A Note on soShearEnergy of Jahangir graphs

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1 **Abstract :**

Let $G = (V, E)$ be a finite connected graph. A set $D \subset V$ is a dominating set of G if every vertex in V-D is adjacent to some vertex in D. A dominating set D of G is called a minimal dominating set if no proper subset of D is a dominating set. In this paper soShearEnergy of Jahangir graph $J_{2,m}$ is calculated for all possible minimal dominating set. Energy cure for those graphs are ploted. Hardihood + and - of the Jahangir graph $J_{2,m}$ is also calculated.

keywords,: idegree,odegree, oShearEnergy,soShearEnergy
AMS Subject Classification (05Cxx,05C99):

1 introduction

Let $G = (V, E)$ be a finite connected graph. A set $D \subset V$ is a dominating set of G if every vertex in V-D is adjacent to some vertex in D. A dominating set D of G is called a minimal dominating set if no proper subset of D is a dominating set. The notion of shear Energy in terms of idegree and odegree has been introduced in the paper [2]. In this paper soShearEnergy of Jahangir graphs $J_{2,m}$ with respect to the given minimal dominating sets are calculated. The $Hardihood^{+}$ and $Hardihood^{-}$ are also calculated in this paper.

A dominating set D of G is called a minimum dominating set if D is a dominating set with minimum cardinality of a minimal dominating sets of G. A dominating set D of G is called an independent dominating set if the vertices in D are independent. A dominating set D of G is called a maximum independent dominating set if D is a independent dominating set with maximum cardinality . If its cardinality is minimum, it is minimum independent dominating set. Independent sets with some other properties also evolved and soShearEnergy is calculated for these minimal dominating sets.

2 Preliminaries

Definition 2.1.

Let G be a graph and S be a subset of V(G). Let $v \in V - S$ *, the* ***idegree*** *of v with respect to S is the number of neighbours of v in V-S and it is denoted by* $id_S(v)$ *.*

Definition 2.2.

Let G be a graph and S be a subset of V(G). Let $v \in V - S$ *, the* ***odegree*** *of v with respect to S is the number of neighbours of v in V-S be a minimal dominating set, the number of edges which join the vertices of S and is denoted as* od_S *(v).*

Definition 2.3.

Let G be graph and S be a subset of V(G). Let $v \in V - S$ *, the* ***oidegree*** *of v with respect to S is* $od_S(v) - id_S(v)$ *if od* $>$ *id and it is denoted by* $oid_S(v)$

Definition 2.4.

Let G be a graph and D be a dominating set, ***oShearEnergy*** *of a graph with respect to D denoted by* $os\epsilon_D(G)$ *is the summation of all oid if od* $>$ *id or otherwise zero .*

Definition 2.5.

Let G be a graph and D be a minmal dominating set ,then ***ShearEnergy curve*** *is the curve obtained by joining the oShearEnergies of* D_{i-1} *and* D_i *for* $1 \leq i \leq n$ *, taking the number of vertices of* D_i *along the x axis and the oShearEnergy along the y axis*

Definition 2.6.

Let G be a graph and D be a minimal dominating set, ***soShearEnergy*** *of a graph with respect to D is*

$$\sum_{0}^{|V-D|} os\epsilon_{D_i}(G)$$

where $D_{i+1} = D_i \cup V_{i+1}, V_{i+1}$ *is a singleton vertex with minimum oidegree of* $V - D_i$ *and* D_0 *is a minimal dominating set where* $0 \leq i \leq |V - D|$ *, it is denoted by* $sos\epsilon_D(G)$

Definition 2.7.

Let G be a graph and $MDS(G)$ *is the set of all minimal dominating set of G , then* $Hardihood^+$ *of a graph G is* $max\{so\epsilon_{(MDS(G))}(G)\}$ *is denoted as* $HD^+(G)$ *.*

Definition 2.8.

Let G be a graph and $MDS(G)$ *is the set of all minimal dominating set of G , then* $Hardihood^-$ *of a graph G is is* $min\{so\epsilon_{(MDS(G))}(G)\}$ *is denoted as* $HD^-(G)$ *.*

Given below is the modified steps in the algorithm find in [2] to find the soShearEnergy of any given graph with respect to the given dominating set. Using this algorithm soSherEnergy is found out.

Algorithm

5. Find the vertex with minimum positive oidegree and the number of vertices with minimum positive oidegree.

6.a) If the number of vertices with minimum oidegree is 1, then shift the vertex to the dominating set

Else

Find the vertex with maximum idegree among the vertex with minimumoidegree and shift it to the dominating set.

b) If no such positive oidegree exists shift a vertex with oidegree 0 that also has the maximum idegree among the vertex with oidegree zero to the set D otherwise shift a vertex with minimum negative oidegree to the dominating set.

Remark 2.9. *The number of iterations needed to find the soEnergy of a graph is* $|V-D|+1$.

3 Minimal dominating sets of Jahangir graphs

Definition 3.1. ***Jahangir graph*** $J_{n,m}$ *for* $m \geq 3$ *, is a graph on nm+1 vertices consisting of a cycle* C_{nm} *with one additional vertex which is adjacent to m vertices of* C_{nm} *at distance n to each other on* C_{nm} *.*

Let v_{2m+1} be the center vertex and $v_1, v_2, ..., v_{2m}$ be vertices of the cycle C_{2m} where the vertex v_s with $s \cong 1(mod2)$ is incident with the center vertex v_{2m+1} .

Let $S_0 = \{v_1, v_5, ..., V_{2m-3}, v_{2m+1}\}$ be the connected dominating set, $S_1 = \{v_2, v_4, ...v_{2m}, v_{2m+1}\}$ be the maximum independent dominating set.Independent dominating set without the center point, Minimum independent dominating set can aslo be calculated. These are possible minimal dominating sets of Jahangir graph $J_{2,m}$.

Theorem 3.2. *Let* $J_{n,m}$ *for* $m \geq 3$ *be the graph and D be the minimum connected dominating set, then the number of iterations is* $nm-\gamma+2$ *.*

Proof: Let $J_{n,m}$ be the Janangir graph. The number of vertices in the Jahangir graph is $nm+1$

i.e., $p = nm+1$

$\implies \quad p-\gamma = nm+1-\gamma$

$\implies \quad p-|D| = nm-\gamma+1$

$\implies \quad |V-D| = nm-\gamma+1$

$\implies \quad |V-D|+1 = nm-\gamma+2$

Hence by remark 1.10 the number of iterations in a Jahangir graph is $nm-\gamma+2$. ■

Corollary 3.3. *Let* $J_{2s,m}$ *for* $m \geq 3$ *be the graph and D be the minimum connected dominating set, then the number of iterations is* $2(sm+1)-\gamma$ *.*

Proof: Let $J_{n,m}$ be the Jahangir graph. Let n=2s, s=1,2,3,...

By theorem 4.1, number of iterations in

a Jahangir graph is $nm - \gamma + 2$
Since n=2s, number of iterations of $J_{2s,m}$ is $2sm - \gamma + 2$
number of iterations of $J_{2s,m}$ is $2(sm + 1) - \gamma$. ■

Corollary 3.4. *Let $J_{2,m}$ for $m \geq 3$ be the graph and D be the minimum connected dominating set, then the number of iterations is $2(m+1) - \gamma$.*

Proof: By putting s=1 in corollary 2.3, number of iteration in a Jahangir graph $J_{2,m}$ is $2(m+1) - \gamma$. ■

Example

In this example soShearEnergy of $J_{2,5}$ is calculated using the algorithm for all possible minimal dominating sets.
The minimum connected dominating set of $J_{2,5}$ is $D = \{v_1, v_5, v_7, v_11\}$ and $V - D = \{v_2, v_3, v_4, v_6, v_8, v_9, v_10\}$. Let us consider $D_0 = D$,then oid s are 0,-1,0,2,0,-1,0. Hence $os\epsilon_{(D_0)}(J_{2,5}) = 2$.
By step 3 of the algorithm, the vertex with minimum positive oid 2 is v_6 . So vertex v_6 is shifted to the set D. Now $D_1 = \{v_1, v_5, v_6, v_7, v_11\}$. oid s are 0,-1,0,0,-1,0 . $os\epsilon_{D_1}(J_{2,5}) = 0$.
By step 4 of the algorithm, all the vertices have zero or negative value, hence oenergy is 0 and by step 6,the vertex with oid zero is v_2 is shifted to the set D.
$D_2 = \{v_1, v_2, v_5, v_6, v_7, v_11\}$ oid_{V-D_2} are1,0,0,-1,0. $os\epsilon_{D_2}(J_{2,5}) = 1$.
By step 3 of the algorithm, the vertex with minimum positive oid is v_3 . So vertex v_3 is shifted to the set D.
$D_3 = \{v_1, v_2, v_3, v_5, v_6, v_7, v_11\}$ oid s are 2,0,-1,0. $os\epsilon_{D_3}(J_{2,5}) = 2$.
By step 3 of the algorithm, the vertex with minimum positive oid is v_4 . So vertex v_4 is shifted to the set D.
$D_4 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_11\}$ oid s are 0,-1,0. $o\epsilon_{D_5}(J_{2,5}) = 0$.
By step 4 of the algorithm, all the vertices have zero or negative values, vertex with zero oid is v_8 . So vertex v_8 is shifted to the set D.
$D_5 = \{1, 2, 3, 4, 5, 6, 7, 8, 11\}$ oid s are 1,0. $o\epsilon_{D_5}(J_{2,5}) = 1$.
By step 3 of the algorithm, the vertex with minimum positive oid is v_9 . So vertex v_9 is shifted to the set D.
$D_6 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 11\}$ oid s are 1,0. $os\epsilon_{D_6}(J_{2,5}) = 0$.
By step 3 of the algorithm, the first vertex with maximum positive oid is v_10 . So vertex v_10 is also shifted to the set D.
$D_7 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ now the set V-D is empty. So $os\epsilon_D(J_{2,5}) = 0$.
By step 9 , $sos\epsilon_{(D)}(J_{2,5})$ =2+0+1+2+0+1+2+0=2+2(1+2+0)=8.
$sos\epsilon_{(D)}(J_{2,5}) = 8$, when D is minimum connected dominating set.

The maximal independent dominating set of $J_{2,5}$ is $D = \{2, 4, 6, 8, 10, 11\}$ and $V - D = \{1, 3, 5, 7, 9\}$. The oid's are 3 are all the vertices ,since id are zero and od are 3 for all the vertices. Since the number of vertices in V-D is 5, $os\epsilon_{(D)} = 5(3)$.

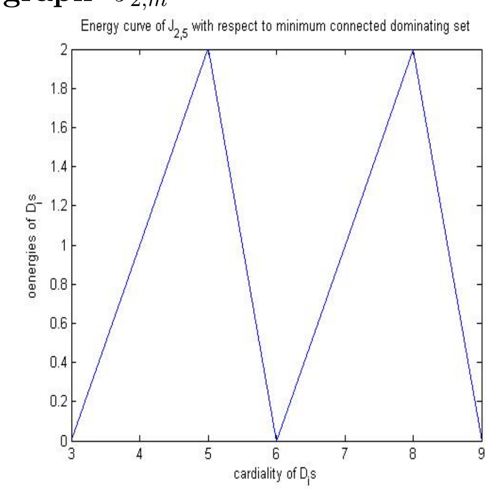
As the number of vertices in V-D decreases till zero. The $sos\epsilon_{(D)}(J_{2,5}) = 3(5+4+3+2+1) = 45$.

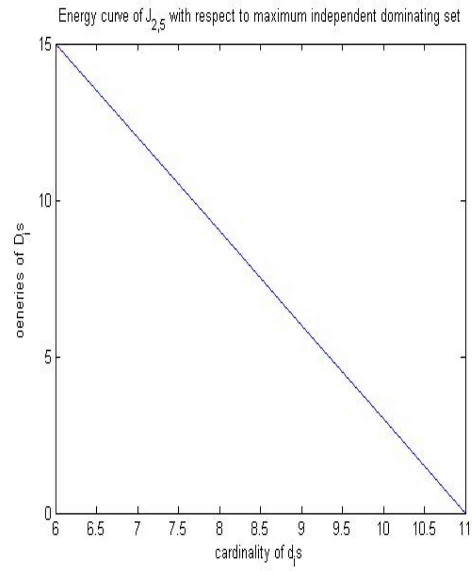
The minimum independent dominating set of $J_{2,5}$ is $D = \{2,5,8,10\}$ and $V - D = \{1,3,4,6,7,9,111\}$ .Applying the algorithm $sos\epsilon_{(D)}(J_{2,5}) = 2+1+1+2+3+1+2+0 = 12$.

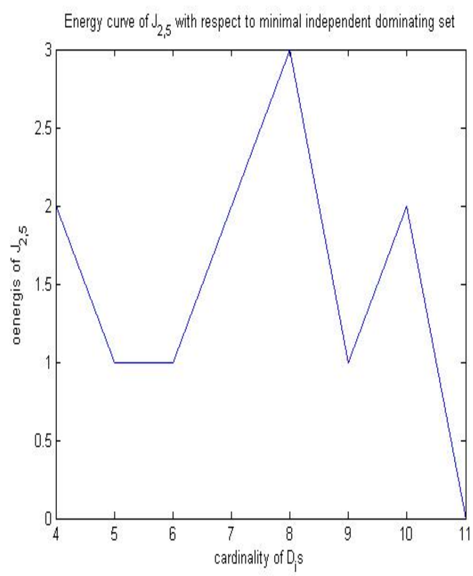
The independent dominating set which is not minimal nor maximum of $J_{2,5}$ with all the vertex with degree 2 is $D = \{1,3,5,7,9\}$ and $V - D = \{2,4,6,8,10,11\}$. Applying the algorithm $sos\epsilon_{(D)}(J_{2,5}) = 15+13+11+9+7+5+0 = 60$.

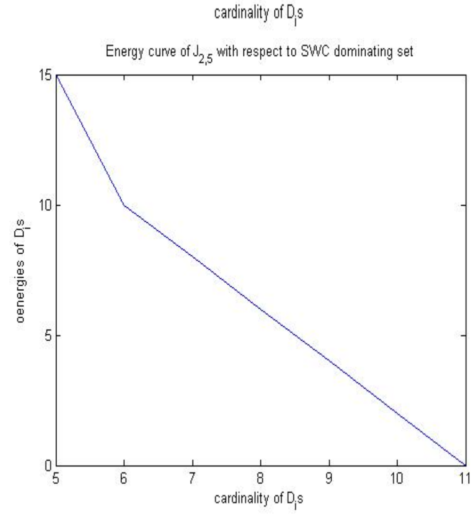
By step 10 of the algorithm, fixing the x axis with the number of vertices of V-D and the Y axis with the oShearEnergy in each stage, the graphs obtained for all the minimal dominating sets are given below.

Energy Curves of Jahangir graph $J_{2,m}$









4 soEnergy of Jahangir graph $J_{2,m}$ with respect to the connected dominating set

Theorem 4.1. *Let $J_{2,m}$ for $m \geq 3$ be the graph and D be the minimum connected dominating set then, $o\epsilon_{J_{2,m}}(D) \in \{0,1,2\} = z_3$.*

proof: Let G be the graph $J_{2,m}$ and D be the minimum connected dominating set with the center vertex.

Case(i) Let m be an odd number,then the minimal connected dominating set have the vertex v_{2m-5} and v_{2m-3} .

Therefore the vertex $v_{2m-4} \in V - D$ have the $id = 0$, $od = 2$ and oid as 2. All other even vertices have id and od 1, all the odd vertices have id 2 , od 1 and oid -1, and $o\epsilon = 2$.

As the vertex v_{2m-4} is shifted to D, there is no change in the id and od .Hence oids are 0 and -1.At this stage $o\epsilon = 0$.

The first vertex with oid value zero is shifted to D, i.e., vertex v_2 is shifted to D. Now $id(v_3) = 1$, $od(v_3) = 2$, $oid(v_3) = 1$, and for all other odd vertices iod is -1 and even vertices it is 0.

At this stage $o\epsilon = 1$.

This process is repeated till cardinality of V-D is zero. therefore the possible $o\epsilon$ are 0,1 and 2.

Case(ii) Let m be an even number. All the even labeled vertices have id , od 1, and $iod = 0$ and odd labeled vertices have id 2 , od 1 and oid -1. Therefore $o\epsilon = 0$.

At this stage the first vertex with oid zero is shifted to D. So the vertex v_3 have id 1 , od 2 and oid 1. all other vertices remains the same. Therefore $o\epsilon = 1$.

At this stage the vertex v_3 is shifted to D. The vertex v_4 have id 0 , od 2 and oid 2. All other vertices remains the same. Therefore $o\epsilon = 2$.

This process is repeated till cardinality of V-D is zero. therefore the possible $o\epsilon$ are 0,1and 2. Therefore $o\epsilon_{J_{2,m}}(D) \in \{0,1,2\} = z_3$. ■

Theorem 4.2. *Let $J_{2,m}$ for $m \geq 3$ be the graph and D be the minimum connected dominating set, then*

$$sos\epsilon_D(J_{2,m}) = \begin{cases} \frac{3m}{2} & \text{if } m \text{ is even and } m \geq 4 \\ 2 + \frac{3m}{2} & \text{if } m \text{ is odd and } m \geq 3. \end{cases}$$

Proof: Let $J_{2,m}$ be the graph, v_{2m+1} be the center vertex and $v_1, v_2, ..., v_{2m}$ be vertices of the cycle C_{2m} where the vertex v_s with $s \cong 1(mod 2)$ is incident with the center vertex v_{2m+1} .

Let D be a minimum connected dominating set containing the center vertex v_{2m+1} , i.e, $\{v_1, v_5, ..., v_{2m-3}, v_{2m+1}\}$.

We know that,in Jahangir graph $J_{2,m}$, degreee of center vertex is m, degrees of vertices $\{v_1, v_3, .., v_{2m-2}\}$ are 3 and for other vertices it is 2.

(i) Let m be odd number.
By the theorem 4.3, for m is odd the $o\epsilon$ begin with 2 . Then it continues as 0,1 and 2 .Two c_n are involved for the occurence of these $os\epsilon$.
Therefore $sos\epsilon_D(J_{2,m}) = 2+\frac{m}{2}(0+1+2)$
$= 2 + 3\frac{m}{2}$.
(ii) Let m be even number.
By theorem 4.3 $os\epsilon$ s are 0,1 and 2. Two C_n are needed for this formation. Therefore $sos\epsilon_D(J_{2,m}) = \frac{m}{2}(0 + 1 + 2)$
$= 3\frac{m}{2}$
Therefore $sos\epsilon_D(J_{2,m}) =$
$$\begin{cases} \frac{3m}{2} & \text{if m is even and } m \geq 4 \\ 2 + \frac{3m}{2} & \text{if m is odd and } m \geq 3. \end{cases} .$$
■

Corollary 4.3. *Let $J_{2,m}$ for $m \geq 3$ be the graph and D be the minimum connected dominating set, then $so\epsilon_{J_{2,m+1}}(D) = 2 + so\epsilon_{J_{2,m}}(D)$, where m is odd and $m \geq 3$.*

Proof: From theorem 3.2, It is clear that $so\epsilon_{J_{2,m+1}}(D) = 2 + \frac{3m}{2}$, where m is odd and $m \geq 3$.
By the theorem 3.2 we know that $so\epsilon_{J_{2,m}}(D) = \frac{3m}{2}$,if m is even .
$\therefore$ $so\epsilon_{J_{2,m+1}}(D) = 2 + so\epsilon_{J_{2,m}}(D)$, where m is odd and $m \geq 4$ ■

5 soShearEnergy of $J_{2,m}$ with respect to independent dominating set

Let D be the maximal independent dominating set and $D = \{v_2, v_4, v_6, ..., v_{2m}, v_{2m+1}\}$.

Theorem 5.1. *Let $J_{2,m}$ for $m \geq 3$ be the graph and D be the maximal independent dominating set, then $o\epsilon_{J_{2,m}}(D) \in \{0, 3, 6, ...3m\}$.*

Proof: Let G be the graph $J_{2,m}$ and D be the maximal independent dominating set. Since D is the maximal independent domianting set $D = \{v_2, v_4, v_6, ..., v_{2m}, v_{2m+1}\}$. Then $V - D = \{v_1, v_3, ...v_{2m-1}\}$. From the construction itself it is clear that od of all the odd labeled vertices are 3 and id is zero. Since id is zero for all the vertices, oid is 3 for all the vertices. It is clear that cardinality of V-D is m. Therefore $os\epsilon = 3m$. As the vertices get shifted to the set D $os\epsilon = 3(m - 1)$ and so on.
The process continues till cardinality of V-D is zero. When the cardinality is zero $os\epsilon = 0$ Therefore $os\epsilon_{J_{2,m}}(D) \in \{0, 3, 6, ...3m\}$ ■

Theorem 5.2. *Let $J_{2,m}$ for $m \geq 3$ be the graph and D be the maximal independent dominating set then, $sos\epsilon_{J_{2,m}}(D) = \frac{3}{2}m(m + 1)$.*

Proof: Let $J_{2,m}$ be the graph , v_{2m+1} be the center vertex and $v_1, v_2, ..., V_{2m}$ be vertices that incident clockwise on cycle C_{2m} so that $deg(v_1) = 3$.
Let D be the maximal independent dominating set containing the center vertex v_{2m+1} also, i.e, $\{v_2, v_4, ..., V_{2m}, v_{2m+1}\}$ then $V - D = \{v_1, v_3, ..., v_{2m-1}\}$. As all these vertices are independent, idegree of these vertices are zero From the label it is clear that all the odd labeled vertices have odegree 3. oid of these vertices are 3. As the cardinality of V-D is m , $os\epsilon_D(J_{2,m}) = 3m$.
As the algorithm proceeds, in each and every step, the vertices are shifted one by one to the set D, then the value of m decreases three by three till cardinality of V-D is zero.
Therefore the value of $os\epsilon_D(J_{2,m})$ are $3(m-1), 3(m-2), ...3, 0$.
By definition, $sos\epsilon_{J_{2,m}}(D) = 3m + 3(m-1) + 3(m-2) + ... + 3(2) + 3 + 0$
$= 3(m + (m-1) + (m-2) + ... + 2 + 1)$
$= \frac{3}{2}m(m+1)$. ■

Let IDWOC be the independent dominating set with out the center vertex and it is not the minimum dominating set. Then the soShearEnergy of Jahangir graph with respect to the dominating set IDWOC is calculated in the formula given below.

Theorem 5.3. *Let $J_{2,m}$ for $m \geq 3$ be the graph and IDWOC be the dominating set then,* $sos\epsilon_{J_{2,m}}(IDWOC) = \sum_{i=0}^{m}(3m-2i)$.

Proof: Let IDWOC be an independent dominating set without center vertex i.e, $\{v_1, v_4, ..., V_{2m-1}\}$ then $V - D = \{v_2, v_4, v_6, ..., v_{2m}, v_{2m+1}\}$. As all these vertices are independent, idegree of these vertices are zero. From the label it is clear that all the even labeled vertices have odegree 2, and the center vertex have odegree m. oid of all the even vertices are 2 and the center vertex is m. As the cardinality of V-D is m+1 , $os\epsilon_D(J_{2,m}) = m + 2m$.
As the algorithm proceeds the vertex of minimum oid is shifted to D and the $os\epsilon_D(J_{2,m}) = 3m - 2$ As the algorithm proceeds in each and every step the vertices are shifted one by one to the set D, then the value of m decreases two by two till cardinality of V-D is zero.
Therefore the value of $os\epsilon_D(J_{2,m})$ are $(3m-2), (3m-4), ...(m+2), m, 0$.
By definition, $sos\epsilon_{J_{2,m}}(ISWC) = (m + 2m) + (3m-2) + (3m-4) + ... + (m+2) + m + 0$.
On Simplification we get , $sos\epsilon_{J_{2,m}}(ISWC) = \sum_{i=0}^{m}(3m-2i)$. ■

Let ${}^4D$ be the minimum independent dominating set. The vertices in the dominating set forms three groups, they are as follows,

1. If m mod 3 = 0, then $D = \{v_2, v_5, v_8, \ldots, v_{2m-4}, v_{2m-1}\}$.
2. If m mod 3 = 1 and m

mod 3 = 2, then $D = \{v_2, v_5, v_8, \ldots, v_{2m-3}, v_{2m}\}$.

Definition 5.4. *Let G be a Jahangir graph , D the dominating set and $(os\epsilon_{D_i})$ be the oShearEnergy sequence of the graph G with respect to the minimal dominating set. The sum of the subsequence of $(os\epsilon_{D_i})$ given by $\sum_{oid(v_2 m+1\leq 0)}(os\epsilon_{D_i})$ is known as $\omega-index$ and denoted by $\omega(m)$. The sum of the subsequence of $(os\epsilon_{D_i})$ given by $\sum_{oid(v_2 m+1\geq 0)}(os\epsilon_{D_i})$ is known as $\Omega - index$ and denoted by $\Omega(m)$.*

Result 5.5. *Let $J_{2,m}$ for $m\geq 3$ be the graph, and ${}^4D$ the minimal dominating set, then series of $\omega-index$ is given by*

1. *Let m mod 3 =0, then*
   (a) *If m mod 2=0, then the sequence begin with 0 followed by (1,0) for $\frac{m}{6}$ number of times.*
   (b) *If m mod 2=1, then the sequence begin with 0 followed by (1,0) for $\lfloor\frac{m}{6}\rfloor$ number of times and then by a (1,1).*
2. *Let m mod 3 =1, then*
   (a) *If m mod 2=0, then the sequence begin with $\lfloor\frac{m}{6}\rfloor + 1$ numbers of (1,0) .*
   (b) *If m mod 2=1, then the sequence begin with $\lfloor\frac{m}{6}\rfloor$ numbers of (1,0) number of times and then by a (1,1).*
3. *Let m mod 3 =2, then*
   (a) *If m mod 2=0, then the sequence begin with 2 followed by $\frac{m}{6}$ numbers of (1,0) .*
   (b) *If m mod 2=1, then the sequence begin with 2 followed by $\lfloor\frac{m}{6}\rfloor$ numbers of (1,0) number of times and then by a (1,1).*

Result 5.6. *Let $J_{2,m}$ for $m\geq 3$ be the graph, and ${}^4D$ the minimal dominating set, then $\Omega - index$ is given by $(2i)4+\sum(2i-1)+(2i-1)$, where $i = 1,2,\ldots$. Further i's are bunch which contains two consecutive m's such as when i=1, m=4,5.*

Result 5.7. *Let $J_{2,m}$ for $m\geq 3$ be the graph, and ${}^4D$ the minimal dominating set, then $sos\epsilon_{J_{2,m}} = \omega(m)+\Omega(m)$*

6 Hardihood of Jahangir graph $J_{2,m}$

Theorem 6.1. *Let $J_{2,m}$ for $m\geq 3$ be the graph,*
$HD^{+}(G) = (m+2m)+(3m-2)+(3m-4)+...+(m+2)+m+0$ when D is the ISWC dominating set
$HD^{-}(G) = sos\epsilon_{J_{2,m}}(D)$ when D is the minimum connected dominating set.

proof: Let G be the Jehangir graph $J_{2.m}$. It is clear that $\frac{3}{2}(m(m+1)) \leq (m+2m)+(3m-2)+(3m-4)+...+$

$(m+2)+m+0$.
Therefore $sos\epsilon_{J_{2,m}}(D)$ is maximum when D is ISWC dominating set.
Hence $HD^{+}(G)=(m+2m)+(3m-2)+(3m-4)+...+(m+2)+m+0$ when D is the ISWC dominating set. When D is minimal dominating set , all the vertices will not have the value 3 as the iodegree so the value is less that $\frac{3}{2}m(m+1)$. Another possible minimal dominating set is connected dominating set which has the value $\frac{m}{2}(0+1+2)$ or $2+\frac{m}{2}(0+1+2)$ ■

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