Eulerian integral associated with product of two multivariable I-functions, generalized Lauricella function and a class of polynomials and expansion of multivariable I-function II

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ABSTRACT
The present paper is evaluated a new Eulerian integral associated with the product of two multivariable I-functions defined by Nambisan et al [2] a generalized Lauricella function , a class of multivariable polynomials and a expansion of multivariable I-function defined by Nambisan et al [2] with general arguments . We will study the case concerning the multivariable H-function defined by Srivastava et al [7] an d the Srivastava-Doust polynomial.

Keywords: Eulerian integral, multivariable I-function, generalized Lauricella function of several variables, multivariable H-function, generalized hypergeometric function, class of polynomials

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1. Introduction

In this paper, we evaluate a new Eulerian integral of most general characters associated with the products of two multivariable I-functions defined by nambisan et al [2] , a serie expansion of multivariable I-function defined by Nambisan et al [2] and a class of polynomials with general arguments.
First time, we define the multivariable I-function by :

$$\bar{I}(z_1''',\cdots,z_v''') = \bar{I}^{0,N:M_1,N_1;\cdots;M_v,N_v}_{P,Q:P_1,Q_1;\cdots;P_v,Q_v}\left(\begin{array}{c|c} \mathrm{z}_1''' & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(v)};A_j)_{N+1,P}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_v''' & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(v)};B_j)_{M+1,Q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};1)_{1,N_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{N_1+1,P_1};\cdots;(c_j^{(v)},\gamma_j^{(v)};1)_{1,N_u},(c_j^{(v)},\gamma_j^{(v)};C_j^{(v)})_{N_v+1,P_v} \\ (\ \bar{d}_j^{(1)},\bar{\delta}_j^{(1)};1)_{1,M_1},(\bar{d}_j^{(1)},\bar{\delta}_j^{(1)};D_j^{(1)})_{M_1+1,Q_1};\cdots;(\bar{d}_j^{(v)},\bar{\delta}_j^{(v)};1)_{1,M_v},(\bar{d}_j^{(v)},\bar{\delta}_j^{(v)};D_j^{(v)})_{M_v+1,Q_v} \end{array}\right) \tag{1.1}$$

$$=\frac{1}{(2\pi\omega)^v}\int_{L_1}\cdots\int_{L_v}\phi_1(s_1,\cdots,s_v)\prod_{i=1}^{v}\xi_i'(s_i)z_i'''^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_v \tag{1.2}$$

where $\phi_1(s_1,\cdots,s_v),\xi_i'(s_i), i=1,\cdots,v$ are given by :

$$\phi_1(s_1,\cdots,s_v)=\frac{1}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{v}\alpha_j^{(i)}s_j\right)\prod_{j=M+1}^{Q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{v}\beta_j^{(i)}s_j\right)} \tag{1.3}$$

$$\xi_i'(s_i) = \frac{\prod_{j=1}^{N_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{M_i}\Gamma\left(\bar{d}_j^{(i)}-\bar{\delta}_j^{(i)}s_i\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{D_j^{(i)}}\left(1-\bar{d}_j^{(i)}+\bar{\delta}_j^{(i)}s_i\right)} \tag{1.4}$$

$i = 1, \cdots, v$

Serie representation

If $z_i''' \neq 0; i = 1, \cdots, v$

$\delta_{h_i}^{(i)}(d_j^{(i)}+k_i) \neq \delta_j^{(i)}(\delta_{h_i}^{(i)}+\eta_i) for j \neq h_i, j, h_i = 1, \cdots, m_i (i = 1, \cdots, v), k_i, \eta_i = 0, 1, 2, \cdots (i = 1, \cdots, v)$, then

$$\bar{I}(z_1''', \cdots, z_v''') = \sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\left[\phi_1\left(\frac{dh_1^{(1)}+k_1}{\delta h_1^{(1)}}, \cdots, \frac{dh_v^{(v)}+k_v}{\delta h_v^{(v)}}\right)\right]\prod_{j\neq h_i i=1}^{r}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\frac{dh_i+k_i}{\delta h_i}} \tag{1.5}$$

This result can be proved on computing the residues at the poles :

$$s_i = \frac{dh_i^{(i)}+k_i}{\delta h_i^{(i)}}, (h_i = 1, \cdots, m_i, k_i = 0, 1, 2, \cdots) for i = 1, \cdots, v \tag{1.6}$$

We may establish the the asymptotic expansion in the following convenient form :

$$\bar{I}(z_1''', \cdots, z_v''') = 0(\,|z_1'''|^{\alpha_1}, \cdots, |z_v'''|^{\alpha_v}\,), max(\,|z_1'''|, \cdots, |z_v'''|\,) \to 0$$

$$\bar{I}(z_1''', \cdots, z_v''') = 0(\,|z_1'''|^{\beta_1}, \cdots, |z_v'''|^{\beta_u}\,), min(\,|z_1'''|, \cdots, |z_v'''|\,) \to \infty$$

where $k = 1, \cdots, v : \alpha_k = min[Re(\bar{d}_j^{(k)}/\bar{\delta}_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will note $\eta_{h_i,k_i} = \dfrac{dh_i^{(i)}+k_i}{\delta h_i}$ $, (h_i = 1, \cdots, m_i, k_i = 0, 1, 2, \cdots) for i = 1, \cdots, v$ (1.7)

The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1, \cdots, z_r) = I_{p,q:p_1,q_1;\cdots;p_r,q_r}^{0,n:m_1,n_1;\cdots;m_r,n_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,p} : \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,q} : \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r} \\ \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r} \end{array}\right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.9}$$

where $\phi(s_1,\cdots,s_r),\theta_i(s_i), i=1,\cdots,r$ are given by :

$$\phi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{n}\Gamma^{A_j}\left(1-aj+\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=1}^{q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \tag{1.10}$$

$$\theta_i(s_i)=\frac{\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)} \tag{1.11}$$

For more details, see Nambisan et al [2].

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U_i=\sum_{j=1}^{p}A_j\alpha_j^{(i)}-\sum_{j=1}^{q}B_j\beta_j^{(i)}+\sum_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=1}^{q_i}D_j^{(i)}\delta_j^{(i)}\leqslant 0, i=1,\cdots,r \tag{1.12}$$

The integral (2.1) converges absolutely if

$|arg(z_k)|<\frac{1}{2}\Delta_k\pi, k=1,\cdots,r$ where

$$\Delta_k=-\sum_{j=n+1}^{p}A_j\alpha_j^{(k)}-\sum_{j=1}^{q}B_j\beta_j^{(k)}+\sum_{j=1}^{m_k}D_j^{(k)}\delta_j^{(k)}-\sum_{j=m_k+1}^{q_k}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n_k}C_j^{(k)}\gamma_j^{(k)}-\sum_{j=n_k+1}^{p_k}C_j^{(k)}\gamma_j^{(k)}>0 \tag{1.13}$$

Consider the second multivariable I-function.

$$I(z'_1,\cdots,z'_s)=I_{p',q':p'_1,q'_1;\cdots;p'_s,q'_s}^{0,n':m'_1,n'_1;\cdots;m'_s,n'_s}\left(\begin{array}{c|c} z'_1 & (a'_j;\alpha'^{(1)}_j,\cdots,\alpha'^{(s)}_j;A'_j)_{1,p'}: \\ \cdot & \\ \cdot & \\ \cdot & \\ z'_s & (b'_j;\beta'^{(1)}_j,\cdots,\beta'^{(s)}_j;B'_j)_{1,q'}: \end{array}\right.$$

$$\left.\begin{array}{c} (c'^{(1)}_j,\gamma'^{(1)}_j;C'^{(1)}_j)_{1,p'_1};\cdots;(c'^{(s)}_j,\gamma'^{(s)}_j;C'^{(s)}_j)_{1,p'_s} \\ \\ (d'^{(1)}_j,\delta'^{(1)}_j;D'^{(1)}_j)_{1,q'_1};\cdots;(d'^{(s)}_j,\delta'^{(s)}_j;D'^{(s)}_j)_{1,q'_s} \end{array}\right) \tag{1.14}$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L'_1}\cdots\int_{L'_s}\psi(t_1,\cdots,t_s)\prod_{i=1}^{s}\xi_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.15}$$

where $\psi(t_1,\cdots,t_s),\xi_i(s_i), i=1,\cdots,s$ are given by :

$$\psi(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{n'}\Gamma^{A'_j}(1-a'_j+\sum_{i=1}^{s}\alpha'^{(i)}_jt_j)}{\prod_{j=n'+1}^{p'}\Gamma^{A'_j}(a'_j-\sum_{i=1}^{s}\alpha'^{(i)}_jt_j)\prod_{j=1}^{q'}\Gamma^{B'_j}(1-b'_j+\sum_{i=1}^{s}\beta'^{(i)}_jt_j)} \tag{1.16}$$

$$\xi_i(s_i) = \frac{\prod_{j=1}^{n'_i}\Gamma^{C'^{(i)}_j}\left(1-c'^{(i)}_j+\gamma'^{(i)}_j t_i\right)\prod_{j=1}^{m'_i}\Gamma^{D'^{(i)}_j}\left(d'^{(i)}_j-\delta'^{(i)}_j t_i\right)}{\prod_{j=n'_i+1}^{p'_i}\Gamma^{C'^{(i)}_j}\left(c'^{(i)}_j-\gamma'^{(i)}_j t_i\right)\prod_{j=m'_i+1}^{q'_i}\Gamma^{D'^{(i)}_j}\left(1-d'^{(i)}_j+\delta'^{(i)}_j t_i\right)} \tag{1.17}$$

For more details, see Nambisan et al [2].

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U'_i = \sum_{j=1}^{p'} A'_j\alpha'^{(i)}_j - \sum_{j=1}^{q'} B'_j\beta'^{(i)}_j + \sum_{j=1}^{p'_i} C'^{(i)}_j\gamma'^{(i)}_j - \sum_{j=1}^{q'_i} D'^{(i)}_j\delta'^{(i)}_j \leqslant 0, i=1,\cdots,s \tag{1.18}$$

The integral (2.1) converges absolutely if

where $|arg(z'_k)| < \frac{1}{2}\Delta'_k\pi, k=1,\cdots,s$

$$\Delta'_k = -\sum_{j=n'+1}^{p'} A'_j\alpha'^{(k)}_j - \sum_{j=1}^{q'} B'_j\beta'^{(k)}_j + \sum_{j=1}^{m'_k} D'^{(k)}_j\delta'^{(k)}_j - \sum_{j=m'_k+1}^{q'_k} D'^{(k)}_j\delta'^{(k)}_j + \sum_{j=1}^{n'_k} C'^{(k)}_j\gamma'^{(k)}_j - \sum_{j=n'_k+1}^{p'_k} C'^{(k)}_j\gamma'^{(k)}_j > 0 \tag{1.19}$$

Srivastava and Garg [5] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u] = \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L} (-L)_{h_1R_1+\cdots+h_uR_u} B(E;R_1,\cdots,R_u)\frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \tag{1.20}$$

The coefficients are $B[E;R_1,\ldots,R_v]$ arbitrary constants, real or complex.

2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [6 ,page 39 eq .30]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^{P}\Gamma(A_j+s_1+\cdots+s_r)}{\prod_{j=1}^{Q}\Gamma(B_j+s_1+\cdots+s_r)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j+s_1+\cdots+s_r)$ are separated from those of $\Gamma(-s_j), j=1,\cdots,r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j=1,\cdots,r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}$$

$$F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right) \tag{2.2}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i,\tau_j,h_j\in\mathbb{C},\lambda_j\in\mathbb{R}^+(i=1,\cdots,k;j=1,\cdots,l)$

$$min(Re(\alpha),Re(\beta))>0,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1,\ \max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,$$

and $F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust[3,page 454] given by :

$$F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\prod_{j=1}^{l}\Gamma(\lambda_j)\prod_{j=1}^{k}\Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}}\int_{L_1}\cdots\int_{L_{l+k}}\frac{\Gamma\left(\alpha+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}{\Gamma\left(\alpha+\beta+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}\prod_{j=1}^{l}\Gamma(\lambda_j+s_j)\prod_{j=1}^{k}\Gamma(-\sigma_j+s_{l+j})$$

$$\prod_{j=1}^{l+k}\Gamma(-s_j)z_1^{s_1}\cdots z_l^{s_l}z_{l+1}^{s_{l+1}}\cdots,z_{l+k}^{s_{l+k}}\,\mathrm{d}s_1\cdots\mathrm{d}s_{l+k} \tag{2.3}$$

Here the contour $L_j's$ are defined by $L_j=L_{w\zeta_j\infty}(Re(\zeta_j)=v_j'')$ starting at the point $v_j''-\omega\infty$ and terminating at the point $v_j''+\omega\infty$ with $v_j''\in\mathbb{R}(j=1,\cdots,l)$ and each of the remaining contour $L_{l+1},\cdots,L_{l+k}$ run from $-\omega\infty$ to $\omega\infty$

(2.2) can be easily established by expanding $\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.4}$$

integrating term by term with the help of the integral given by Saigo and Saxena [3, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [5, page 454].

3. Eulerian integral

In this section , we note :

$$\theta_i=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}},\zeta_j^{(i)}>0(i=1,\cdots,r)\,;\,\theta_i'=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'^{(i)}},\zeta_j'^{(i)}>0(i=1,\cdots,s)$$

$$\theta_i'' = \prod_{j=1}^{l} \left[1 - \tau_j (t-a)^{h_i}\right]^{-\zeta_j''^{(i)}}, \zeta_j''^{(i)} > 0 (i = 1, \cdots, u)$$

$$\theta_i''' = \prod_{j=1}^{l} \left[1 - \tau_j (t-a)^{h_i}\right]^{-\zeta_j'''^{(i)}}, \zeta_j'''^{(i)} > 0 (i = 1, \cdots, v) \tag{3.1}$$

$$X = m_1, n_1; \cdots; m_r, n_r; m_1', n_1'; \cdots; m_s', n_s'; 1, 0; \cdots; 1, 0; 1, 0; \cdots; 1, 0 \tag{3.2}$$

$$Y = p_1, q_1; \cdots; p_r, q_r; p_1', q_1'; \cdots; p_s', q_s'; 0, 1; \cdots; 0, 1; 0, 1; \cdots; 0, 1 \tag{3.3}$$

$$A = (a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}, 0, \cdots, 0, 0, \cdots, 0, 0, \cdots, 0; A_j)_{1,p} \tag{3.4}$$

$$B = (b_j; \beta_j^{(1)}, \cdots, \beta_j^{(r)}, 0 \cdots, 0, 0 \cdots, 0, 0 \cdots, 0; B_j)_{1,q} \tag{3.5}$$

$$A' = (a_j'; 0, \cdots, 0, \alpha_j'^{(1)}, \cdots, \alpha_j'^{(s)}, 0, \cdots, 0, 0, \cdots, 0; A_j')_{1,p'} \tag{3.6}$$

$$B' = (b_j'; 0, \cdots, 0, \beta_j'^{(1)}, \cdots, \beta_j'^{(s)}, 0, \cdots, 0, 0, \cdots, 0; B_j')_{1,q'} \tag{3.7}$$

$$\mathrm{C} = (\mathrm{c}_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \cdots; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,p_r}; (c_j'^{(1)}, \gamma_j'^{(1)}; C_j'^{(1)})_{1,p_1'}; \cdots; (c_j'^{(r)}, \gamma_j^{(s)}; C_j'^{(s)})_{1,p_s'}$$

$$(1, 0; 1); \cdots; (1, 0; 1); (1, 0; 1); \cdots; (1, 0; 1) \tag{3.8}$$

$$D = (\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \cdots; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r}; (\mathrm{d'}_j^{(1)}, \delta_j'^{(1)}; D_j'^{(1)})_{1,q_1'}; \cdots; (d_j'^{(s)}, \delta_j'^{(s)}; D_j'^{(s)})_{1,q_s'};$$

$$(0, 1; 1); \cdots; (0, 1; 1); (0, 1; 1); \cdots; (0, 1; 1) \tag{3.9}$$

$$K_1 = (1 - \alpha - \sum_{i=1}^{u} R_i a_i - \sum_{i=1}^{v} \eta_{G_i,g_i} a_i'; \mu_1, \cdots, \mu_r, \mu_1', \cdots, \mu_s', h_1, \cdots, h_l, 1, \cdots, 1; 1) \tag{3.10}$$

$$K_2 = (1 - \beta - \sum_{i=1}^{u} R_i b_i - \sum_{i=1}^{v} \eta_{G_i,g_i} b_i'; \rho_1, \cdots, \rho_r, \rho_1', \cdots, \rho_s', 0, \cdots, 0, 0 \cdots, 0; 1) \tag{3.15}$$

$$K_j = [1 - \lambda_j - \sum_{i=1}^{u} R_i \zeta_j''^{(i)} - \sum_{i=1}^{v} \eta_{G_i,g_i} \zeta_j'''^{(i)}; \zeta_j^{(1)}, \cdots, \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots, \zeta_j'^{(s)},$$

$$0, \cdots, \underset{\mathrm{j}}{1}, \cdots, 0, 0 \cdots, 0; 1]_{1,l} \tag{3.16}$$

$$K_j' = [1 + \sigma_j - \sum_{i=1}^{u} R_i \lambda_j''^{(i)} - \sum_{i=1}^{v} \eta_{G_i,g_i} \lambda_j'''^{(i)}; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)},$$

$$0, \cdots, 0, 0 \cdots, \underset{\mathrm{j}}{1}, \cdots, 0; 1]_{1,k} \tag{3.17}$$

$$L_1 = (1 - \alpha - \beta - \sum_{i=1}^{u} R_i (a_i + b_i) - \sum_{i=1}^{v} (a_i' + b_i') \eta_{G_i,g_i}; \mu_1 + \rho_1, \cdots, \mu_r + \rho_r, \mu_1' + \rho_1', \cdots, \mu_r' + \rho_r',$$

$$h_1, \cdots, h_l, 1, \cdots, 1; 1) \tag{3.18}$$

$$L_j = [1 - \lambda_j - \sum_{i=1}^{u} R_i \zeta_j''^{(i)} - \sum_{i=1}^{s} \zeta_j'''^{(i)} \eta_{G_i,g_i}; \zeta_j^{(1)}, \cdots, \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots, \zeta_j'^{(s)}, 0, \cdots, 0, 0 \cdots, 0; 1]_{1,l} \tag{3.19}$$

$$L'_j=[1+\sigma_j-\sum_{i=1}^{u}R_i\lambda''^{(i)}_j-\sum_{i=1}^{v}\lambda'''^{(i)}_j\eta_{G_i,g_i};\lambda_j^{(1)},\cdots,\lambda_j^{(r)},\lambda'^{(1)}_j\cdots,\lambda'^{(s)}_j,0,\cdots,0,0,\cdots,0;1]_{1,k} \tag{3.20}$$

$$P_1=(b-a)^{\alpha+\beta-1}\left\{\prod_{j=1}^{h}(af_j+g_j)^{\sigma_j}\right\} \tag{3.21}$$

$$B_{u,v}=(b-a)^{\sum_{i=1}^{v}(a'_i+b'_i)\eta_{G_i,g_i}+\sum_{i=1}^{u}(a_i+b_i)R_i}\left\{\prod_{j=1}^{h}(af_j+g_j)^{-\sum_{i=1}^{v}\lambda'''_i\eta_{g_i,h_i}-\sum_{i=1}^{u}\lambda''_iR_i}\right\} \tag{3.22}$$

$$B_u=\frac{(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!} \tag{3.23}$$

$$\mathfrak{A}=A,A';\mathfrak{B}=B,B' \tag{3.24}$$

We have the general Eulerian integral.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_u}\left(\begin{array}{c} \mathrm{z}''_1\theta''_1(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda''^{(1)}_j}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}''_u\theta''_u(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda''^{(u)}_j}\end{array}\right)$$

$$\bar{I}\left(\begin{array}{c} \mathrm{z}'''_1\theta'''_1(t-a)^{a'_1}(b-t)^{b'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'''^{(1)}_j}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}'''_v\theta'''_v(t-a)^{a'_v}(b-t)^{b'_v}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'''^{(v)}_j}\end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda^{(1)}_j}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda^{(r)}_j}\end{array}\right)$$

$$I\left(\begin{array}{c} z_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{K_k}B_uB_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$I_{p+p'+l+k+2,q+q'+l+k+1;Y}^{0,n+n'+l+k+2;X}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \\ \cdot\,\cdot\,\cdot & \mathfrak{A}\ ,\ \mathrm{K}_1,K_2,K_j,K_j':C \\ \cdot\,\cdot\,\cdot & \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & \cdot \\ \cdot\,\cdot\,\cdot & \cdot \\ \cdot\,\cdot\,\cdot & \cdot \\ \frac{z_s'(b-a)^{\mu_s'+\rho_s'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdot\,\cdot\,\cdot & \cdot \\ \cdot\,\cdot\,\cdot & \\ \tau_l(b-a)^{h_l} & \\ \frac{(b-a)f_1}{af_1+g_1} & \mathfrak{B}\ ,\ \mathrm{L}_1,L_j,L_j':D \\ \cdot\,\cdot\,\cdot & \\ \cdot\,\cdot\,\cdot & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \tag{3.25}$$

where $\mathfrak{A},\mathfrak{B},C,D,X,K_1,K_2,K_j,K_j',L_1,L_j,L_j'$, $\mathfrak{B},\mathfrak{B}',P_1,P_u,B_u$ and $\mathfrak{B}_1$ are defined above.

Provided that

(A) $a,b\in\mathbb{R}(a<b);\mu_i,\mu_u',\rho_i,\rho_u',\lambda_j^{(i)},\lambda_j'^{(u)},h_v\in\mathbb{R}^+$, $f_i,g_j,\tau_v,\sigma_j,\lambda_v\in\mathbb{C}$ $(i=1,\cdots,r;j=1,\cdots;k;$

$u=1,\cdots,s;v=1,\cdots,l),a_i,b_i,\lambda_j''^{(i)},\zeta_j''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,u;j=1,\cdots,k)$

$a_i',b_i',\lambda_j'''^{(i)},\zeta_j'''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,v;j=1,\cdots,k)$

(B) $m_j,n_j,p_j,q_j(j=1,\cdots,r),n,p,q\in\mathbb{N}^*$;$\delta_j^{(i)}\in\mathbb{R}_+(j=1,\cdots,q_i;i=1,\cdots,r)$

$\alpha_j^{(i)}\in\mathbb{R}_+(j=1,\cdots,p;i=1,\cdots,r),\beta_j^{(i)}\in\mathbb{R}_+(j=1,\cdots,q;i=1,\cdots,r),\gamma_j^{(i)}\in\mathbb{R}_+(j=1,\cdots,p_i;i=1,\cdots,r)$

$a_j(j=1,\cdots,p),b_j(j=1,\cdots,q),c_j^{(i)}(j=1,\cdots,p_i,i=1,\cdots,r),d_j^{(i)}(j=1,\cdots,q_i,i=1,\cdots,r)\in\mathbb{C}$

The exposants $A_j(j=1,\cdots,p), B_j(j=1,\cdots,q), C_j^{(i)}(j=1,\cdots,p_i;i=1,\cdots,r), D_j^{(i)}(j=1,\cdots,q_i;i=1,\cdots,r)$

of various gamma function involved in (1.3) and (1.4) may take non integer values.

$m'_j,n'_j,p'_j,q'_j(j=1,\cdots,s),n',p',q'\in\mathbb{N}^*$;$\delta_j'^{(i)}\in\mathbb{R}_+(j=1,\cdots,q'_i;i=1,\cdots,s)$

$\alpha_j'^{(i)}\in\mathbb{R}_+(j=1,\cdots,p';i=1,\cdots,s),\beta_j'^{(i)}\in\mathbb{R}_+(j=1,\cdots,q';i=1,\cdots,r),\gamma_j'^{(i)}\in\mathbb{R}_+(j=1,\cdots,p'_i;i=1,\cdots,s)$

$a'_j(j=1,\cdots,p'),b'_j(j=1,\cdots,q'),c_j'^{(i)}(j=1,\cdots,p'_i,i=1,\cdots,s),d_j'^{(i)}(j=1,\cdots,q'_i,i=1,\cdots,s)\in\mathbb{C}$

The exposants

$A'_j(j=1,\cdots,p'),B'_j(j=1,\cdots,q'),C_j'^{(i)}(j=1,\cdots,p'_i;i=1,\cdots,s),D_j'^{(i)}(j=1,\cdots,q'_i;i=1,\cdots,s)$

of various gamma function involved in (1.9) and (1.10) may take non integer values.

(C) $\max\limits_{1\leqslant j\leqslant k}\left\{\left|\dfrac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,\ \max\limits_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1$

(D) $Re\big[\alpha+\sum\limits_{j=1}^{v}a'_j\min\limits_{1\leqslant k\leqslant M_i}\dfrac{\bar{d}_k^{(j)}}{\bar{\delta}_k^{(j)}}+\sum\limits_{j=1}^{r}\mu_j\min\limits_{1\leqslant k\leqslant m_i}\dfrac{d_k^{(j)}}{\delta_k^{(j)}}+\sum\limits_{j=1}^{s}\mu'_i\min\limits_{1\leqslant k\leqslant m'_i}\dfrac{d_k'^{(j)}}{\delta_k'^{(j)}}\big]>0$

$$Re\big[\beta+\sum_{j=1}^{v}b'_j\min_{1\leqslant k\leqslant M_i}\frac{\bar{d}_k^{(j)}}{\bar{\delta}_k^{(j)}}+\sum_{j=1}^{r}\rho_j\min_{1\leqslant k\leqslant m_i}\frac{d_k^{(j)}}{\delta_k^{(j)}}+\sum_{j=1}^{s}\rho'_j\min_{1\leqslant k\leqslant m'_i}\frac{d_k'^{(j)}}{\delta_k'^{(j)}}\big]>0$$

(E) $Re\left(\alpha+\sum\limits_{i=1}^{v}\eta_{G_i,g_i}a'_i+\sum\limits_{i=1}^{u}R_ia_i+\sum\limits_{i=1}^{r}\mu_is_i+\sum\limits_{i=1}^{s}t_i\mu'_i\right)>0$

$$Re\left(\beta+\sum_{i=1}^{v}\eta_{G_i,g_i}b'_i+\sum_{i=1}^{u}R_ib_i+\sum_{i=1}^{r}v_is_i+\sum_{i=1}^{s}t_i\rho'_i\right)>0$$

$$Re\left(\lambda_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda_j'''^{(i)}+\sum_{i=1}^{u}R_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\zeta_j^{(i)}+\sum_{i=1}^{s}t_i\zeta_j'^{(i)}\right)>0(j=1,\cdots,l);$$

$$Re\left(-\sigma_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}+\sum_{i=1}^{u}R_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\lambda_j^{(i)}+\sum_{i=1}^{s}t_i\lambda_j'^{(i)}\right)>0(j=1,\cdots,k);$$

(F) $U_i=\sum\limits_{j=1}^{p}A_j\alpha_j^{(i)}-\sum\limits_{j=1}^{q}B_j\beta_j^{(i)}+\sum\limits_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)}-\sum\limits_{j=1}^{q_i}D_j^{(i)}\delta_j^{(i)}\leqslant 0,i=1,\cdots,r$

$$U'_i=\sum_{j=1}^{p'}A'_j\alpha_j'^{(i)}-\sum_{j=1}^{q'}B'_j\beta_j'^{(i)}+\sum_{j=1}^{p'_i}C_j'^{(i)}\gamma_j'^{(i)}-\sum_{j=1}^{q'_i}D_j'^{(i)}\delta_j'^{(i)}\leqslant 0,i=1,\cdots,s$$

(G) $\Delta_k=-\sum\limits_{j=n+1}^{p}A_j\alpha_j^{(k)}-\sum\limits_{j=1}^{q}B_j\beta_j^{(k)}+\sum\limits_{j=1}^{m_k}D_j^{(k)}\delta_j^{(k)}-\sum\limits_{j=m_k+1}^{q_k}D_j^{(k)}\delta_j^{(k)}+\sum\limits_{j=1}^{n_k}C_j^{(k)}\gamma_j^{(k)}-\sum\limits_{j=n_k+1}^{p_k}C_j^{(k)}\gamma_j^{(k)}$

$$-\mu_i - \rho_i - \sum_{l=1}^{k} \lambda_j^{(i)} - \sum_{l=1}^{l} \zeta_j^{(i)} > 0 \quad (i = 1, \cdots, r)$$

$$\Delta'_k = -\sum_{j=n'+1}^{p'} A'_j \alpha'^{(k)}_j - \sum_{j=1}^{q'} B'_j \beta'^{(k)}_j + \sum_{j=1}^{m'_k} D'^{(k)}_j \delta'^{(k)}_j - \sum_{j=m'_k+1}^{q'_k} D'^{(k)}_j \delta'^{(k)}_j + \sum_{j=1}^{n'_k} C'^{(k)}_j \gamma'^{(k)}_j - \sum_{j=n'_k+1}^{p'_k} C'^{(k)}_j \gamma'^{(k)}_j$$

$$-\mu'_i - \rho'_i - \sum_{l=1}^{k} \lambda'^{(i)}_j - \sum_{l=1}^{l} \zeta'^{(i)}_j > 0 \quad (i = 1, \cdots, s)$$

(H) $$\left| arg \left(z_i \prod_{j=1}^{l} \left[1 - \tau_j (t-a)^{h_i}\right]^{-\zeta_j^{(i)}} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j^{(i)}} \right) \right| < \frac{1}{2} \Delta_i \pi \quad (a \leqslant t \leqslant b; i = 1, \cdots, r)$$

$$\left| arg \left(z'_i \prod_{j=1}^{l} \left[1 - \tau'_j (t-a)^{h'_i}\right]^{-\zeta'^{(i)}_j} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda'^{(i)}_j} \right) \right| < \frac{1}{2} \Delta'_i \pi \quad (a \leqslant t \leqslant b; i = 1, \cdots, s)$$

(I) The multiple series occuring on the right-hand side of (3.25) is absolutely and uniformly convergent.

Proof

To prove (3.25), first, we express in serie the multivariable I-function defined by Nambisan et al [2] with the help of (1.5), a class of multivariable polynomials defined by Srivastava et al $S_L^{h_1,\cdots,h_u}[.]$ in serie with the help of (1.20), the I-functions of r-variables and s-variables defined by Nambisan et al [2] in terms of Mellin-Barnes type contour integral with the help of (1.8) and (1.15) respectively. Now collect the power of $\left[1 - \tau_j (t-a)^{h_i}\right]$ with $(i = 1, \cdots, r; j = 1, \cdots, l)$ and collect the power of $(f_j t + g_j)$ with $j = 1, \cdots, k$. Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the $(r + s + k + l)$ dimensional Mellin-Barnes integral in multivariable I-function defined by Nambisan et [2], we obtain the equation (3.20).

Remarks

If a) $\rho_1 = \cdots, \rho_r = \rho'_1 = \cdots, \rho'_s = 0$; b) $\mu_1 = \cdots, \mu_r = \mu'_1 = \cdots, \mu'_s = 0$, we obtain the similar formulas that (3.25) with the corresponding simplifications.

4. Particular cases

a) If $A_j = B_j = C_j^{(i)} = D_j^{(i)} = A'_j = B'_j = C'^{(i)}_j = D'^{(i)}_j = 1$, the multivariable I-function defined by Nambisan et al [2] reduces to multivariable H-function defined by Srivastava et al [7].

We the following generalized Eulerian integral concerning the multivariable H-function :

$$\int_a^b (t-a)^{\alpha-1} (b-t)^{\beta-1} \prod_{j=1}^{l} \left[1 - \tau_j (t-a)^{h_i}\right]^{-\lambda_j} \prod_{j=1}^{k} (f_j t + g_j)^{\sigma_j}$$

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u} \begin{pmatrix} z''_1 \theta''_1 (t-a)^{a_1} (b-t)^{b_1} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda''^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ z''_u \theta''_u (t-a)^{a_u} (b-t)^{b_u} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda''^{(u)}_j} \end{pmatrix}$$

$$\bar{I}\left(\begin{array}{c} z_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ . \\ . \\ . \\ z_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{array}\right)$$

$$H\left(\begin{array}{c} z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ . \\ . \\ . \\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array}\right)$$

$$H\left(\begin{array}{c} z_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ . \\ . \\ . \\ z_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{K_k}B_uB_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$H^{0,n+n'+l+k+2;X}_{p+p'+l+k+2,q+q'+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathfrak{A}\ ,\mathrm{K}_1,K_2,K_j,K_j':C \\ \cdots & . \\ \cdots & . \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & . \\ \frac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z_s'(b-a)^{\mu_s'+\rho_s'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & . \\ \tau_1(b-a)^{h_1} & \\ \cdots & \\ \cdots & \\ \tau_l(b-a)^{h_l} & \mathfrak{B}\ ,\mathrm{L}_1,L_j,L_j':D \\ \frac{(b-a)f_1}{af_1+g_1} & \\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (4.1)$$

under the same notations and conditions that (3.25) with $A_j = B_j = C_j^{(i)} = D_j^{(i)} = A'_j = B'_j = C'^{(i)}_j = D'^{(i)}_j = 1$

b) If $B(L; R_1, \cdots, R_u) = \dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_u\theta_j^{(u)}} \prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j} \cdots \prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi'_j+\cdots+m_u\psi_j^{(u)}} \prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j} \cdots \prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}}$ (4.2)

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [4]. We have

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}}\left(\begin{array}{c} z''_1\theta''_1(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z''_u\theta''_u(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(u)}} \end{array}\right.$$

$$\left.\begin{array}{c} [(\text{-L});\text{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}]:[(b');\phi'];\cdots;[(b^{(u)});\phi^{(u)}] \\ [(\text{c});\psi',\cdots,\psi^{(u)}]:[(d');\delta'];\cdots;[(d^{(u)});\delta^{(u)}] \end{array}\right)$$

$$\bar{I}\left(\begin{array}{c} z'''_1\theta'''_1(t-a)^{a'_1}(b-t)^{b'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z'''_v\theta'''_v(t-a)^{a'_v}(b-t)^{b'_v}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{array}\right)$$

$$I\left(\begin{array}{c} z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array}\right)$$

$$I\left(\begin{array}{c} z'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t$$

$$= P_1 \sum_{h_1=1}^{M_1} \cdots \sum_{h_v=1}^{M_v} \sum_{k_1=0}^{\infty} \cdots \sum_{k_v=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \prod_{i=1}^{v} \frac{(-)^{k_i}}{\delta h_i^{(i)} k_i!} z_i'''^{\eta_{h_i,k_i}} \prod_{k=1}^{u} z''^{K_k} B_u' B_{u,v} [\phi_1 (\eta_{h_1,k_1}, \cdots, \eta_{h_r,k_r})]_{j \neq h_i}$$

$$I^{0,n+n'+l+k+2;X}_{p+p'+l+k+2,q+q'+l+k+1;Y} \left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathfrak{A} \ , \mathrm{K}_1, K_2, K_j, K_j' : C \\ \cdot \cdot \cdot & \cdot \\ \cdot \cdot \cdot & \cdot \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & \cdot \\ \cdot \cdot \cdot & \cdot \\ \cdot \cdot \cdot & \cdot \\ \frac{z_s'(b-a)^{\mu_s'+\rho_s'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & \cdot \\ \tau_1(b-a)^{h_1} & \\ \cdot \cdot \cdot & \\ \cdot \cdot \cdot & \\ \tau_l(b-a)^{h_l} & \\ \frac{(b-a)f_1}{af_1+g_1} & \mathfrak{B} \ , \mathrm{L}_1, L_j, L_j' : D \\ \cdot \cdot \cdot & \\ \cdot \cdot \cdot & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array} \right) \quad (4.3)$$

under the same conditions and notations that (3.25)

where $B_u' = \frac{(-L)_{h_1R_1+\cdots+h_uR_u} B(E; R_1, \cdots, R_u)}{R_1! \cdots R_u!}$, $B[E; R_1, \ldots, R_v]$ is defined by (4.2)

Remark:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable I-functions and a class of multivariable polynomials defined by Srivastava et al [5].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable I-functions defined by Nambisan et al [2], a expansion of multivariable I-function defined by Nambisan et al [2] and a class of multivariable polynomials defined by Srivastava [5] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

REFERENCES

[1] B. L. J. Braaksma, “Asymptotic expansions and analytic continuations for a class of Barnes integrals,”Compositio Mathematical, vol. 15, pp. 239–341, 1964.

[2] Prathima J. Nambisan V. and Kurumujji S.K. A Study of I-function of Several Complex Variables, International Journalof Engineering Mathematics Vol(2014) , 2014 page 1-12

[3] Saigo M. and Saxena R.K. Unified fractional integral formulas for the multivariable H-function I. J.Fractional Calculus 15 (1999), page 91-107.

[4] Srivastava H.M. and Daoust M.C. Certain generalized Neumann expansions associated with Kampé de Fériet function. Nederl. Akad. Wetensch. Proc. Ser A72 = Indag Math 31(1969) page 449-457.

[5] Srivastava H.M. And Garg M. Some integral involving a general class of polynomials and multivariable H-function. Rev. Roumaine Phys. 32(1987), page 685-692.

[6] Srivastava H.M. and Karlsson P.W. Multiple Gaussian Hypergeometric series. Ellis.Horwood. Limited. New-York, Chichester. Brisbane. Toronto , 1985.

[7] H.M. Srivastava and R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.

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