A Series of Nested Two, Three Designs and Group Divisible Designs using Hadamard Matrices

Roshni Tiwari[1], H.L. Sharma[1] and S.S. Gautam[2]

[1]*Dept. of Agril.Econ. and F.M., JNKVV, Jabalpur, Madhya Pradesh (M.P.) India*
[2]*Dept. of Mathematics and Statistics, JNKVV, Jabalpur, Madhya Pradesh (M.P.) India*
[3]*Dept of Statistics, M.G.C.G.V.V. Chitrakoot, Satna , Madhya Pradesh (M.P.) India*

Abstract- *A series of nested two, three and group divisible designs using HadamardMatrices has been constructed. Some examples are also added.*
Keywords- *BIB designs, Group divisible designs, Nested designs.*

I. Introduction

The use of block designs is widely recognized in many fields of experimental material. However, there may be more sources of variation than can be eliminated by ordinary block designs. For such situations, Preece [7] introduced nested balanced incomplete block (NBIBD) designs, while Singh and Dey [9] and Srivastava [10] introduced balanced incomplete block designs with nested rows and columns (BIBRC).

Gupta and Kageyama [4] followed by Das *et al*. [2] proposed the use of NBIBDs with k_2 =2 for dialed cross experiments in plant-breeding investigations of v cultivars. Morgan *et al.* [6] have given a generalized recursive construction that enables us to make not only NBIBD but multiply nested BIBD.

Group divisible (GD) designs constitute the largest, simplest and perhaps the most important type of 2-assocoate partially balanced incomplete block (PBIBD) designs. Clatworthy [1] tabulated 443 parameters combinations of GD designs with there solutions

Recently, Greig and Rees [3] have given the BIBI designs for many sets of treatments but not given in terms of Hadamard Matrices. The objective of the present paper is to construct a series of nested 2 and 3 – designs with the help of Hadamard Matrices.

II. Definitions and Notations

For the definitions and notations the reader is referred to Raghavarao [8], Preece [7] and Hedayat and Wallis [5].

A. Nested balanced incomplete block design (NBIBD)

It is a block design for v treatments, each replicated r times, with two systems of blocks, such that

(i) The second system is nested within the first, with each block from the first system (subsequently called 'block') containing exactly m blocks from the second system (called sub-blocks);
(ii) ignoring the second system leaves a BIB design with parameters (v,b_1,r,k_1,λ_1) and
(iii) ignoring the first system leaves a BIB design with parameters $(v, b_2,r,k_2, \lambda_2)$.

The integers v, b_1, b_2, r, k_1, k_2, λ_1, λ_2 are called the parameters of an NBIB design. Thus, we have the following relationships among the above parameters

$vr = b_1k_1= b_2k_2= b_1 m k_2$
$\lambda_1 (v-1) = r (k_1-1)$
$\lambda_2 (v-1) = r (k_2-1)$
Hence, we have
$(v-1)(\lambda_1 -m \lambda_2) = (m-1) r$

B. Group Divisible Designs

A group divisible (GD) is an arrangement of v (= mn) treatments in b blocks such that each block consists of k (<v) distinct treatments; each treatment is replicated r-times, and the set of treatments can be partitioned into m (≥2) groups of n (≥2) treatments each, any two distinct treatments occurring together in λ_1 blocks if they belong to the same group, and in λ_2 blocks if they belong to different groups. Furthermore, if r- λ_1=0, the GD is said to be singular; if r- λ_1 >0 and rk-vλ_2 =0, it is called semi-regular (SR); and if r- λ_1>0 and rk-v λ_2>0, it is called regular (R).

C. Hadamard Matrix

A square matrix H of order t whose entries are +1 or -1 is called a Hadamard matrix of order t provided that its rows are pairwise orthogonal i.e.,

$$HH' = tI$$

A Hadamard matrix is said to be normalized when all the entries of its first row and first column are +1. The core of a Hadamard matrix is obtained by deleting the first row and first column of its normalized form.

III. Method of construction of Nested 2 and 3 – Designs

The designs under consideration here are a generalization of BIB designs in which a single set of v treatments to be selected from BIB design on the basis of Hadamard matrix. Then these v treatments are replaced by s sets of v as per the chosen block size. We keep these s sets of v on the same block set. Let X is the total s sets of v. Also let X' be another s sets of v which are composed of complement of X leading to one to one correspondence to the treatments with invariant treatment ∞.

We develop, then each set by taking modulo 4t-1, which is the number of treatments of normalized Hadamard matrix of order 4t (Here, we take t≥2). The design

D= [X∪X'], where ∪ stands for union

is considered to be the nested 2- and 3- design with the following parameters.

v= 4t,

$$b_1 = \binom{2t}{t-i}(4t-1),$$

$$r = \binom{2t-1}{t+i}(4t-1)$$

k_1=2(t-i)

$$\lambda_{21} = \binom{2t-1}{t+i}(2t-2i-1)$$

$$\lambda_{31} = \binom{2t-2}{t+i}(2t-2i-1) \quad \ldots(3.1)$$

For,

v= 4t,

$$b_2 = 2\binom{2t}{t-i}(4t-1),$$

$$r = \binom{2t-1}{t+i}(4t-1)$$

k_2= (t-i)

$$\lambda_{22} = \binom{2t-1}{t+i}(t-i-1)$$

$$\lambda_{32} = \binom{2t-3}{t+i}(t-1)$$

where, i = 0,1,2,…,t-2, and t≥2

By deleting one complete replicate from the above constructed nested design with parameters (3.1), we get a group divisible design with the following parameters:

v= 4t,

$$b_1 = \left(\binom{2t}{t-i} - 2\right)(4t-1)$$

$$r = \left(\binom{2t-1}{t+i} - 1\right)(4t-1)$$

$$k_1 = 2(t-i)$$

$$\lambda_1 = \binom{2t-1}{t+i}(2t-2i-1) - 1$$

$$\lambda_2 = \binom{2t-1}{t+i}(2t-2i-1)$$

for i = 0,1,2,…,(t-2) where t≥2

Illustrative Examples

The nested 2 and 3 - designs for some values of t (t≤8) are given below:

v	b_1	b_2	R	k_1	k_2	λ_{21}	λ_{31}	λ_{22}	λ_{32}
8	42	84	21	4	2	9	3	3	N.A.
12	220	440	110	6	3	50	20	20	3
16	1050	2100	525	8	4	245	105	105	15

Their initial blocks are given as
For v =8, k_1 =4, k_2= 2
[(2,6) (4,5)] [(4,6) (2,5)] [(5,6) (2,4)][(∞,3) (0,1)] [(0,3) (∞,1)] [(1,3) (∞,0)] mod 7

For v = 12, k_1 = 6, k_2= 3
[(1,2,4) (5,6,10)] [(1,2,5) (4,6,10)] [(1,2,6)(4,5,10)] [(1,2,10) (4,5,6)] [(1,4,5) (2,6,10)] [(1,4,10) (2,5,6)] [(1,5,6) (2,4,10)] [(1,5,10)(2,4,6)] [(1,6,10) (2,4,5)] [(1,4,6) (2,5,10)] [(0,3,7) (8,9,∞)] [(0,3,8) (7,9,∞)] [(0,3,9) (7,8,∞)] [(0,3,∞) (7,8,9)] [(0,7,8) (3,9,∞)] [(0,7,∞) (3,8, 9)] [(0,8,9) (3,7,∞)] [(0,8,∞) (3,7, 9)] [(0,9,∞) (3,7, 8)] [(0,7,9) (3,8,∞)] mod 11

For v =12, k_1 = 4, k_2 = 2 does not exist but it doubles, exist.
For GD design when v= 8, we have
v=8, b=36, r=18, k=4, λ_1=8, λ_2=9 as group divisible design. The association scheme is

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