Radio mean D-distance number of cycle-related graphs

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Abstract

A Radio Mean D-distance labeling of a connected graph G is an injective map f from the vertex set V(G) to the ℕ such that for two distinct vertices u and v of G, $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^D(G)$, where $d^D(u, v)$ denotes the D-distance between u and v and $diam^D(G)$ denotes the D-diameter of G. The radio mean D-distance number of f, $rmn^D(f)$ is the maximum label assigned to any vertex of G. The radio mean D-distance number of G, $rmn^D(G)$ is the minimum value of $rmn^D(f)$ taken over all radio mean D-distance labeling f of G. In this paper we find the radio mean D-distance number of cycle-related graphs.

Key words . D-distance, Radio D-distance coloring, Radio D-distance number, Radio mean D- distance, Radio mean D-distance number.

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1. Introduction

By a graph G = (V, E) we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively.

Let G be a connected graph of diameter d and let k an integer such that $1 \leq k \leq d$. A radio k-coloring of G is an assignment f of colors (positive integers) to the vertices of G such that $d(u, v) + |f(u) - f(v)| \geq 1 + k$ for every two distinct vertices u, v of G. The radio k-coloring number $rc_k(f)$ of a radio k-coloring f of G is the maximum color assigned to a vertex of G. The radio k-chromatic number $rc_k(G)$ is $\min\{rc_k(f)\}$ over all radio k-colorings f of G. A radio k-coloring f of G is a minimum radio k-coloring if $rc_k(f) = rc_k(G)$. A set S of positive integers is a radio k-coloring set if the elements of S are used in a radio k-coloring of some graph G and S is a minimum radio k-coloring set if S is a radio k-coloring set of a minimum radio k-coloring of some graph G. The radio 1-chromatic number $rc_1(G)$ is then the chromatic number $\chi(G)$. When k = Diam(G), the resulting radio k-coloring is called radio coloring of G. The radio number of G is defined as the minimum span of a radio coloring of G and is denoted as rn(G).

Radio labeling (multi-level distance labeling) can be regarded as an extension of distance two labeling which is motivated by the channel assignment problem introduced by Hale [6]. Chartrand et al. [2] introduced the concept of radio labeling of graph. Chartrand et al. [3] gave the upper bound for the radio number of Path. The exact value for the radio number of Path and Cycle was given by Liu and Zhu [10]. However Chartrand et al. [2] obtained different values than Liu and Zhu [10]. They found the lower and upper bound for the radio number of Cycle. Liu [9] gave the lower bound for the radio number of Tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O.Togni [8]. M.M.Rivera et al. [20] gave the radio number of $C_n \times C_n$, the Cartesian product of C_n. In [4] C.Fernandez et al. found the radio number for complete graph, star graph, complete bipartite graph, wheel graph and gear graph. M.T.Rahim and I.Tomescu [18] investigated the radio number of Helm Graph. The radio number for the generalized prism graphs were presented by Paul Martinez et.al. in [11].

The concept of D-distance was introduced by D. Reddy Babu et al. [19, 20, 21]. If u, v are vertices of a connected graph *G* the *D*-length of a connected u-v path *s* is defined as $\ell^D(s) = \ell(s) + deg(v) + deg(u) + \sum deg(w)$ where the sum runs over all intermediate vertices *w* of *s* and $\ell(s)$ is the length of the path. The *D*-distance, $d^D(u, v)$ between two vertices u, v of a connected graph G is defined a $d^D(u, v) = \min\{\ell^D(s)\}$ where the minimum is taken over all u-v paths *s* in *G*. In other words, $d^D(u, v) = \min\{\ell(s) + deg(v) + deg(u) + \sum deg(w)\}$ where the sum runs over all intermediate vertices *w* in *s* and minimum is taken over all u-v paths *s* in *G*.

In [12], we introduce the concept of Radio D-distance. The radio D-distance coloring is a function $f : V(G) \rightarrow \mathbb{N} \cup \{0\}$ such that $d^D(u, v) + |f(u) - f(v)| \geq diam^D(G) + 1$. It is denoted by $rn^D(G)$. A radio D-distance coloring f of G is a minimum radio D-distance coloring if $rn^D(f) = rn^D(G)$, where $rn^D(G)$ is called radio D-distance number.

Radio mean labeling was introduced by R. Ponraj et al [15,16,17]. A radio mean labeling is a one to one mapping f from V(G) to $\mathbb{N}$ satisfying the condition

$$d(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam(G) \quad (1.1)$$

for every u, v ∈ V (G). The span of a labeling f is the maximum integer that f maps to a vertex of G. The radio mean number of G, rmn(G) is the lowest span taken over all radio mean labelings of the graph G. The condition (1.1) is called radio mean condition.

In [14], we introduce the concept of Radio mean D-distance labeling. A radio mean D-distance labeling is a one to one mapping f from V(G) to $\mathbb{N}$ satisfying the condition

$$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^D(G) \quad (1.2)$$

for every u, v ∈ V(G). The span of a labeling f is the maximum integer that f maps to a vertex of G. The radio mean D-distance number of G, $rmn^D(G)$ is the lowest span taken over all radio mean D-distance labelings of the graph G. The condition (1.2) is called radio mean D-distance condition. In this paper we determine the radio mean D-distance number of cycle-related graphs. The function $f : V(G) \rightarrow \mathbb{N}$ always represents injective map unless otherwise stated.

2 . Main Result

Theorem 2.1.

The radio mean D-distance number of a cycle,

$$rmn^D(C_n) \leq \begin{cases} 3 \ if \ n = 3. \\ 5\left(\frac{n}{2}\right) - 7, \ n \geq 10 \ if \ n \ is \ even. \\ 5\left(\frac{n-1}{2}\right) - 6, \ n \geq 9 \ if \ n \ is \ odd. \end{cases}$$

Proof .

It is obvious that $diam^D(C_n) = 3\left(\frac{n}{2}\right) + 2$(if n is even) and $diam^D(C_n) = 3\left\{\frac{(n-1)}{2}\right\} + 2$(if n is odd). Let $V(C_n) = \{v_1, v_2, v_3, \ldots, v_n\}$. Define the function f as

if n is even

$f(v_{2i-1}) = x_i, \quad 1 \leq i \leq n - 5.$

$f(v_{n+2-2i}) = x_{n-5+i}, \quad 1 \leq i \leq n - 5.$

$f(x_i) = 3\left(\frac{n}{2}\right) - 7 + i, 1 \leq i \leq n.$

if n is odd

$f(v_{2i-1}) = x_i, \quad 1 \leq i \leq n - 5.$

$f(v_{n+1-2i}) = x_{n-5+i}, \quad 1 \leq i \leq n - 5.$

$f(v_n) = x_n$

$f(x_i) = 3\left(\frac{n-1}{2}\right) - 7 + i, 1 \leq i \leq n.$

Case 1. n is even, We must show that the radio mean D-distance condition

$$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(C_n) + 1 = 3\left(\frac{n}{2}\right) + 3,$$

for every pair of vertices (u, v) where u ≠ v.

If v_i and v_j are adjacent, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil$

$$\geq 5 + \left\lceil \frac{3\left(\frac{n}{2}\right)+i-7+3\left(\frac{n}{2}\right)+j-7}{2} \right\rceil \geq 3\left(\frac{n}{2}\right) + 3.$$

If v_i and v_j are not adjacent $|i - j| \leq \frac{n}{2}$,

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil$

$$\geq 3\left(\frac{n}{2}\right) + 2 + \left\lceil \frac{3\left(\frac{n}{2}\right)+i-7+3\left(\frac{n}{2}\right)+j-7}{2} \right\rceil \geq 3\left(\frac{n}{2}\right) + 3$$

Case 2. n is odd, We must show that the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$

$$\geq diam^D(C_n) + 1 = 3\left\{\frac{(n-1)}{2}\right\} + 3$$

If v_i and v_j are adjacent,

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil$

$$\geq 5 + \left\lceil \frac{3\left(\frac{n-1}{2}\right)+i-6+3\left(\frac{n-1}{2}\right)+j-6}{2} \right\rceil \geq 3\left\{\frac{(n-1)}{2}\right\} + 3.$$

If v_i and v_j are not adjacent $|i - j| \leq \frac{n}{2}$,

$$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 3\left\{\frac{(n-1)}{2}\right\} + 3$$

$$+ \left\lceil \frac{3\left(\frac{n-1}{2}\right)+i-7+3\left(\frac{n-1}{2}\right)+j-7}{2} \right\rceil \geq 3\left\{\frac{(n-1)}{2}\right\} + 3.$$

Therefore,

$$rmn^D(C_n) \leq \begin{cases} 3 \ if \ n = 3. \\ 5\left(\frac{n}{2}\right) - 7, \ n \geq 10 \ if \ n \ is \ even. \\ 5\left(\frac{n-1}{2}\right) - 6, \ n \geq 9 \ if \ n \ is \ odd. \end{cases}$$

□

Note. $rmn^D(C_n) \leq \begin{cases} 4\left(\frac{n}{2}\right) - 2, \ n = 4, 6, 8. \\ 4\left(\frac{n-1}{2}\right) - 2, n = 5, 7. \end{cases}$

Theorem 2.2.
The radio mean D-distance number of a wheel graph,

$$rmn^D(W_n) \leq \begin{cases} 4 \text{ if } n = 3. \\ 7 \text{ if } n = 4, 5. \\ 3\left(\frac{n-1}{2}\right) + 3 \text{ if } n \text{ is odd}, n \geq 7 \\ 3\left(\frac{n}{2}\right) + 2 \text{ if } n \text{ is even}, n \geq 6 \end{cases}$$

Proof .

It is obvious that $diam^D(W_n) = n + 8 (n \geq 6)$. Let $V(W_n) = \{v_0, v_1, v_2, v_3, \ldots, v_n\}$, where v_0 is the central vertex. Define the function f as

if n is even

$f(v_0) = x_0$.

$f(v_{2i-1}) = x_i$, $1 \leq i \leq \left(\frac{n}{2}\right)$.

$f(v_{n-2i}) = x_{\left(\frac{n}{2}\right)+i}$, $1 \leq i \leq \left(\frac{n}{2}\right) - 1$.

$f(v_n) = x_n$

$$f(x_i) = \begin{cases} 5, i = 0. \\ \left(\frac{n}{2}\right) + 2 + i, 1 \leq i \leq n \end{cases}$$

if n is odd

$f(v_0) = x_0$.

$f(v_{2i-1}) = x_i$, $1 \leq i \leq \left(\frac{n-1}{2}\right)$.

$f(v_{n+1-2i}) = x_{\left(\frac{n-1}{2}\right)+i}$, $1 \leq i \leq \left(\frac{n-1}{2}\right)$.

$f(v_n) = x_n$

$$f(x_i) = \begin{cases} 5, i = 0. \\ \left(\frac{n-1}{2}\right) + 2 + i, 1 \leq i \leq n \end{cases}$$

Case 1. n is even, We must show that the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(G) + 1 = n + 9$, for every pair of vertices (u, v) where $u \neq v$.

If v_0 and v_i are adjacent $d^D(v_0, v_i) + \left\lceil \frac{f(v_0)+f(v_i)}{2} \right\rceil$

$$\geq n + 4 + \left\lceil \frac{5+\left(\frac{n}{2}\right)+2+i}{2} \right\rceil \geq n + 9$$

for (v_i, v_j), $|i - j| \leq \frac{n}{2}$

$$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq n + 8 + \left\lceil \frac{\left(\frac{n}{2}\right)+2+i+\left(\frac{n}{2}\right)+2+j}{2} \right\rceil \geq n + 9$$

If v_i and v_j are adjacent

$$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 7 + \left\lceil \frac{\left(\frac{n}{2}\right)+2+i+\left(\frac{n}{2}\right)+2+j}{2} \right\rceil \geq n + 9.$$

Case 2. n is odd, We must show that the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(G) + 1 = n + 9$, for every pair of vertices (u, v) where $u \neq v$.

If v_0 and v_i are adjacent

$$d^D(v_0, v_i) + \left\lceil \frac{f(v_0)+f(v_i)}{2} \right\rceil \geq n + 4 + \left\lceil \frac{5+\left(\frac{n-1}{2}\right)+2+i}{2} \right\rceil \geq n + 9$$

for (v_i, v_j), $|i - j| \leq \frac{n}{2}$

$$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq n + 8 + \left\lceil \frac{\left(\frac{n-1}{2}\right)+2+i+\left(\frac{n-1}{2}\right)+2+j}{2} \right\rceil \geq n + 9$$

If v_i and v_j are adjacent

$$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 7 + \left\lceil \frac{\left(\frac{n-1}{2}\right)+2+i+\left(\frac{n-1}{2}\right)+2+j}{2} \right\rceil \geq n + 9$$

Therefore,

$$rmn^D(W_n) \leq \begin{cases} 4 \text{ if } n = 3. \\ 7 \text{ if } n = 4, 5. \\ 3\left(\frac{n-1}{2}\right) + 3 \text{ if } n \text{ is odd}, n \geq 7 \\ 3\left(\frac{n}{2}\right) + 2 \text{ if } n \text{ is even}, n \geq 6 \end{cases}$$

□

Note. The Gear graph is obtained by inserting a vertex in each edge of the cycle of the wheel.

Theorem 2.3.
The radio mean D-distance number of a gear graph,

$$rmn^D(G_n) \leq \begin{cases} 11, n = 3. \\ 16, n = 4. \\ 17, n = 5. \\ 3n + 4, n \geq 6. \end{cases}$$

Proof .

It is clear that $diam^D(G_n) = n + 14 (n \geq 6)$. Let $\{v_0, v_1, v_2, v_3, \ldots, v_n\}$ and $\{u_1, u_2, u_3, \ldots, u_n\}$ are the vertices set, where v_0 is the central vertex. Define the function f as $f(v_0) = 4$, $f(u_i) = n + i + 4$, $1 \leq i \leq n$, $f(v_i) = 2n + i + 4$, $1 \leq i \leq n$. We must show that the radio mean D-distance condition

$$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(G_n) + 1 = n + 15,$$

for every pair of vertices (u, v) where $u \neq v$.

If v_0 and v_i are adjacent

$$d^D(v_0, v_i) + \left\lceil \frac{f(v_0)+f(v_i)}{2} \right\rceil \geq n + 4 + \left\lceil \frac{4+2n+i+4}{2} \right\rceil \geq n + 15.$$

If v_0 and u_i are not adjacent

$$d^D(v_0, u_i) + \left\lceil \frac{f(v_0)+f(u_i)}{2} \right\rceil \geq n + 7 + \left\lceil \frac{4+n+i+4}{2} \right\rceil \geq n + 15.$$

Check the pair of (u_i, u_j) $|i - j| \leq \frac{n}{2}$

$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq$

$n + 14 + \left\lceil \frac{n+i+4+n+j+4}{2} \right\rceil \geq n + 15.$

Check the pair of (v_i, v_j) $|i-j| \leq \frac{n}{2}$

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq$

$n + 8 + \left\lceil \frac{2n+i+4+2n+j+4}{2} \right\rceil \geq n + 15.$

If v_i and u_j are adjacent

Check the pair of (v_i, u_j)

$d^D(v_i, u_j) + \left\lceil \frac{f(v_i)+f(u_j)}{2} \right\rceil \geq$

$6 + \left\lceil \frac{2n+i+4+n+j+4}{2} \right\rceil \geq n + 15.$

Therefore,

$$rmn^D(G_n) \leq \begin{cases} 11, n = 3. \\ 16, n = 4. \\ 17, n = 5. \\ 3n+4, n \geq 6. \end{cases}$$

□

Note. The Helm H_n is obtained from a wheel by attaching a pendant edge at each vertex of the n-cycle.

Theorem 2.4.

The radio mean D-distance number of a Helm graph,

$$rmn^D(H_n) \leq \begin{cases} 10, n = 3. \\ 16, n = 4. \\ 17, n = 5. \\ 2(n+4), n \geq 6. \end{cases}$$

Proof .

It is clear that $diam^D(H_n) = n + 14 (n \geq 6)$. Let $\{v_0, v_1, v_2, v_3, \ldots, v_n\}$and $\{u_1, u_2, u_3, \ldots, u_n\}$ are the vertex set, where v_0 is the central vertex and also $\{u_1, u_2, u_3, \ldots, u_n\}$ are the pendent vertices. Define the function f as $f(v_0) = 7$, $f(u_i) = i + 8$, $1 \leq i \leq n$,

$f(v_i) = n + i + 8$, $1 \leq i \leq n$.

We must show that the radio mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(G) + 1 = n + 15$, for every pair of vertices (u, v) where $u \neq v$.

If v_0 and v_i are adjacent

$d^D(v_0, v_i) + \left\lceil \frac{f(v_0)+f(v_i)}{2} \right\rceil \geq$

$n + 5 + \left\lceil \frac{7+n+i+8}{2} \right\rceil \geq n + 15$

If v_0 and u_i are not adjacent

$d^D(v_0, u_i) + \left\lceil \frac{f(v_0)+f(u_i)}{2} \right\rceil \geq$

$n + 7 + \left\lceil \frac{7+i+8}{2} \right\rceil \geq n + 15.$

Check the pair of (u_i, u_j) $|i-j| \leq \frac{n}{2}$

$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq$

$n + 14 + \left\lceil \frac{i+8+j+8}{2} \right\rceil \geq n + 15.$

Check the pair of (v_i, v_j) $|i-j| \leq \frac{n}{2}$

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq$

$n + 10 + \left\lceil \frac{n+i+8+n+j+8}{2} \right\rceil \geq n + 15.$

Check the pair of (v_i, u_j)

$d^D(v_i, u_j) + \left\lceil \frac{f(v_i)+f(u_j)}{2} \right\rceil \geq$

$n + 12 + \left\lceil \frac{n+i+8+j+8}{2} \right\rceil \geq n + 15.$

Therefore,

$$rmn^D(H_n) \leq \begin{cases} 10, n = 3. \\ 16, n = 4. \\ 17, n = 5. \\ 2(n+4), n \geq 6. \end{cases}$$

□

Theorem 2.5.

The radio mean D-distance number of a fan graph,

$$rmn^D(F_n) \leq \begin{cases} 3, \ n = 1. \\ n+4, \ n \geq 2. \end{cases}$$

Proof .

It is clear that $diam^D(F_n) = n + 8 (n \geq 2)$. Let $V(F_n) = \{v_0, v_1, v_2, v_3, \ldots, v_n\}$, where v_0 is the central vertex. Define the function f as $f(v_{i-1}) = i + 3$, $1 \leq i \leq n+1$. We must show that the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(G) + 1 = n + 9$, for every pair of vertices (u, v) where $u \neq v$.

If v_0 and v_i are adjacent

$d^D(v_0, v_i) + \left\lceil \frac{f(v_0)+f(v_i)}{2} \right\rceil \geq$

$n + 4 + \left\lceil \frac{3+i+3}{2} \right\rceil \geq n + 9$

$d^D(v_0, v_j) + \left\lceil \frac{f(v_0)+f(v_j)}{2} \right\rceil \geq$

$n + 3 + \left\lceil \frac{3+j+3}{2} \right\rceil \geq n + 9$

If v_i and v_j are adjacent

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq$

$6 + \left\lceil \frac{i+3+j+3}{2} \right\rceil \geq n + 9$

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq$

$5 + \left\lceil \frac{i+3+j+3}{2} \right\rceil \geq n + 9$

If v_i and v_j are not adjacent

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq$

$n + 8 + \left\lceil \frac{i+3+j+3}{2} \right\rceil \geq n + 9$

Therefore,

$$rmn^D(F_n) \leq \begin{cases} 3, \ n = 1. \\ n+4, \ n \geq 2. \end{cases}$$

□

Note. The graph $C_n^{(t)}$ denoting the one point union of t copies cycle C_n. The graph $C_3^{(t)}$ (or $K_3^{(t)}$) is called friendship graph.

Theorem 2.6.

The radio mean D-distance number of a friendship graph, $rmn^D(C_3^{(t)}) \leq \begin{cases} 7, t = 2. \\ 11, t = 3. \\ 4t-3, t \geq 4. \end{cases}$

Proof .

It is clear that $diam^D(C_3^{(t)}) = 2t + 6$ $(t \geq 2)$. Let $(C_3^{(t)}) = \{v_0, v_1, v_2, v_3, \ldots, v_{2t}\}$, where v_0 is the central vertex. Define the function f as

$$f(v_0) = x_0$$
$$f(v_{2i-1}) = x_i,\ 1 \leq i \leq 2t.$$
$$f(v_{2(t+1)-2i}) = x_{n+i},\ 1 \leq i \leq 2t.$$
$$f(x_i) = \begin{cases} 3, i = 0. \\ 2t + i - 3, 1 \leq i \leq t. \\ 3t + i - 3, 1 \leq i \leq t. \end{cases}$$

We must show that the radio mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(G) + 1 = 2t + 7$, for every pair of vertices (u, v) where $u \neq v$.

If v_0 and v_i are adjacent

$$d^D(v_0, v_i) + \left\lceil \frac{f(v_0)+f(v_i)}{2} \right\rceil \geq$$
$$2t + 3 + \left\lceil \frac{3+2t+i-3}{2} \right\rceil \geq 2t + 7$$
$$d^D(v_0, v_i) + \left\lceil \frac{f(v_0)+f(v_i)}{2} \right\rceil \geq$$
$$2t + 3 + \left\lceil \frac{3+3t+i-3}{2} \right\rceil \geq 2t + 7$$

If v_i and v_j are adjacent

$$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 5 + \left\lceil \frac{2t+i-3+3t+j-3}{2} \right\rceil = 13$$

If v_i and v_j are not adjacent

$$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq$$
$$2t + 6 + \left\lceil \frac{2t+i-3+2t+j-3}{2} \right\rceil \geq 2t + 7$$
$$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq$$
$$2t + 6 + \left\lceil \frac{3t+i-3+3t+j-3}{2} \right\rceil \geq 2t + 7$$

Therefore,

$$rmn^D(C_3^{(t)}) \leq \begin{cases} 7, n = 2. \\ 11, n = 3. \\ 4t-3, n \geq 4. \end{cases}$$

□

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