On characterization of fuzzy η-irresolute functions Where η∈ {α, semi, pre}

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Abstract

In this paper we characterize the C-fuzzy α-irresolute function, C-fuzzy irresolute function, C-fuzzy pre-irresolute function.

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Key Words: *Fuzzy topology, C-fuzzy α-irresolute, C-fuzzy irresolute and C-fuzzy pre-irresolute*.

1. Introduction

Sutha. et. al. introduced and studied the concepts of C-fuzzy α-open sets, C-fuzzy semi-open sets, C-fuzzy pre-open sets and C-fuzzy α-closed sets, C-fuzzy semi-closed sets, C-fuzzy pre-closed sets. In this paper we characterize the C-fuzzy α-irresolute function, C-fuzzy irresolute function, C-fuzzy pre-irresolute function The concepts that are needed in this paper are discussed in the second section. The third section is devoted to applications of C-fuzzy α-open, C-fuzzy semi-open, C-fuzzy pre-open to fuzzy irresolute functions Throughout this paper (X, τ) is a fuzzy topological space in the sense of Chang[4].

2. Preliminaries

Definition 2.1

Let (X, τ) be a fuzzy topological space. Then a fuzzy subset λ of X is called

(i) fuzzy α-open in (X, τ) if $\lambda \leq Int\ Cl\ Int\ \lambda$ **[2]**

(ii) fuzzy semi-open in (X, τ) if $\lambda \leq Cl\ Int\ \lambda$ **[1]**

(iii) fuzzy pre-open in (X, τ) if $\lambda \leq Int\ Cl\ \lambda$ **[2]**

A fuzzy subset λ of a fuzzy topological space (X, τ) is said to be fuzzy α-closed (resp. fuzzy semi closed, fuzzy pre closed) if 1−λ is fuzzy α-open (resp. fuzzy semi-open, fuzzy pre-open).

Definition 2.2[7]

Let λ be a fuzzy subset of a fuzzy topological space (X, τ). Let C: [0, 1] →[0, 1] be a complement function. Then λ is

(i) C-fuzzy α-open if C(λ) is fuzzy α-closed,
(ii) C-fuzzy semi-open if C(λ) is fuzzy semi-closed,
(iii) C-fuzzy pre-open if C(λ) is fuzzy pre-closed,

Definition 2.3[8]

f is said to be fuzzy irresolute if $f^{-1}(\lambda)$ is fuzzy semi-closed in (X, τ) for each fuzzy semi-closed set λ in (Y, σ).

Definition 2.4[6]
f is said to be fuzzy α-irresolute if $f^{-1}(\lambda)$ is fuzzy α-closed in (X, τ) for each fuzzy α-closed set λ in (Y, σ).

Definition 2.5[4]
f is said to be fuzzy pre-irresolute if $f^{-1}(\lambda)$ is fuzzy pre-closed in (X, τ) for each fuzzy pre-closed set λ in (Y, σ).

Lemma 2.6
Suppose f is a function from X to Y. Then $f^{-1}(C\mu) = Cf^{-1}(\mu)$ for any fuzzy subset μ of Y.

3. On fuzzy η-irresolute functions

Theorem 3.1
Suppose f : X →Y be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] → [0, 1] satisfies the involutive condition. Then f is fuzzy α-irresolute if and only if $f^{-1}(\lambda)$ is C-fuzzy α-open in X for every C-fuzzy α-open set λ in Y.

Proof
Suppose f is fuzzy α-irresolute. Let λ be C-fuzzy α-open in Y. Then by Definition 2.2(i), Cλ is fuzzy α-closed in Y. Then by using Definition 2.4, $f^{-1}(C\lambda)$ is fuzzy α-closed in X. By using Lemma 2.6, $C(f^{-1}(\lambda))$ is fuzzy α-closed in X. By Definition 2.2(i), $f^{-1}(\lambda))$ is C-fuzzy α-open in X. Conversly, assume that, $f^{-1}(\lambda)$ is C-fuzzy α-open in X. Let λ be fuzzy α-closed in Y. Since C satisfies the involutive condition, C(Cλ) is fuzzy α-closed. Again by Definition 2.2(i), Cλ is C-fuzzy α-open in Y. By our assumption, $f^{-1}(C\lambda)$ is C-fuzzy α-open in X. Then by using Lemma 2.6, $C(f^{-1}(\lambda))$ is C-fuzzy α-open in X. Again by Definition 2.2(i), $C(C(f^{-1}(\lambda)))$ is fuzzy α-closed in X. Since C satisfies the involutive condition, $f^{-1}(\lambda))$ is fuzzy α-closed in X. By Definition 2.4, f is fuzzy α-irresolute.

Theorem 3.2
Suppose f : X →Y be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] → [0, 1] satisfies the involutive condition. Then f is fuzzy irresolute if and only if $f^{-1}(\lambda)$ is C-fuzzy semi-open in X for every C-fuzzy semi-open set λ in Y.

Proof
Suppose f is fuzzy irresolute. Let λ be C-fuzzy semi-open in Y. Then by Definition 2.2(ii), Cλ is fuzzy semi-closed in Y. Then by using Definition 2.3, $f^{-1}(C\lambda)$ is fuzzy semi-closed in X. By using Lemma 2.6, $C(f^{-1}(\lambda))$ is fuzzy semi-closed in X. By Definition 2.2(ii), $f^{-1}(\lambda))$ is C-fuzzy semi-open in X. Conversly, assume that, $f^{-1}(\lambda)$ is C-fuzzy semi-open in X. Let λ be fuzzy semi-closed in Y. Since C satisfies the involutive condition, C(Cλ) is fuzzy semi-closed. Again by Definition 2.2(ii), Cλ is C-fuzzy semi-open in Y. By our assumption, $f^{-1}(C\lambda)$ is C-fuzzy semi-open in X. Then by using Lemma 2.6, $C(f^{-1}(\lambda))$ is C-fuzzy semi-open in X. Again by Definition 2.2(ii), $C(C(f^{-1}(\lambda)))$ is fuzzy semi-closed in X. Since C satisfies the involutive condition, $f^{-1}(\lambda))$ is fuzzy semi-closed in X. By Definition 2.3, f is fuzzy irresolute.

Theorem 3.3
Suppose f : X →Y be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] → [0, 1] satisfies the involutive condition. Then f is fuzzy pre-irresolute if and only if $f^{-1}(\lambda)$ is C-fuzzy pre-open in X for every C-fuzzy pre-open set λ in Y.

Proof
Suppose f is fuzzy pre-irresolute. Let λ be C-fuzzy pre-open in Y. Then by Definition 2.2(iii), Cλ is fuzzy pre-closed in Y. Then by using Definition 2.5, $f^{-1}(C\lambda)$ is fuzzy pre-closed in X. By using Lemma 2.6, $C(f^{-1}(\lambda))$ is fuzzy pre-closed in X. By Definition 2.2(iii), $f^{-1}(\lambda))$ is C-fuzzy pre-open in X. Conversly, assume that, $f^{-1}(\lambda)$ is C-fuzzy pre-open in X. Let λ be fuzzy pre-closed in Y. Since C satisfies the involutive condition, C(Cλ) is fuzzy pre-closed. Again by Definition 2.2(iii), Cλ is C-fuzzy pre-open in Y. By our assumption, $f^{-1}(C\lambda)$ is C-fuzzy pre-open in X. Then by using Lemma 2.6, $C(f^{-1}(\lambda))$ is C-fuzzy pre-open in X. Again by Definition 2.2(iii), $C(C(f^{-1}(\lambda)))$ is fuzzy pre-closed in X. Since C satisfies the involutive condition, $f^{-1}(\lambda))$ is fuzzy pre-closed in X. By Definition 2.5, f is fuzzy pre-irresolute.

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