as Gupta and Sridhar [27]. Abel and Veena [28]. Abel et al. [18] studied the flow and heat transfer of a viscoelastic fluid immersed in a porous medium over a nonisothermal stretching sheet and the fluid viscosity may assumed to vary as a function of temperature. The present work deals with Casson fluid flow towards a vertical plate embedded in porous

medium in presence of heat source/sink. The governing boundary layer equation have been transfomeed to a two-point boundary value problem using a local similarity approach and these have been solved numerically. The effects of various embedded parameters on fluid velocity, temperature and concentration have been shown graphically. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.



Fig 1.Physical model of a boundary layer flow over a vertical stretching surface.

We consider steady two-dimensional forced convection flow of a viscous incompressible fluid past heated stretching sheet immersed in a porous medium along a vertical plate in pressure of Cassen fluid. The x-axis is taken along the plate and y-axis is normal to the plate and flow is confined in half plane g > 0 as shown in Fig. 1. The present paper is concerned primarily with mathematical formulation based on Darcy's law, where the effects of a solid boundary and the inertial effects are neglected. These effects become more significant near boundary and in a media with high porosity [30].

In this paper, we shall limit our consideration to flows where the non-linear farchheimer form is neglected but the linear during term describing the distributed body force extracted by porous medium is retained. Here, we have assumed that Reynolds number is very small (typically < 10) [31].

We assume that the rheological equation of state for an isotropic and incompressible flow of a cusson fluid can be written as

$$\tau_{ij} = \begin{cases} (\mu_{\rm B} + \tau_{\rm y} / \sqrt{2\pi}) 2e_{ij} & \pi > \pi c \\ \bar{(\mu_{\rm B} + \tau_{\rm y} / \sqrt{2\pi c})} 2e_{ij} & \pi < \pi c \end{cases}$$

where $\mu_{\rm B}$ is plastic dynamics viscosity of the casson fluid, $\tau_{\rm y}$ is the yield stress of fluid, π is the product of component of deformation rate with itself, namely, $\pi = e_{ij}e_{ij}$, e_{ij} is the (i,j) the component of the deformation rate of $\pi_{\rm c}$ is the critical value π .

The governing equations of continuity and energy equation are given by (with the application of Darcy's law)

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{v}}{\partial \mathbf{y}} = 0 \qquad \dots (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y} + \frac{\mu}{\rho}\left(1 + \frac{1}{\beta_1}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{\rho K}u + g\beta(T - T_{\infty}) \qquad \dots (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p}(T - T_\infty) \qquad \dots (3)$$

The boundary conditions are

$$u = cx; v = 0, T = T_w \text{ at } y = 0$$
 ...(4)

$$u \to 0; T \to T_{\infty} \text{ as } y \to \infty$$
 ...(5)

where u and v are components of velocity respectively in x and y directions, T is the temperature, K is the coefficient of thermal diffusivity, Q_0 is the dimensional heat generation/absorption (>0/<0), C_p is coefficient of the specific heat, ρ is the fluid density, μ is the coefficient of fluid viscosity, k is the permeability of the porous medium, β is the thermal expansion coefficient, β_1 is Casson parameter, c (>0) is constant, T_w is the uniform wall temperature, T_{∞} is the temperature far away from the sheet and y is gravitational acceleration.

Proceeding with the analysis, we introduce the following dimensionless variable f and θ as well as the similarity variable η

$$\eta = y \sqrt{\frac{c}{v}}; \ \psi(x, y) = \sqrt{vc} x f(\eta)$$

$$\theta(\eta) = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}$$
...(6)

where $\boldsymbol{\psi}$ is the stream function.

The temperature dependent fluid viscosity is given by [32]

$$\mu = \mu^* [a + b(T_w - T)]$$

where μ^* is the constant value of coefficient of viscosity far away from the sheet and a,b are constant and b(> 0), stream function defined by

$$\mathbf{u} = \frac{\partial \Psi}{\partial \mathbf{y}}; \mathbf{v} = -\frac{\partial \Psi}{\partial \mathbf{x}} \qquad \dots (7)$$

Equation of continuity is satisfied and equation (2) and (3) reduce to

$$[a + A(1-\theta)] \left[1 + \frac{1}{\beta_1} \right] f^{'''} + f f^{''} - A f^{''} \theta' - f^{'2} - k [a + A(1-\theta)] f^{'} + G_T \theta = 0$$
...(8)

and

$$\frac{1}{\Pr} \Theta'' + f\Theta' + \lambda \Theta = 0 \qquad \dots (9)$$

The boundary conditions are

$$f' = 0; f = 0; \theta = 1$$
 at $\eta = 0$...(10)

and

$$f \to 0; \theta \to 0 \text{ as } \eta \to \infty$$
 ...(11)

Here

A = b (T_w - T_∞), v^{*} =
$$\frac{\mu^*}{\rho}$$
 Swati Mukhopadhyay et al. [37]

A is viscosity parameter.

$$k = \frac{v}{ck}$$
 is the permeability parameter

$$Pr = \frac{v^*}{k}$$
 is the Prandtl number and

$$\lambda = \frac{Q_0}{\rho C_p c}$$
 is the heat-source or sink parameter

$$G_{T} = \frac{g\beta(T_{w} - T_{\infty})x^{3}}{v^{2}}$$
 Thermal Grashof number

It is noted that when $G_T = 0$ and $\beta_1 \rightarrow \infty$ this paper reduces to the Swati Mukhapadhyay [37]. In order to assess the accuracy of the numerical method, comparison with these obtained by Swati Mukhapadhyay [37] are shown in Table I. The results show good agreement.

3. Results and Discussion

The system of non-linear ordinary differential equation (8) and (9) with boundary condition (10) and (11) is solved numerically using the shooting technique with fourth-order Runge-Kutta scheme. We guessing of

f''(0) and $\theta'(0)$ by showing technique until the boundary conditions at infinity are satisfied. The step size

 $\Delta \eta = 0.001$ is used while obtaining the numerical solution and accuracy up to seventh decimal place which very sufficient for convergence.

In this method, we choose suitable finite value which dependent on the values of the parameter used. The computations were done by a programme which uses a symbolic and computational computer language matlab.



(b)

(a)

Fig2. Velocity profiles for several values of viscosity variable parameter A (a) in case of non-porous medium and in absence of heat source/sink (a=1,k=0,Pr=0.5, λ =0,Gt=0, β 1=

 ∞)(b)in case of porous medium and in presence of heat source /sink (a=1,k=0.1,Pr=0.5,\lambda=0.1,Gt=0,\beta1=\infty)



Fig 3 Velocity profiles for several values of viscosity variable parameter A in case of non-porous medium and in absence of heat source/sink ($a=1,k=0,Pr=0.5,\lambda=0,Gt=0,\beta=\infty$)



(a)



(b)

Fig 4 Velocity profiles for several values of permeability parameter k1 (a) in case of uniform viscosity and in absence of heat source/sink (a=1,A=0,Pr=0.5, λ =0,Gt=0, β 1= ∞) (b) In case of variable viscosity and in absence of heat source /sink (a=1,A=0,Pr=0.5, λ =0.1,Gt=0, β 1= ∞)



(a)





Fig 5 Temperature distribution for several values of permeability parameter k (a) in case of uniform viscosity and in absence of heat source/sink(a=1,A=0,Pr=0.5, λ =0,Gt=0, β 1= ∞)(b) In case of variable viscosity and in absence of heat source /sink (a=1,A=1,Pr=0.5, λ =0.1,Gt=0, β 1= ∞)



Fig 6 Temperature distribution for several values of heat source/sink parametery in case of porous medium and variable viscosity, $(a=1,A=1,k=0.1,Pr=0.5,Gt=0,\beta 1=\infty)$



Fig 7 Velocity profile for several values of prandtl number Pr in presence of porous medium, variable viscosity, heat source/sink and(a=1,k=0.1,A=1, λ =0.1,Gt=0, β 1= ∞)



Fig 8 Temperature distribution for several values of prandtl number Pr in presence of porous medium, variable viscosity, heat source/sink (a=1,k=0.1,A=1, λ =0.1,Gt=0, β 1= ∞)



(b)

Fig 9 (a) Velocity profile (b) Temperature distribution for several values of Grashof number (Gt) in presence of porous medium, variable viscosity, heat source/sink with in both case ($a=1,k=0.1,Pr=0.5,\lambda=0.1,A=1,\beta=\infty$))



(b)

Fig 10 (a)Velocity profile (b) Temperature distribution for several values of casson parameter β 1 in presence of porous medium, variable viscosity, heat source/sink with in both cases (a=1,k=0.1,Pr=0.5,λ=0.1,A=1,Gt=1)

The effects of temperature-dependent fluid viscosity on velocity distribution and heat transfer in case of non-porous media in absence of any heat source/sink. In fig. 2(a) and 2(b) velocity profiles are shown for different values of A. Fig. 2(a) shown that fluid velocity is found to decrease up to the crossing over point with the increase in A but after the crossing over point it increases with increasing A. In Fig. 2(b) velocity field is found decay with increasing value of η for all values of A considered. The combined effects of heat source and permeability parameter, in absence of thermal

Grashof number and Cassan parameter at infinity are considered in this case. Fig.3 shown temperature distribution for several values of viscosity variation parameter A in case of Grashof number at zero and Cassan parameter at infinity with non-porous medium and absence of heat source/sink.In this case temperature decrease with increase of A. Now we concentrate on velocity and temperature distribution for the variation of permeability parameter k_1 of the porous medium without heat generation or absorption with Pr = 0.5, $G_T = 0$, $\beta_1 = \infty$. Fig. 4(a) and 4(b) demonstrate the effects of permeability parameter k₁ on velocity field in the absence (A = 0) and presence (A = 1) of temperature dependent fluid viscosity parameter A respectively with increasing k₁, fluid velocity is found to decrease [fig. 4(a) and 4(b)] is the porosity of the medium increases, the value of k_1 decreases. For decreasing k₁ fluid gets more space to flow as a consequence its velocity increases. Fig. 5(a) and 5(b) exhibit that $\theta(\eta)$ is boundary layer increases with increasing permeability parameter k₁ in both the cases the thermal boundary layer thickness thinner with the decreasing becomes the permeability parameter k₁. The effect of increasing values of k_1 opposes the flow in the boundary layer region, which results in more heat transfer from the sheet to the fluid. This is because of presence of the porous medium μ to increase the resistance to the fluid motion, this causes the fluid velocity to decrease (Fig. 4(a) and Fig. 4(b)) and due to which there is rise in the temperature in the boundary layer (Fig. 5(a) and 5(b)). In Fig. 6, effects of heat source/sink on the temperature field is shown, taking fixed values for the parameters A = 2, $k_1 =$ 0.1, Pr = 0.5, $\beta_1 = \infty$ and $G_T = 0$ and various values for internal heat generation/absorption parameter λ . In this case, temperature field increasing with the increase of heat source parameter λ . It is observed that the thermal boundary layer generates energy, which ceases the temperature profiles to increase with the increasing values of λ . The internal heat generation/absorption enhances/damps the heat transport. The heat generation source leads to a larger thermal diffusion layer that may increase thermal boundary layer thickness, on the contrary, the thermal boundary layer thickness decreases for heat absorption sink. Fig.7 and Fig.8 depict the velocity and temperature profiles for the effects of Prandtl number Pr on momentum and heat transfer. Fluid velocity decreases with increasing Prandtl number. An increase in Prandtl number reduces the thermal boundary layer thickness. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. It can be noticed that as Pr decrease the thickness of the thermal boundary layer becomes greater than the thickness of the velocity boundary layer according to the well-

known relation
$$\frac{\delta T}{\delta} \simeq \frac{1}{Pr}$$
 where δ_T is the

thickness of the velocity boundary layer and δ is the thickness of the velocity boundary layer, so the thickness of the thermal boundary layer increases as Prandtl number Pr decreases, and hence temperature profile decreases with increase of Prandtl number Pr. Fig. 8 implies that the increase of Prandtl number Pr results in a decrease of temperature distribution at a particular point. This is due to the fact that there would be a decrease of thermal boundary layer thickness with increasing values of Prandtl number Pr. Temperature distribution asymptotically approaches to zero in the free streum region. In heat transfer problems, the Prandtl number Pr controls the relative thickening of momentum and thermal boundary layers when Prandtl number Pr is small, it means that heat diffuses quickly compared to the velocity (momentum), which means that for liquid metals, the thickness of the thermal boundary layer is much bigger than the momentum boundary layer. Fluids with lower Prandtl number will passes higher thermal conductivities (and thicker thermal boundary layer structures) so that heat can diffuse from the sheet faster than for higher Pr fluids (thinner boundary layers). Hence Prandtl number can be used to increase the rate of cooling in conducting flows. In Fig. 9(b) it is observed that an increase in thermal Grashof number cause a decrease in the thermal boundary layer thickness and consequently the fluid temperature decreases due to buoyancy effect. However, opposite true in velocity profiles. The influence of the Cassan parameter on the velocity and temperature profiles is shown in Fig. 10(a) and Fig. 10(b) when a = 1; A = 1; $k_1 = 0.1$; $\lambda = 0.1$; $G_T = 0$ and Pr = 0.5. The magnitude of the velocity is greater in the case of a Casson fluid when compared with a viscous fluid. Hence in general, with an increase in β_1 , the velocity of the fluid decreases for a stretching sheet. However, for a stretching sheet the opposite is true in temperature profiles case.

Table 1: Values of skin-friction [-f''(0)] and wall temperature gradient $[-\theta'(0)]$ for values of k1 with a = 1; A = 0; $\lambda = 0$, Pr = 1, $\beta_1 = \infty$ and $G_T = 0$.

K1	[-f''(0)] [37] Swati	Present study	[-θ'(0)] [37] Swati	Present study
1.0	1.414213	1.414209	0.500001	0.500003
2.0	1.732051	1.732050	0.447553	0.447555

4. References

- 1. Crane L J (1970) Flow past a stretching plate. Z Angew Math Phys 21: 645-647
- Gupta P S, Gupta A S (1977) Heat and mass transfer on a stretching sheet with suction or blowing. Can J ChemEng 55: 744-746
- Chen C K, Char M I (1988) Heat transfer of a continuous stretching surface with suction or blowing. J Math Anal Atrp/135: 568-586
- 4. Data B K, Ray P, Gupta A S (1985) Temperature field in the flow over a stretching sheet with uniform heat flux. IntCommun Heat Mass Transf 12: 89-94
- Ishak A, Nazar R, Pop I (2006) Mixed convection boundary layer in the stagnation point flow toward a stretching vertical sheet. Meccanica 41: 509-518
- Ishak A, Nazar R, Pop I (2007) Mixed convection on the stagnation point flow towards a vertical continuously stretching sheet. ASME J Heat transfer 129: 1087-1090
- Ishak A, Nazar R, Pop I (2008) Mixed convection stagnation point flow of a micropolar fluid towards a stretching sheet. Meccanica 43: 411-418
- Ishak A, Nazar R, Pop I (2009) Boundary layer flow and heat transfer over an unsteady stretching vertical surface. Meccanica44 : 369-375. doi:10.1007/S 11012-008-9176-0
- 9. Mahapatra T K, Dholey S, Gupta A S (2007) Momentum and heat transfer in the magnetohydrodynamic stagnation-point flow of a viscoelastic fluid toward a stretching surface. Meccanica 42: 263-272
- Bautros Y Z, Abd-el-Malek M B, Badran N A, Hassan H S (2006) Lie-group method of solution for steady two dimensional boundary-layer stagnation-point flow towards a heated stretching sheet placed in porous medium. Meccanica41: 681-691
- Pal D, Hiremath P S (2010) Computation modeling of heat transfer over an unsteady stretching surface embedded in porous medium. Meccanica 46: 349-357 doi:10.1007/S 11012-010-9313-0
- Pol D (2009) Heat and mass transfer in stagnation-point flow towards a stretching surface in the presence of buoyancy force and thermal radiation. Meccanica 44: 145-158 doi:10-1007/S 11012-008-9155-1.
- Aziz R C, Hashim I, Alomari A R (2011) Thin film flow and heat transfer on an unsteady stretching sheet with internal heating. Meccanica 46: 349-357 doi:10.1007/S 11012-010-9313-0
- Horwing H, Gerstan K (1986) The effect variable properties on laminar boundary layer flow. Warme-Stoffubertrag 20: 47-57
- Lai F C, Kulacki F A (1990) The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium. Int J Heat Mass transfer 33: 1028-1031
- Pop I, Gorla R S K, Rashidi M (1992) The effect of variable viscosity on flow and heat transfer to a continuous moving flat plate. Int J EngSci 30(1): 1-6
- Chaim T C (1996) Heat transfer with variable thermal conductivity in a stagnation-point flow towards a stretching sheet. IntCommun Heat Mass Transfer 23: 239-248
- Abel M S, Khan S K, Prasad K V (2002) Study of viscoelastic fluid and heat transfer over a stretching sheet with variable viscosity. Int J Non-Linear Mech 37: 81-88
- Garg J, Kassay D K, Tadjerun H, Zebib A (1982) The effects of significant viscosity variation on convective heat transport in water saturated porous medium. J. Fluid Mech 117: 233-249

- Mehta K N, Sood S (1992) Transient free convection flow with temperature dependent viscosity in a fluid saturated porous medium. Int. J. EngSci 30: 1083-1087
- Mukhopadhyay S, Layek G C, Samad S A (2005) Studying MHD boundary layer flow over a heated stretching sheet with variable viscosity. Int. J Heat Mass transfer 48: 4460-4466
- El-Aziz M A (2007) Temperature dependent viscosity and thermal conductivity effects on combined heat and mass transfer in MHD three-dimensional flow over a stretching surface with Ohmic heating. Meccanica 42: 375-386
- Danpapat B S, Santra B, Vajravelu K (2007) The effects of variable fluid properties and thermo capillarity on the flow of a thin film on an unsteady sheet. Int J Heat Mass Transfer 50: 991-996
- Salem A M (2007) Variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet. PhysLettA 369: 315-322
- Mukhopadyay S, Layak G C (2008) Effect of thermal radiation and variable fluid viscosity of free convective and heat transfer past a porous stretching surface. Int J Heat and Mass Trans 2167-2178
- Prasad K V, Pal D, Umesh U, PrasannaRao N S (2009) The effect of variable viscosity on MHD viscoelastic fluid flow and heat transfer over a stretching sheet. Comun Nonlinear SciNumerSimul.doi 10.1016/J.Consns.2009.04.003
- Gupta R K, Sridhar T (1985) Visco-elastic effects in non-Newtonian flow through porous media, RheolActa 24: 148-511
- Abel S, Veen P H (1998) Visco-elastic flulid flow and heat transfer in a porous medium over a stretching sheet. Int J Non-Linear Mech 33: 531-538
- Vajravelu K, Hadjinicolauu D (1993) Heat transfer in viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. Int. Commun Heat mass transfer 20: 417-430
- Vafai K, Tien C L (1981) Boundary and Inertia effects on flow and heat transfer in porous media. Int J Heat Mass transfer 24: 195-204
- 31. Takhur H S, Bhargava R, Rawat S, Beg T A, Beg A (2007) Finite element modeling up laminar flow of a third grade fluid in a Darcy forchheimer porous medium with suction effects. Int J App MechEng 12(1): 215-233
- Batchelor G K (1967) An introduction to fluid dynamics. Cambridge University Press, London, pp.597
- Saikrishnan P, Ray S (2003) Non-uniform slot injection (suction) into steady laminar water boundary layers over (i) a cylinder (ii) a sphere. Int J EngSci 41: 1351-1365
- 34. Bird R B, Stewart W E, Lightfoot E N (1960) Transport Phenomena. Wiley, New York
- Cortell R (2005) Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing. Fluid Dyn Res 37: 231-245
- 36. Subhas Abel M, Nandeppanavar M M, Malkhud M B(2010) Hydromagnetic boundary layer flow and heat transfer in viscoelastic fluid over a continuously moving permeable stretching surface with non-uniform heat source/sink embedded in fluid saturated porous medium. ChemEngCommun 197(5): 633-655.doi:10.1080/00986440903287742
- 37. Swati Mukhopadhyay, G C Layak (2011) Effects of variable fluid viscosity and flow past a heated stretching sheet embedded in a porous medium in presence of heat source/sink. Meccanica (2012) 47: 863-876 DOI 10.1007β 11012-011-9457-6