Eulerian integral associated with product of two multivariable I-functions, a class of polynomials and the multivariable $\bar{I}$-function defined by Nambisan II

$F.Y.\ AYANT^1$

1 Teacher in High School , France

ABSTRACT
The present paper is evaluated a new Eulerian integral associated with the product of two multivariable I-functions defined by Prasad [1] a generalized Lauricella function , a class of multivariable polynomials and Multivariable I-function defined by Nambisan [2] with general arguments . We will study the case concerning the multivariable H-function defined by Srivastava et al [7] and Srivastava-Daoust polynomial [4].

Keywords: Eulerian integral, multivariable I-function, generalized Lauricella function of several variables, multivariable H-function, generalized hypergeometric function, class of polynomials

2010 Mathematics Subject Classification. 33C60, 82C31

1. Introduction

In this paper, we evaluate a new Eulerian integral of most general characters associated with the products of two multivariable I-functions defined by Prasad [1] , a expansion serie of multivariable I-function defined by Nambisan et al [2] and a class of polynomials with general arguments.
First time, we define the multivariable $\bar{I}$-function by :

$$I(z_1''',\cdots,z_v''')=\bar{I}^{0,N:M_1,N_1;\cdots;M_v,N_v}_{P,Q:P_1,Q_1;\cdots;P_v,Q_v}\left(\begin{array}{c|c} \mathrm{z}_1''' & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(v)};A_j)_{N+1,P}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_v''' & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(v)};B_j)_{M+1,Q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};1)_{1,N_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{N_1+1,P_1};\cdots;(c_j^{(v)},\gamma_j^{(v)};1)_{1,N_u},(c_j^{(v)},\gamma_j^{(v)};C_j^{(v)})_{N_v+1,P_v} \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};1)_{1,M_1},(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{M_1+1,Q_1};\cdots;(d_j^{(v)},\delta_j^{(v)};1)_{1,M_v},(d_j^{(v)},\delta_j^{(v)};D_j^{(v)})_{M_v+1,Q_v} \end{array}\right) \tag{1.1}$$

$$=\frac{1}{(2\pi\omega)^v}\int_{L_1}\cdots\int_{L_v}\phi_1(s_1,\cdots,s_v)\prod_{i=1}^{v}\xi_i'(s_i)z_i''^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_v \tag{1.2}$$

where $\phi_1(s_1,\cdots,s_v),\xi_i'(s_i),i=1,\cdots,v$ are given by :

$$\phi_1(s_1,\cdots,s_v)=\frac{1}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{v}\alpha_j^{(i)}s_j\right)\prod_{j=M+1}^{Q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{v}\beta_j^{(i)}s_j\right)} \tag{1.3}$$

$$\xi_i'(s_i) = \frac{\prod_{j=1}^{N_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)} \tag{1.4}$$

$i=1,\cdots,v$

Serie representation

If $z_i''' \neq 0; i=1,\cdots,v$

$\delta_{h_i}^{(i)}(d_j^{(i)}+k_i) \neq \delta_j^{(i)}(\delta_{h_i}^{(i)}+\eta_i) for j \neq h_i, j, h_i = 1,\cdots,m_i (i=1,\cdots,v)$,$k_i,\eta_i = 0,1,2,\cdots (i=1,\cdots,v)$, then

$$\bar{I}(z_1''',\cdots,z_v''') = \sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\left[\phi_1\left(\frac{dh_1^{(1)}+k_1}{\delta h_1^{(1)}},\cdots,\frac{dh_v^{(v)}+k_v}{\delta h_v^{(v)}}\right)\right]\prod_{j\neq h_i i=1}^{r}\frac{(-)^{k_i}}{\delta h_i^{(i)} k_i!} z_i'''^{\frac{dh_i+k_i}{\delta h_i}} \tag{1.5}$$

This result can be proved on computing the residues at the poles :

$$s_i = \frac{dh_i^{(i)}+k_i}{\delta h_i^{(i)}}, (h_i = 1,\cdots,m_i, k_i = 0,1,2,\cdots) for i = 1,\cdots,v \tag{1.6}$$

We may establish the the asymptotic expansion in the following convenient form :

$$\bar{I}(z_1''',\cdots,z_v''') = 0(\,|z_1'''|^{\alpha_1},\cdots,|z_v'''|^{\alpha_v}\,), max(\,|z_1'''|,\cdots,|z_v'''|\,) \to 0$$

$$\bar{I}(z_1''',\cdots,z_v''') = 0(\,|z_1'''|^{\beta_1},\cdots,|z_v'''|^{\beta_u}\,), min(\,|z_1'''|,\cdots,|z_v'''|\,) \to \infty$$

where $k=1,\cdots,v : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j=1,\cdots,m_k$ and

$$\beta_k = max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,n_k$$

We will note $\eta_{h_i,k_i} = \dfrac{dh_i^{(i)}+k_i}{\delta h_i}$, $(h_i = 1,\cdots,m_i, k_i = 0,1,2,\cdots) for i = 1,\cdots,v$ (1.7)

The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}}^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha_{2j}',\alpha_{2j}'')_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta_{2j}',\beta_{2j}'')_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)})_{1,p_r} : (a_j^{(1)},\alpha_j^{(1)})_{1,p^{(1)}};\cdots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta_{rj}^{(1)},\cdots,\beta_{rj}^{(r)})_{1,q_r} : (b_j^{(1)},\beta_j^{(1)})_{1,q^{(1)}};\cdots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}} \end{array}\right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1, \cdots, s_r) \prod_{i=1}^{r} \theta_i(s_i) z_i^{s_i} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.9}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i \pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)} \right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)} \right) \tag{1.10}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}), max(|z_1|, \cdots, |z_r|) \to 0$$

$$I(z_1, \cdots, z_r) \; = 0(|z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}), min(|z_1|, \cdots, |z_r|) \to \infty$$

where $k = 1, \cdots, r : \alpha_k' = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k' = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

Condider a second multivariable I-function defined by Prasad [1]

$$I(z_1', \cdots, z_s') = I^{0,n_2';0,n_3';\cdots;0,n_r':m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)}}_{p_2',q_2',p_3',q_3';\cdots;p_s',q_s':p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)}} \left(\begin{array}{c|c} \mathrm{z'}_1 & (\mathrm{a'}_{2j}; \alpha_{2j}'^{(1)}, \alpha_{2j}'^{(2)})_{1,p_2'}; \cdots ; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z'}_s & (\mathrm{b'}_{2j}; \beta_{2j}'^{(1)}, \beta_{2j}'^{(2)})_{1,q_2'}; \cdots ; \end{array} \right.$$

$$\left. \begin{array}{c} (\mathrm{a'}_{sj}; \alpha_{sj}'^{(1)}, \cdots, \alpha_{sj}'^{(s)})_{1,p_s'} : (a_j'^{(1)}, \alpha_j'^{(1)})_{1,p'^{(1)}}; \cdots ; (a_j'^{(s)}, \alpha_j'^{(s)})_{1,p'^{(s)}} \\ \\ (\mathrm{b'}_{sj}; \beta_{sj}'^{(1)}, \cdots, \beta_{sj}'^{(s)})_{1,q_s'} : (b_j'^{(1)}, \beta_j'^{(1)})_{1,q'^{(1)}}; \cdots ; (b_j'^{(s)}, \beta_j'^{(s)})_{1,q'^{(s)}} \end{array} \right) \tag{1.11}$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\psi(t_1,\cdots,t_s)\prod_{i=1}^{s}\xi_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.12}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

where $|argz'_i| < \frac{1}{2}\Omega'_i\pi$,

$$\Omega'_i = \sum_{k=1}^{n'^{(i)}}\alpha'^{(i)}_k - \sum_{k=n'^{(i)}+1}^{p'^{(i)}}\alpha'^{(i)}_k + \sum_{k=1}^{m'^{(i)}}\beta'^{(i)}_k - \sum_{k=m^{(i)}+1}^{q'^{(i)}}\beta'^{(i)}_k + \left(\sum_{k=1}^{n'_2}\alpha'^{(i)}_{2k} - \sum_{k=n_2+1}^{p'_2}\alpha'^{(i)}_{2k}\right)$$

$$+\cdots+\left(\sum_{k=1}^{n'_s}\alpha'^{(i)}_{sk} - \sum_{k=n'_s+1}^{p'_s}\alpha'^{(i)}_{sk}\right) - \left(\sum_{k=1}^{q'_2}\beta'^{(i)}_{2k} + \sum_{k=1}^{q'_3}\beta'^{(i)}_{3k} + \cdots + \sum_{k=1}^{q'_s}\beta'^{(i)}_{sk}\right) \tag{1.13}$$

where $i = 1,\cdots,s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z'_1,\cdots,z'_s) = 0(\,|z'_1|^{\alpha'_1},\cdots,|z'_s|^{\alpha'_s}\,)\,,max(\,|z'_1|\,,\cdots,|z'_s|\,) \rightarrow 0$$

$$I(z'_1,\cdots,z'_s)\ = 0(\,|z'_1|^{\beta'_1}\,,\cdots,|z'_s|^{\beta'_s}\,)\,,min(\,|z'_1|\,,\cdots,|z'_s|\,) \rightarrow \infty$$

where $k = 1,\cdots,z : \alpha''_k = min[Re(b'^{(k)}_j/\beta'^{(k)}_j)], j = 1,\cdots,m'_k$ and

$$\beta''_k = max[Re((a'^{(k)}_j - 1)/\alpha'^{(k)}_j)], j = 1,\cdots,n'_k$$

where $k = 1,\cdots,z : \alpha''_k = min[Re(b'^{(k)}_j/\beta'^{(k)}_j)], j = 1,\cdots,m'_k$ and

$$\beta''_k = max[Re((a'^{(k)}_j - 1)/\alpha'^{(k)}_j)], j = 1,\cdots,n'_k$$

Srivastava and Garg [5] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]= \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \tag{1.14}$$

The coefficients are $B[E;R_1,\ldots,R_v]$ arbitrary constants, real or complex.

2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [6 ,page 39 eq .30]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^{P}\Gamma(A_j+s_1+\cdots+s_r)}{\prod_{j=1}^{Q}\Gamma(B_j+s_1+\cdots+s_r)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j+s_1+\cdots+s_r)$ are separated from those of $\Gamma(-s_j), j=1,\cdots,r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j=1,\cdots,r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}\mathrm{d}t\ =(b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}$$

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right) \tag{2.2}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i,\tau_j,h_j\in\mathbb{C},\lambda_j\in\mathbb{R}^+(i=1,\cdots,k;j=1,\cdots,l)$

$$min(Re(\alpha),Re(\beta))>0,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1,\ \max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,$$

and $F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust[3,page 454] given by :

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\prod_{j=1}^{l}\Gamma(\lambda_j)\prod_{j=1}^{k}\Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}}\int_{L_1}\cdots\int_{L_{l+k}}\frac{\Gamma\left(\alpha+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}{\Gamma\left(\alpha+\beta+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}\prod_{j=1}^{l}\Gamma(\lambda_j+s_j)\prod_{j=1}^{k}\Gamma(-\sigma_j+s_{l+j})$$

$$\prod_{j=1}^{l+k}\Gamma(-s_j)z_1^{s_1}\cdots z_l^{s_l}z_{l+1}^{s_{l+1}}\cdots,z_{l+k}^{s_{l+k}}\,\mathrm{d}s_1\cdots\mathrm{d}s_{l+k} \tag{2.3}$$

Here the contour $L_j's$ are defined by $L_j = L_{w\zeta_j\infty}(Re(\zeta_j)=v_j'')$ starting at the point $v_j''-\omega\infty$ and terminating at the point $v_j''+\omega\infty$ with $v_j''\in\mathbb{R}(j=1,\cdots,l)$ and each of the remaining contour $L_{l+1},\cdots,L_{l+k}$ run from $-\omega\infty$ to $\omega\infty$

(2.2) can be easily established by expanding $\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.4}$$

integrating term by term with the help of the integral given by Saigo and Saxena [3, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [5, page 454].

3. Eulerian integral

In this section , we note :

$$\theta_i=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}},\zeta_j^{(i)}>0(i=1,\cdots,r)\,;\,\theta_i'=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'^{(i)}},\zeta_j'^{(i)}>0(i=1,\cdots,s)$$

$$\theta_i''=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j''^{(i)}},\zeta_j''^{(i)}>0(i=1,\cdots,u)$$

$$\theta_i'''=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'''^{(i)}},\zeta_j'''^{(i)}>0(i=1,\cdots,v) \tag{3.1}$$

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};p_2',q_2';p_3',q_3';\cdots;p_{s-1}',q_{s-1}';0,0;\cdots;0,0;0,0;\cdots;0,0 \tag{3.2}$$

$$V=0,n_2;0,n_3;\cdots;0,n_{r-1};0,n_2';0,n_3';\cdots;0,n_{s-1}';0,0;\cdots;0,0;0,0;\cdots;0,0 \tag{3.3}$$

$$X=m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)};m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)};1,0;\cdots;1,0;1,0;\cdots;1,0 \tag{3.4}$$

$$Y=p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)};p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)};0,1;\cdots;0,1;0,1;\cdots;0,1 \tag{3.5}$$

$$A=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)});\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)})\,;(a_{2k}';\alpha_{2k}'^{(1)},\alpha_{2k}'^{(2)});\cdots;$$

$$(a_{(s-1)k}';\alpha_{(s-1)k}'^{(1)},\alpha_{(s-1)k}'^{(2)},\cdots,\alpha_{(s-1)k}'^{(s-1)}) \tag{3.6}$$

$$;(b_{2k}';\beta_{2k}'^{(1)},\beta_{2k}'^{(2)});\cdots;B=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)});\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)})$$

$$(b_{(s-1)k}';\beta_{(s-1)k}'^{(1)},\beta_{(s-1)k}'^{(2)},\cdots,\beta_{(s-1)k}'^{(s-1)}) \tag{3.7}$$

$$A=(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)},0,\cdots,0,0\cdots,0,0,\cdots,0) \tag{3.8}$$

$$A' = (a'_{sk}; 0, \cdots, 0, \alpha'^{(1)}_{sk}, \alpha'^{(2)}_{sk}, \cdots, \alpha'^{(s)}_{sk}, 0, \cdots, 0, 0, \cdots, 0) \tag{3.9}$$

$$B = (b_{rk}; \beta^{(1)}_{rk}, \beta^{(2)}_{rk}, \cdots, \beta^{(r)}_{rk}, 0, \cdots, 0, 0, \cdots, 0, 0, \cdots, 0) \tag{3.10}$$

$$B' = (b'_{sk}; 0, \cdots, 0, \beta'^{(1)}_{sk}, \beta'^{(2)}_{sk}, \cdots, \beta'^{(s)}_{sk}, 0, \cdots, 0, 0, \cdots, 0) \tag{3.11}$$

$$A' = (a^{(1)}_k, \alpha^{(1)}_k)_{1,p^{(1)}}; \cdots; (a^{(r)}_k, \alpha^{(r)}_k)_{1,p^{(r)}}; (a'^{(1)}_k, \alpha^{(1)}_k)_{1,p^{(1)}}; \cdots; (a'^{(s)}_k, \alpha'^{(s)}_k)_{1,p'^{(s)}};$$

$$(1,0); \cdots; (1,0); (1.0); \cdots; (1.0) \tag{3.12}$$

$$B' = (b^{(1)}_k, \beta^{(1)}_k)_{1,q^{(1)}}; \cdots; (b^{(r)}_k, \beta^{(r)}_k)_{1,q^{(r)}}; (b'^{(1)}_k, \beta'^{(1)}_k)_{1,q'^{(1)}}; \cdots; (b'^{(s)}_k, \beta'^{(s)}_k)_{1,q'^{(s)}};$$

$$(0,1); \cdots; (0,1); (0,1); \cdots; (0,1) \tag{3.13}$$

$$K_1 = (1 - \alpha - \sum_{i=1}^{u} R_i a_i - \sum_{i=1}^{v} \eta_{G_i,g_i} a'_i; \mu_1, \cdots, \mu_r, \mu'_1, \cdots, \mu'_s, h_1, \cdots, h_l, 1, \cdots, 1) \tag{3.14}$$

$$K_2 = (1 - \beta - \sum_{i=1}^{u} R_i b_i - \sum_{i=1}^{v} \eta_{G_i,g_i} b'_i; \rho_1, \cdots, \rho_r, \rho'_1, \cdots, \rho'_s, 0, \cdots, 0, 0 \cdots, 0) \tag{3.15}$$

$$K_j = [1 - \lambda_j - \sum_{i=1}^{u} R_i \zeta''^{(i)}_j - \sum_{i=1}^{v} \eta_{G_i,g_i} \zeta'''^{(i)}_j; \zeta^{(1)}_j, \cdots, \zeta^{(r)}_j, \zeta'^{(1)}_j \cdots, \zeta'^{(s)}_j,$$

$$0, \cdots, \underset{\text{j}}{1}, \cdots, 0, 0 \cdots, 0]_{1,l} \tag{3.16}$$

$$K'_j = [1 + \sigma_j - \sum_{i=1}^{u} R_i \lambda''^{(i)}_j - \sum_{i=1}^{v} \eta_{G_i,g_i} \lambda'''^{(i)}_j; \lambda^{(1)}_j, \cdots, \lambda^{(r)}_j, \lambda'^{(1)}_j \cdots, \lambda'^{(s)}_j,$$

$$0, \cdots, 0, 0 \cdots, \underset{\text{j}}{1}, \cdots, 0]_{1,k} \tag{3.17}$$

$$L_1 = (1 - \alpha - \beta - \sum_{i=1}^{u} R_i(a_i + b_i) - \sum_{i=1}^{v} (a'_i + b'_i) \eta_{G_i,g_i}; \mu_1 + \rho_1, \cdots, \mu_r + \rho_r, \mu'_1 + \rho'_1, \cdots, \mu'_r + \rho'_r,$$

$$h_1, \cdots, h_l, 1, \cdots, 1) \tag{3.18}$$

$$L_j = [1 - \lambda_j - \sum_{i=1}^{u} R_i \zeta''^{(i)}_j - \sum_{i=1}^{s} \zeta'''^{(i)}_j \eta_{G_i,g_i}; \zeta^{(1)}_j, \cdots, \zeta^{(r)}_j, \zeta'^{(1)}_j \cdots, \zeta'^{(s)}_j, 0, \cdots, 0, 0 \cdots, 0]_{1,l} \tag{3.19}$$

$$L'_j = [1 + \sigma_j - \sum_{i=1}^{u} R_i \lambda''^{(i)}_j - \sum_{i=1}^{v} \lambda'''^{(i)}_j \eta_{G_i,g_i}; \lambda^{(1)}_j, \cdots, \lambda^{(r)}_j, \lambda'^{(1)}_j \cdots, \lambda'^{(s)}_j, 0, \cdots, 0, 0, \cdots, 0]_{1,k} \tag{3.20}$$

$$P_1=(b-a)^{\alpha+\beta-1}\left\{\prod_{j=1}^{h}(af_j+g_j)^{\sigma_j}\right\} \tag{3.21}$$

$$B_{u,v}=(b-a)^{\sum_{i=1}^{v}(a'_i+b'_i)\eta_{G_i,g_i}+\sum_{i=1}^{u}(a_i+b_i)R_i}\left\{\prod_{j=1}^{h}(af_j+g_j)^{-\sum_{i=1}^{v}\lambda_i'''\eta_{g_i,h_i}-\sum_{i=1}^{u}\lambda_i''R_i}\right\} \tag{3.22}$$

$$B_u=\frac{(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!} \tag{3.23}$$

$$\mathfrak{A}=A,A';\mathfrak{B}=B,B' \tag{3.24}$$

We have the general Eulerian integral.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_u}\left(\begin{array}{c} z_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(u)}} \end{array}\right)$$

$$\bar{I}\left(\begin{array}{c} z_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{array}\right)$$

$$I\left(\begin{array}{c} z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array}\right)$$

$$I\left(\begin{array}{c} z_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t$$

$$=P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{R_k}B_uB_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$I_{U:p_r+p'_s+l+k+2,q_r+q'_s+l+k+1;Y}^{V;0,n_r+n'_s+l+k+2;X}\left(\begin{array}{c|c}\frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathrm{A}\ ;\ \mathrm{K}_1,K_2,K_j,K'_j,\mathfrak{A}:A'\\ \cdots & \cdot\\ \cdots & \cdot\\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot\\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(1)}_j}} & \cdot\\ \cdots & \cdot\\ \cdots & \cdot\\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(s)}_j}} & \cdot\\ \tau_1(b-a)^{h_1} & \cdot\\ \cdots & \cdot\\ \cdots & \cdot\\ \tau_l(b-a)^{h_l} & \cdot\\ \frac{(b-a)f_1}{af_1+g_1} & \mathrm{B}\ ;\ \mathrm{L}_1,L_j,L'_j:D_1,\mathfrak{B}:B'\\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \tag{3.25}$$

We obtain the I-function of $r+s+k+l$ variables. The quantities $U,V,X,Y,A,B,K_1,K_2,K_j,K'_j,\mathfrak{A},\mathfrak{A}',\mathfrak{A}_1,L_1,L_j,L'_j$, $\mathfrak{B},\mathfrak{B}',P_1,B_u,B_{u,v}$ and $\mathfrak{B}_1$ are defined above.

Provided that

(A) $a,b\in\mathbb{R}(a<b);\mu_i,\mu'_u,\rho_i,\rho'_u,\lambda_j^{(i)},\lambda_j'^{(u)},h_v\in\mathbb{R}^+$, $f_i,g_j,\tau_v,\sigma_j,\lambda_v\in\mathbb{C}$ $(i=1,\cdots,r;j=1,\cdots;k;$

$u=1,\cdots,s;v=1,\cdots,l),a_i,b_i,\lambda_j''^{(i)},\zeta_j''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,u;j=1,\cdots,k)$

$a'_i,b'_i,\lambda_j'''^{(i)},\zeta_j'''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,v;j=1,\cdots,k)$

(B) $a_{ij},b_{ik},\in\mathbb{C}\ (i=1,\cdots,r;j=1,\cdots,p_i;k=1,\cdots,q_i);a_j^{(i)},b_j^{(k)}\in\mathbb{C}$

$(i=1,\cdots,r;j=1,\cdots,p^{(i)};k=1,\cdots,q^{(i)})$

$a'_{ij},b'_{ik},\in\mathbb{C}\ (i=1,\cdots,s;j=1,\cdots,p'_i;k=1,\cdots,q'_i);a_j'^{(i)},b_j'^{(k)},\in\mathbb{C}$

$(i=1,\cdots,r;j=1,\cdots,p'^{i)};k=1,\cdots,q'^{(i)})$

$\alpha_{ij}^{(k)},\beta_{ij}^{(k)}\in\mathbb{R}^+$ ($(i=1,\cdots,r,j=1,\cdots,p_i,k=1,\cdots,r)$; $\alpha_j^{(i)},\beta_i^{(i)}\in\mathbb{R}^+\ (i=1,\cdots,r;j=1,\cdots,p_i)$

$\alpha_{ij}'^{(k)},\beta_{ij}'^{(k)}\in\mathbb{R}^+$ ($(i=1,\cdots,s,j=1,\cdots,p'_i,k=1,\cdots,s)$; $\alpha_j'^{(i)},\beta_i'^{(i)}\in\mathbb{R}^+\ (i=1,\cdots,s;j=1,\cdots,p'_i)$

(C) $\max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1$

(D) $Re[\alpha+\sum_{j=1}^{v}a'_j\min_{1\leqslant k\leqslant M_i}\frac{d_k^{(j)}}{\delta_k^{(j)}}+\sum_{j=1}^{r}\mu_j\min_{1\leqslant k\leqslant m^{(i)}}\frac{b_k^{(j)}}{\beta_k^{(j)}}+\sum_{j=1}^{s}\mu'_i\min_{1\leqslant k\leqslant m'^{(i)}}\frac{b_k'^{(j)}}{\beta_k'^{(j)}}]>0$

$$Re[\beta+\sum_{j=1}^{v}b'_j\min_{1\leqslant k\leqslant M_i}\frac{d_k^{(j)}}{\delta_k^{(j)}}+\sum_{j=1}^{r}\rho_j\min_{1\leqslant k\leqslant m^{(i)}}\frac{b_k^{(j)}}{\beta_k^{(j)}}+\sum_{j=1}^{s}\rho'_j\min_{1\leqslant k\leqslant m'^{(i)}}\frac{b_k'^{(j)}}{\beta_k'^{(j)}}]>0$$

(E) $Re\left(\alpha+\sum_{i=1}^{v}\eta_{G_i,g_i}a'_i+\sum_{i=1}^{u}R_ia_i+\sum_{i=1}^{r}\mu_is_i+\sum_{i=1}^{s}t_i\mu'_i\right)>0$

$$Re\left(\beta+\sum_{i=1}^{v}\eta_{G_i,g_i}b'_i+\sum_{i=1}^{u}R_ib_i+\sum_{i=1}^{r}v_is_i+\sum_{i=1}^{s}t_i\rho'_i\right)>0$$

$$Re\left(\lambda_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda_j'''^{(i)}+\sum_{i=1}^{u}R_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\zeta_j^{(i)}+\sum_{i=1}^{s}t_i\zeta_j'^{(i)}\right)>0(j=1,\cdots,l);$$

$$Re\left(-\sigma_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}+\sum_{i=1}^{u}R_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\lambda_j^{(i)}+\sum_{i=1}^{s}t_i\lambda_j'^{(i)}\right)>0(j=1,\cdots,k);$$

(F) $\Omega_i=\sum_{k=1}^{n^{(i)}}\alpha_k^{(i)}-\sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)}+\sum_{k=1}^{m^{(i)}}\beta_k^{(i)}-\sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)}+\left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)}-\sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right)+\cdots+$

$$\left(\sum_{k=1}^{n_s}\alpha_{sk}^{(i)}-\sum_{k=n_s+1}^{p_s}\alpha_{sk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_s}\beta_{sk}^{(i)}\right)-\mu_i-\rho_i$$

$$-\sum_{l=1}^{k}\lambda_j^{(i)}-\sum_{l=1}^{l}\zeta_j^{(i)}>0\quad(i=1,\cdots,r)$$

$$\Omega'_i=\sum_{k=1}^{n'^{(i)}}{\alpha'_k}^{(i)}-\sum_{k=n'^{(i)}+1}^{p'^{(i)}}{\alpha'_k}^{(i)}+\sum_{k=1}^{m'^{(i)}}{\beta'_k}^{(i)}-\sum_{k=m^{(i)}+1}^{q'^{(i)}}{\beta'_k}^{(i)}+\left(\sum_{k=1}^{n'_2}{\alpha'_{2k}}^{(i)}-\sum_{k=n_2+1}^{p'_2}{\alpha'_{2k}}^{(i)}\right)+$$

$$\cdots+\left(\sum_{k=1}^{n'_s}{\alpha'_{sk}}^{(i)}-\sum_{k=n'_s+1}^{p'_s}{\alpha'_{sk}}^{(i)}\right)-\left(\sum_{k=1}^{q'_2}{\beta'_{2k}}^{(i)}+\sum_{k=1}^{q'_3}{\beta'_{3k}}^{(i)}+\cdots+\sum_{k=1}^{q'_s}{\beta'_{sk}}^{(i)}\right)-\mu'_i-\rho'_i$$

$$-\sum_{l=1}^{k}{\lambda'_j}^{(i)}-\sum_{l=1}^{l}{\zeta'_j}^{(i)}>0\quad(i=1,\cdots,s)$$

(G) $\left|arg\left(z_i\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(i)}}\right)\right|<\frac{1}{2}\Omega_i\pi\ \ (a\leqslant t\leqslant b;i=1,\cdots,r)$

$$\left|arg\left(z'_i\prod_{j=1}^{l}\left[1-\tau'_j(t-a)^{h'_i}\right]^{-\zeta_j'^{(i)}}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(i)}}\right)\right|<\frac{1}{2}\Omega'_i\pi\quad(a\leqslant t\leqslant b;i=1,\cdots,s)$$

(H) (I) The multiple series occuring on the right-hand side of (3.25) is absolutely and uniformly convergent.

Proof

To prove (3.24), first, we express in serie the multivariable I-function defined by Nambisan et al [2] with the help of (1.5), a class of multivariable polynomials defined by Srivastava $S_L^{h_1,\cdots,h_u}[.]$ in serie with the help of (1.14), the I-functions of r-variables and s-variables defined by Prasad [1] in terms of Mellin-Barnes type contour integral with the help of (1.8) and (1.11) respectively. Now collect the power of $\left[1-\tau_j(t-a)^{h_i}\right]$ with $(i=1,\cdots,r;j=1,\cdots,l)$ and collect the power of (f_jt+g_j) with $j=1,\cdots,k$. Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the $(r+s+k+l)$ dimensional Mellin-Barnes integral in multivariable I-function defined by Prasad [1], we obtain the equation (3.24).

Remarks

If a) $\rho_1=\cdots,\rho_r=\rho'_1=\cdots,\rho'_s=0$; b) $\mu_1=\cdots,\mu_r=\mu'_1=\cdots,\mu'_s=0$, we obtain the similar formulas that (3.25) with the corresponding simplifications.

4. Particular cases

a) If $U=V=A=B=0$, **the multivariable I-function defined by Prasad reduces to multivariable H-function defined by Srivastava et al [7] and we obtain :**

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_u}\begin{pmatrix} \mathrm{z}''_1\theta''_1(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda''^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}''_u\theta''_u(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^k(f_jt+g_j)^{-\lambda''^{(u)}_j} \end{pmatrix}$$

$$\bar{I}\begin{pmatrix} \mathrm{z}'''_1\theta'''_1(t-a)^{a'_1}(b-t)^{b'_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda'''^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'''_v\theta'''_v(t-a)^{a'_v}(b-t)^{b'_v}\prod_{j=1}^k(f_jt+g_j)^{-\lambda'''^{(v)}_j} \end{pmatrix}$$

$$H\begin{pmatrix} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^k(f_jt+g_j)^{-\lambda^{(r)}_j} \end{pmatrix}$$

$$H\left(\begin{array}{c} z'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ z'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'^{(s)}_j} \end{array}\right)\mathrm{d}t$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{R_k}B_uB_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$H^{0,n_r+n'_s+l+k+2;X}_{p_r+p'_s+l+k+2,q_r+q'_s+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \\ \cdots & \mathrm{K}_1, K_2, K_j, K'_j, \mathfrak{A} : A' \\ \cdots & \cdot \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(1)}_j}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(s)}_j}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \tau_l(b-a)^{h_l} & \cdot \\ \frac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdots & \mathrm{L}_1, L_j, L'_j : D_1, \mathfrak{B} : B' \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (4.1)$$

under the same notations and conditions that (3.25) with $U = V = A = B = 0$

b) If $B(L; R_1, \cdots, R_u) = \dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_u\theta_j^{(u)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi'_j+\cdots+m_u\psi_j^{(u)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}}$ (4.2)

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [4]. We have

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$F^{1+\bar{A}:B';\cdots;B^{(u)}}_{\bar{C}:D';\cdots;D^{(u)}}\left(\begin{array}{c} \mathrm{z}_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(u)}} \end{array}\right|$$

$$\left.\begin{array}{c}[(\text{-L});\mathrm{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}]:[(b');\phi'];\cdots;[(b^{(u)});\phi^{(u)}] \\ [(\mathrm{c});\psi',\cdots,\psi^{(u)}]:[(d');\delta'];\cdots;[(d^{(u)});\delta^{(u)}]\end{array}\right)$$

$$\bar{I}\left(\begin{array}{c} \mathrm{z}_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t$$

$$=P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{R_k}B_u'B_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$I_{U:p_r+p'_s+l+k+2,q_r+q'_s+l+k+1;Y}^{V;0,n_r+n'_s+l+k+2;X}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathrm{A}\ ;\ \mathrm{K}_1,K_2,K_j,K'_j,\mathfrak{A}:A' \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(1)}_j}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(s)}_j}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \tau_l(b-a)^{h_l} & \cdot \\ \frac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdots & \mathrm{B}\ ;\ \mathrm{L}_1,L_j,L'_j:D_1,\mathfrak{B}:B' \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \tag{4.3}$$

under the same conditions and notations that (3.25)

where $B'_u = \dfrac{(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!}$, $B[E;R_1,\ldots,R_v]$ is defined by (4.2)

Remark:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable I-functions defined by Prasad [1] and a class of multivariable polynomials defined by Srivastava et al [5].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable I-functions defined by Prasad [1],a expansion of multivariable I-function defined by Nambisan et al [2] and a class of multivariable polynomials defined by Srivastava et al [4] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

REFERENCES

[1] Y.N. Prasad , Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , page 231-237.

[2] Prathima J. Nambisan V. and Kurumujji S.K. A Study of I-function of Several Complex Variables, International Journalof Engineering Mathematics Vol(2014) , 2014 page 1-12

[3] Saigo M. and Saxena R.K. Unified fractional integral formulas for the multivariable H-function I. J.Fractional Calculus 15 (1999), page 91-107.

[4] Srivastava H.M. and Daoust M.C. Certain generalized Neumann expansions associated with Kampé de Fériet function. Nederl. Akad. Wetensch. Proc. Ser A72 = Indag Math 31(1969) page 449-457.

[5] Srivastava H.M. And Garg M. Some integral involving a general class of polynomials and multivariable H-function. Rev. Roumaine Phys. 32(1987), page 685-692.

[6] Srivastava H.M. and Karlsson P.W. Multiple Gaussian Hypergeometric series. Ellis.Horwood. Limited. New-York, Chichester. Brisbane. Toronto , 1985.

[7] H.M. Srivastava and R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.

Personal adress : 411 Avenue Joseph Raynaud
Le parc Fleuri , Bat B
83140 , Six-Fours les plages
Tel : 06-83-12-49-68
Department : VAR
Country : FRANCE