Upper and Lower Semi Strong Na Continuous Multifunctions

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Abstract *— The aim of this paper is to introduce semi strong na continuous multifunctions and to obtain new results.*

Keywords — *multifunction, na continuous, feebly open, semi open*

I. INTRODUCTION

One of the most important and basic topics in the theory of classical point set topology and several branches of mathematics, which has been investigated by many authors, is the continuity of functions. This concept has been extended to the setting of multifunctions. Multifunction or multivalued mapping has many applications in mathematical programming, probability, statistics, differential inclusions, fixed point theorems and even in economics.

In 1979 S.N. Maheshwari et al. [14] defined feebly open sets and showed that feebly open and α-open sets are equivalent. In 1986 G. Chae et al. [6] defined a function f:X→Y to be na continuous if the inverse image of each feebly open sets is δ-open. In 1989 Mahmoud et al. [16] defined a function f:X→Y to be strongly na continuous if the inverse image of every semiopen set is δ-open. In the present paper we extended these functions to multifunctions and named as semi strong na continuous multifunctions and investigated the characterizations and properties of it.

II. PRELIMINARIES

Let (X,τ) be a topological space and A a subset of X. The closure (resp. the interior) of A is denoted by A^- (resp. A°). A subset A is defined to be regular open [20] (resp. regular closed) if A= $A^{-\circ}$ (resp. $A=A^{\circ -}$). The δ-interior [21] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by A_δ° A subset A is called δ-open [21] if $A=A_\delta^\circ$, i.e, a set is δ-open, if it is the union of regular open sets. The complement of a δ-open set is called δ-closed. Alternatively, a subset A is called δ-closed [21] if $A=A_\delta^-$, where $A_\delta^-=\{x\in X: A\cap U^{-\circ}\neq\emptyset, U\in\tau$ and $x\in U\}$. A subset A is defined to be semi-open [9] (α-open [10], preopen [17]) if $A\subset A^{\circ -}$ ($A\subseteq A^{\circ-\circ}$, $A\subseteq A^{-\circ}$). The complement of a semi-open (α-open, preopen) set is called semi-closed (α-closed, preclosed). The family of all regular open (δ-open, semi-open, α-open) sets of X containing a point x∈X is denoted by RO(X,x) (δO(X,x), SO(X,x), αO(X,x)). The intersection of all semi-closed (resp. δ-closed) sets of X containing A is called the semi-closure (δ-closure) of A and is denoted by A_s^- (resp. A_δ^-).

For a space (X,τ), the collection of all δ-open sets of (X,τ) forms a topology for X which is usually called the semiregularization of τ and is denoted by τ_s. In general $\tau_s\subseteq\tau$ and if $\tau_s=\tau$ then (X,τ) is called a semiregular space.

By a multifunction F:X→Y, we mean a point-to-set correspondence from X into Y and assume that F(x) ≠ ∅ for all x∈X. For a multifunction F:X→Y, following [4, 5] we shall denote the upper and lower inverse of a set B of Y, $F^+(B)=\{x\in X:F(x)\subseteq B\}$ and $F^-(B)=\{x\in X:F(x)\cap B\neq\emptyset\}$, respectively. For each A⊂X, $F(A)=\cup_{x\in A}F(x)$. F is defined to be a surjection if F(X)=Y or equivalently if for each y∈Y there exists a point x∈X such that y∈F(x). Moreover F:(X,τ)→(Y,ϑ) is called upper semi continuous [18] (renamed upper continuous [13]) (resp. lower semi continuous [18] (renamed lower continuous [13])) if $F^+(V)$(resp.$F^-(V)$) is open in X for each open set V of Y.

III.CHARACTERIZATIONS

Definition 3.1:
A multifunction F:(X,τ)→(Y,ϑ) is called to be:

1) upper semi-strong na continuous (briefly u.s.st.na.c.) if for
each x∈X and each semi-open set V of Y such that $x\in F^+(V)$, there exists U∈RO(X,x) such that $U\subseteq F^+(V)$,

2) lower semi-strong na continuous (briefly l.s.st.na.c.) if for
each x∈X and each semi-open set V of Y such that $x\in F^-(V)$, there exists U∈RO(X,x) such that $U\subseteq F^-(V)$,

3) semi-strong na continuous if it is both upper semi strong na continuous and lower semi strong na continuos..

Definition 3.2 [2]:
A multifunction F:(X,τ)→(Y,ϑ) is defined to be:

1) upper strongly continuous if $F^+(V)$ is clopen in X for each subset V of Y,

2) lower strongly continuous if $F^-(V)$ is clopen in X for each subset V of Y,

3) strongly continuous if it is both upper strongly continuous and lower strongly continuous.

Definition 3.3:
A multifunction $F:(X,\tau)\to(Y,\vartheta)$ is defined to be:

1) upper pre strong na continuous (briefly u.p.st.na.c.)[23] (upper na continuous [22], upper super continuous [1]) if for each $x\in X$ and each preopen (α-open, open) set V of Y such that $x\in F^+(V)$, there exists $U\in\delta O(X,x)$ such that $U\subseteq F^+(V)$,

2) lower pre strong na continuous (briefly l.p.st.na.c.)[23] (lower na continuous [22], lower super continuous [1]) if for each $x\in X$ and each preopen (α-open, open) set V of Y such that $x\in F^-(V)$, there exists $U\in\delta O(X,x)$ such that $U\subseteq F^-(V)$,

3) pre strong na continuous (briefly p.st.na.c.)[23] (na continuous [22], super continuous [1]) if it is both u.s.st.na.c. (na continuous, super continuous) and l.s.st.na.c. (na continuous, super continuous).

Remark 3.1:
For a multifunction $F:(X,\tau)\to(Y,\vartheta)$ the following implications hold:

u.st.c. → u.s.st.na.c. → u.na.c. → u.sup.c → u.c.
↘ ↗
u.p.st.na.c.

l.st.c. → l.s.st.na.c. → l.na.c. → l.sup.c → l.c.
↘ ↗
l.p.st.na.c.

In general the converses are not true.

Example 3.1:
Let $X=\{a,b,c\}$, $Y=\{1,2,3\}$, $\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\},\{b,c\}\}$, $\vartheta=\{\emptyset,Y,\{1\},\{1,2\}\}$. Define a multifunction $F:(X,\tau)\to(Y,\vartheta)$ as follows:
$F(a)=\{1,2\}$, $F(b)=\{3\}$, $F(c)=\{2,3\}$. Then F is u.s.st.na.c. but not upper strongly continuous.

Example 3.2:
Let $X=\{a,b,c\}$, $Y=\{0,1,2\}$, $\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$, $\vartheta=\{\emptyset,Y,\{1\},\{2,3\}\}$. Define a multifunction $F:(X,\tau)\to(Y,\vartheta)$ as follows:
$F(a)=\{1,2\}$, $F(b)=\{2,3\}$, $F(c)=\{1,3\}$. Then F is upper na continuous but not u.s.st.na.c.

Theorem 3.1:
The following conditions are equivalent for a multifunction $F:(X,\tau)\to(Y,\vartheta)$:

1. F is u.s.st.na.c.,
2. For each $x\in X$ and each $V\in SO(Y)$ such that $x\in F^+(V)$, there exists $U\in\delta O(X,x)$ such that $U\subseteq F^+(V)$,
3. $F^+(V)\in\delta O(X)$ for each $V\in SO(Y)$,
4. $F^-(F)\in\delta C(X)$ for each $F\in SC(Y)$,
5. $F(\bar{A}_\delta)\subseteq\overline{[F(A)]}_\delta$ for any subset A of X,
6. $\overline{[F^-(B)]}_\delta\subseteq F^-(\bar{B}_\delta)$ for any subset B of Y.

Proof:
1) $\Longrightarrow$ 2) Let $x\in X$ and $V\in SO(Y)$ such that $x\in F^+(V)$. By (1), there exists $U_x\in RO(X,x)$ such that $U_x\subseteq F^+(V)$. Set $U=\cup\{U_x: U_x\in RO(X,x)\}$ for each $x\in X$. Then U is a δ-open set in X and $U\subseteq F^+(V)$.

2) $\Longrightarrow$ 3) Obvious.

3) $\Longrightarrow$ 4) Obvious.

4) $\Longrightarrow$ 5) For any subset A of X, $\overline{[F(A)]}_s$ is a semi-closed set in Y. $A\subseteq F^-(\overline{[F(A)]}_s)$ and by (4), $F^-(\overline{[F(A)]}_s)$ is δ-closed. Hence $\bar{A}_\delta\subseteq F^-(\overline{[F(A)]}_s)$ and $F(\bar{A}_\delta)\subseteq\overline{[F(A)]}_s$.

5) $\Longrightarrow$ 6) Let $B\subseteq Y$. Then $F^-(V)\subseteq X$. By using (5) we obtain $F(\overline{[F^-(B)]}_\delta)\subseteq\bar{B}_s$. Hence $\overline{[F^-(B)]}_\delta)\subseteq F^-(\bar{B}_s)$.

6) $\Longrightarrow$ 1) Let $V\in SO(Y)$ such that $x\in F^+(V)$. Then $Y-V$ is a semi-closed set. By (6), $\overline{[F^-(Y-V)]}_\delta\subseteq F^-(\overline{(Y-V)}_s$. Hence $F^+(V)$ is a δ-open set in X and there exists $U\in RO(X,x)$ such that $U_x\subseteq F^+(V)$. This shows that F is u.s.st.na.c.

Theorem 3.2:
The following conditions are equivalent for a multifunction $F:(X,\tau)\to(Y,\vartheta)$:

1. F is l.s.st.na.c.,
2. For each $x\in X$ and each $V\in SO(Y)$ such that $x\in F^-(V)$,
there exists $U\in\delta O(X,x)$ such that $U\subseteq F^-(V)$,
3. $F^-(V)\in\delta O(X)$ for each $V\in SO(Y)$,
4. $F^+(F)\in\delta C(X)$ for each $F\in SC(Y)$,
5. $F(\bar{A}_\delta)\subseteq\overline{[F(A)]}_s$ for any subset A of X,
6. $\overline{[F^+(B)]}_\delta\subseteq F^+(\bar{B}_s)$ for any subset B of Y.

Definition 3.4:
Let D be a directed set. A net $(x_\lambda)_{\lambda\in D}$ in X is defined to be δ-converges [6] to a point x in X if the net is eventually in each regular open set containing x.

Theorem 3.3:

F is u.s.st.na.c (l.s.st.na.c.) if and only if for each x∈X and each net (x_λ) δ-converging to x, F(x_λ) is eventually in each semi-open set of Y containing F(x).

Proof:

Neccesity. Let F be u.s.st.na.c, (x_λ) δ-converges to a point x∈X and V∈SO(Y) such that F(x)⊆V. Since F is u.s.st.na.c. there exists U∈RO(X,x) such that U⊆ $F^+(V)$. Since (x_λ) δ-converges to x, (x_λ) is eventually in U. Hence F(x_λ) is eventually in V.

Sufficiency. Suppose that F is not an u.s.st.na.c. multifunction. Then, there exists a point x and a semi-open set V with x∈ $F^+(V)$ such that U⊈$F^+(V)$ for each U∈RO(X,x). Let x_u ∈U and x_u ∉$F^+(V)$. Then for the regular open neighbourhood net (x_u), (x_u) is δ-convergent to x but F(x_u) is not eventually in V. This is a contradiction. Hence F is u.s.st.na.c.

The proof for l.s.st.na.c. multifunctions is similar.

IV. Some Properties of Semi-Strong Na Continuous Multifunctions

Lemma 4.1 [7]:

If A is a dense or open subset of (X,τ) and U∈RO(X) then U∩A is a regular open set in subspace A.

Theorem 4.1:

If F:(X,τ) → (Y,ϑ) is an u.s.st.na.c.(l.s.st.na.c.) multifunction and A is an open subset of (X,τ) then the restriction F_A :(A, τ_A) → (Y,ϑ) is an u.s.st.na.c. (l.s.st.na.c.) multifunction.

Proof: The proof is obvious by Lemma 4.1.

Theorem 4.2:

If F:(X,τ) → (Y,ϑ) and G:(Y,ϑ) → (Z, σ) are u.s.st.na.c (l.s.st.na.c.) multifunctions then, GoF is an u.s.st.na.c. (l.s.st.na.c.) multifunction.

Lemma 4.2:

Let $\{X_\lambda: \lambda \in D\}$ be a family of spaces and U_{λ_i} be a subset of X_{λ_i} for each i=1,2,...,n. Then U=$\prod_{i=1}^{n} U_{\lambda_i} \times \prod_{\lambda\neq\lambda_1} X_\lambda$ is δ- open[6] (resp. semi-open[11]) in $\prod_{\lambda\in D} X_\lambda$ if and only if $U_{\lambda_i} \in$ δO(X_{λ_i}) (resp. $U_{\lambda_i} \in$ SO(X_{λ_i})) for each i=1,2,...,n.

Theorem 4.3:

Let $F_\lambda:(X_\lambda, \tau_\lambda) \to (Y_\lambda, \vartheta_\lambda)$ be a multifunction for each $\lambda \in D$ and F: $\prod X_\lambda \to \prod Y_\lambda$ be a multifunction defined by F($\{x_\lambda\}$)=$\{F_\lambda(x_\lambda)\}$ for each $x_\lambda \in \prod X_\lambda$. If F is an u.s.st.na.c.
(l.s.st.na.c.) then F_λ is u.s.st.na.c. (l.s.st.na.c.) for each $\lambda \in$D.

Proof: Let V_λ ∈SO(Y_λ). Then by Lemma 4.2, $%

V=$V_\lambda \times \prod_{\lambda\neq\beta} Y_\beta$ is semi-open in $\prod Y_\lambda$ and since F is u.s.st.na.c. F^+(V)= $F^+(V_\lambda \times \prod_{\lambda\neq\beta} Y_\beta)$= $F^+(V_\lambda) \times F^+(\prod_{\lambda\neq\beta} Y_\beta)$= $F^+(V_\lambda) \times \prod_{\lambda\neq\beta} X_\beta$ δ -open in $\prod X_\lambda$. From Lemma 4.2, $F^+(V_\lambda) \in$ δO(X). Therefore F_λ is u.s.st.na.c.

The proof for l.s.st.na continuity is similar.

Theorem 4.4:

Suppose that (X,τ) and (X_α, τ_α) are topological spaces for each $\alpha \in$J. Let F: X $\to \prod_{\alpha\in J} X_\alpha$ be a multifunction from X to product space $\prod_{\alpha\in J} X_\alpha$ and let $P_\alpha: \prod_{\alpha\in J} X_\alpha \to X_\alpha$ be the projection for each $\alpha \in$ J. If F is u.s.st.na.c. (l.s.st.na.c.), then P_α oF is u.s.st.na.c. (l.s.st.na.c.) for each $\alpha \in$ J.

Proof: Let V_{α_0}be a semi-open set in $(X_{\alpha_0}, \tau_{\alpha_0})$ for any $\alpha_0 \in$J. Then $(P_{\alpha_0}\text{oF})^+(V_{\alpha_0})$= $F^+(P^+_{\alpha_0}(V_{\alpha_0}))$= $F^+(V_{\alpha_0} \times \prod_{\alpha\neq\alpha_0} X_\alpha)$ ((P_{α_0} oF) $^-$ (V_{α_0})= $F^-(P^-{}_{\alpha_0}(V_{\alpha_0}))$= F^- $(V_{\alpha_0} \times \prod_{\alpha\neq\alpha_0} X_\alpha)$). By Lemma 4.2, $V_{\alpha_0} \times \prod_{\alpha\neq\alpha_0} X_\alpha$ is a semi-open set and since F is u.s.st.na.c. (l.s.st.na.c.), $F^+(V_{\alpha_0} \times \prod_{\alpha\neq\alpha_0} X_\alpha)$
(resp. F^- $(V_{\alpha_0} \times \prod_{\alpha\neq\alpha_0} X_\alpha)$ is δ-open in X. This shows that P_{α_0} oF is u.s.st.na.c. (l.s.st.na.c.). Hence, P_{α_0} oF is u.s.st.na.c. (l.s.st.na.c.) for each $\alpha \in$J.

Recall that for a multifunction F: X →Y, the graph multifunction G_F: X →X×Y of F is defined as follows: G_F (x)={x} × F(x) for every x ∈ X and the subset {{x}×F(x):x∈X}⊆X×Y is called the multigraph of F and is denoted by G(F).

Lemma 4.3 [12]:

For a multifunction F: X →Y, the following holds:

1) $G_F{}^+$(A×B)=A∩F^+(B)

2) $G_F{}^-$ (A×B)=A∩ F^- (B)

for any subset A⊆X and B⊆Y.

Theorem 4.5:

Let F:(X,τ)→(Y,ϑ) be a multifunction. Then F is u.s.st.na.c. if the graph multifunction G_F is u.s.st.na.c.

Proof: Let x∈X and V be any semi-open set in Y such that
F(x) ⊆V. Then X×V is a semi-open set in X×Y. Since {x}×F(x) ⊆X×V, G_F(x)⊆ X×V. By the upper semi-strong na continuity of G_F, there exists U∈ δO(X,x) such that U⊆ $G_F{}^+$(X×V). By using Lemma 4.3, we obtain U⊆ F^+(V). Therefore, F is u.s.st.na.c.

Theorem 4.6:

Let F:(X,τ)→(Y,ϑ) be a multifunction. Then F is l.s.st.na.c. if the graph multifunction G_F is l.s.st.na.c.

Proof: Let x∈X and V be any semi-open set in Y such that

F(x) $\cap$V$\neq \emptyset$. Then X$\times$V is a semi-open set in X$\times$Y. G_F (x) $\cap$ (X $\times$ V)=({x} $\times$ F(x)) $\cap$ (X $\times$ V) =x $\times$ (F(x) $\cap$V)$\neq \emptyset$. Since G_F is l.s.st.na.c. there exists U$\in$ δO(X,x) such that U$\subseteq$ G_F^- (X$\times$V). By Lemma 4.3, we obtain U$\subseteq$ F^- (V). This shows that F is l.s.st.na.c.

Definition 4.1 [1]:

For a multifunction F:(X,τ)→(Y,ϑ) the multigraph G(F)={(x,y):y∈F(x), x∈X} is defined to be δ-closed in X× Y if for each (x,y) ∈X×Y-G(F), there exist U∈ δO(X,x) and an open set V of Y containing y such that (U×V)∩ G(F)=∅.

Definition 4.2:

A space X is defined to be nearly compact [19] (semi-compakt [8]) if every regular open (semi-open) cover of X has a finite subcover.

Theorem 4.7:

If F:(X,τ) → (Y,ϑ) is an u.s.st.na.c. multifunction such that F(x) is semi compact for each x∈X and (Y,ϑ) is a Hausdorff space, then G(F) is δ-closed in X×Y.

Proof: Let (x,y) ∈X×Y-G(F) then y∈Y-F(x). Since Y is a Hausdorff space for each s∈F(x), there exist disjoint open sets U_s and V_s of Y such that s∈ U_s and y∈ V_s. Since every open set is a semi-open set, {U_s : s∈F(x)} is a semi-open cover of F(x) and since F(x) is semi-compact for each x∈X, there exist finite number points $s_1, s_2, \ldots, s_n$ in F(x) such that F(x) ⊆∪{U_{s_i} : i=1,2, ... ,n}. Use U=∪{U_{s_i} : i=1,2, ... ,n} and V=∪{V_{s_i} : i=1,2, ... ,n}. Then U is a semi-open and V is an open set in Y such that F(x)⊆U and y∈V, U∩V=∅. Since F is an u.s.st.na.c. multifunction, F^+(U) ∈ δ O(X,x). Hence we have, (F^+(U) ×V))∩G(F)=∅.

Theorem 4.8:

Let F:(X,τ) →(Y,ϑ) be an u.s.st.na.c. surjective multifunction such that F(x) is semi-compact for each x∈X. If X is a nearly compact space then, Y is semi-compact.

Proof: Let $\{S_\lambda : \lambda \in \Lambda\}$ be a semi-open cover of Y. Since F(x) is semi-compact for each x∈X, there exists a finite subset Λ_x of Λ such that F(x) $\subseteq \cup_{\lambda\in\Lambda_x} S_\lambda$. Use $S_x = \cup_{\lambda\in\Lambda_x} S_\lambda$. Since F is an u.s.st.na.c. multifunction, there exists R_x ∈RO(X,x) such that F(R_x) ⊆S_x. The family {R_x: x∈X} is a regular open cover of X and since X is a nearly compact space, there exist finite number of points $x_1, x_2, \ldots, x_n$ in X such that X=$\cup_{i=1}^{n} R_{x_i}$.

Hence we have, Y=F(X)=F($\cup_{i=1}^{n} R_{x_i}$) =$\cup_{i=1}^{n} F(R_{x_i})$ $\subseteq \cup_{i=1}^{n} S_{x_i} = \cup_{i=1}^{n} \cup_{\lambda\in\Lambda_{x_i}} S_{\lambda_i}$. This shows that Y is semi-compact.

Definition 4.3:

A topological space X is defined to be a semi-Normal space [15] if for any disjoint closed subsets K and F of X there exist two disjoint semi-open sets U and V such that K⊆U, F⊆ V.

Recall that a multifunction F:(X,τ)→(Y,ϑ) is said to be point closed if for each x∈X, F(x) is closed.

Theorem 4.9:

Let F and G be u.s.st.na.c and point closed multifunctions from a topological space (X,τ) to a semi-Normal space (Y,ϑ). Then the set K={x∈X: F(x)∩G(x)≠ ∅} is δ-closed in X.

Proof: Let x∈X-K. Then F(x)∩G(x)=∅. Since F and G are point closed multifunctions, F(x) and G(x) are closed sets and Y is a semi-Normal space, there exist disjoint semi-open sets U and V containing F(x) and G(x) respectively. Since F and G are u.s.st.na.c. multifunctions, F^+(U) and G^+(V) are δ-open sets containing x. Use H= F^+(U)∪ G^+(V). Then H is a δ-open set containing x and H∩K=∅. Hence K is δ-closed in X.

Theorem 4.10:

Let F:(X,τ) → (Y,ϑ) be an u.s.st.na.c. and point closed multifunction from a topological space X to a semi-Normal space Y and F(x) ∩F(y)= ∅ for each distinct pair x,y∈X. Then X is a δ-Hausdorff space.

Proof: Let x and y be any two distinct points in X. Then F(x) ∩F(y)= ∅. Since F is point closed, F(x) and F(y) are closed sets and since Y is a semi-Normal space, there exist disjoint semi-open sets S and T containing F(x) and F(y), respectively. Since F is u.s.st.na.c, F^+(S) and F^+(T) are disjoint δ-open sets containing x and y, respectively. This shows that X is a δ-Hausdorff space.

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