Eulerian integral associated with product of two multivariable A-functions, generalized Lauricella function and a class of polynomial and the multivariable I-function defined by Nambisan I

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ABSTRACT
The present paper is evaluated a new Eulerian integral associated with the product of two multivariable A-functions defined by Gautam et al [1] a generalized Lauricella function , a class of multivariable polynomials and multivariable I-function defined by Nambisan [2] with general arguments . We will study the case concerning the multivariable H-function defined by Srivastava et al [7] and Srivastava-Doust polynomial [4].

Keywords: Eulerian integral, multivariable I-function, generalized Lauricella function of several variables, multivariable H-function, generalized hypergeometric function, class of polynomials,multivariable A-function.

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1. Introduction

In this paper, we evaluate a new Eulerian integral of most general characters associated with the products of two multivariable A-functions defined by Gautam et [1] , a serie expansion of multivariable I-function defined by Nambisan et al [2] and a class of polynomials with general arguments.
First time, we define the multivariable I-function by :

$$\bar{I}(z_1''',\cdots,z_v''') = \bar{I}^{0,N:M_1,N_1;\cdots;M_v,N_v}_{P,Q:P_1,Q_1;\cdots;P_v,Q_v}\left(\begin{array}{c|c} z_1''' & (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(v)};A_j)_{N+1,P} : \\ \cdot & \\ \cdot & \\ \cdot & \\ z_v''' & (b_j;\beta_j^{(1)},\cdots,\beta_j^{(v)};B_j)_{M+1,Q} : \end{array}\right.$$

$$\left.\begin{array}{c} (c_j^{(1)},\gamma_j^{(1)};1)_{1,N_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{N_1+1,P_1};\cdots;(c_j^{(v)},\gamma_j^{(v)};1)_{1,N_u},(c_j^{(v)},\gamma_j^{(v)};C_j^{(v)})_{N_v+1,P_v} \\ (d_j^{(1)},\delta_j^{(1)};1)_{1,M_1},(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{M_1+1,Q_1};\cdots;(d_j^{(v)},\delta_j^{(v)};1)_{1,M_v},(d_j^{(v)},\delta_j^{(v)};D_j^{(v)})_{M_v+1,Q_v} \end{array}\right) \quad (1.1)$$

$$= \frac{1}{(2\pi\omega)^v}\int_{L_1}\cdots\int_{L_v}\phi_1(s_1,\cdots,s_v)\prod_{i=1}^{v}\xi_i'(s_i)z_i'''^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_v \quad (1.2)$$

where $\phi_1(s_1,\cdots,s_v),\xi_i'(s_i),i=1,\cdots,v$ are given by :

$$\phi_1(s_1,\cdots,s_v) = \frac{1}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{v}\alpha_j^{(i)}s_j\right)\prod_{j=M+1}^{Q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{v}\beta_j^{(i)}s_j\right)} \quad (1.3)$$

$$\xi_i'(s_i) = \frac{\prod_{j=1}^{N_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)} \tag{1.4}$$

$i=1,\cdots,v$

Serie representation

If $z_i''' \neq 0; i=1,\cdots,v$

$\delta_{h_i}^{(i)}(d_j^{(i)}+k_i) \neq \delta_j^{(i)}(\delta_{h_i}^{(i)}+\eta_i) for j \neq h_i, j, h_i = 1,\cdots,m_i(i=1,\cdots,v)$,$k_i,\eta_i=0,1,2,\cdots(i=1,\cdots,v)$, then

$$\bar{I}(z_1''',\cdots,z_v''') = \sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\left[\phi_1\left(\frac{dh_1^{(1)}+k_1}{\delta h_1^{(1)}},\cdots,\frac{dh_v^{(v)}+k_v}{\delta h_v^{(v)}}\right)\right]_{j\neq h_i}\prod_{i=1}^{r}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\frac{dh_i+k_i}{\delta h_i}} \tag{1.5}$$

This result can be proved on computing the residues at the poles :

$$s_i = \frac{dh_i^{(i)}+k_i}{\delta h_i^{(i)}}, (h_i=1,\cdots,m_i, k_i=0,1,2,\cdots) for i=1,\cdots,v \tag{1.6}$$

We may establish the the asymptotic expansion in the following convenient form :

$$\bar{I}(z_1''',\cdots,z_v''') = 0(\,|z_1'''|^{\alpha_1},\cdots,|z_v'''|^{\alpha_v}\,), max(\,|z_1'''|,\cdots,|z_v'''|\,) \to 0$$

$$\bar{I}(z_1''',\cdots,z_v''') = 0(\,|z_1'''|^{\beta_1},\cdots,|z_v'''|^{\beta_u}\,), min(\,|z_1'''|,\cdots,|z_v'''|\,) \to \infty$$

where $k=1,\cdots,v : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j=1,\cdots,m_k$ and

$$\beta_k = max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,n_k$$

We will note $\eta_{h_i,k_i} = \dfrac{dh_i^{(i)}+k_i}{\delta h_i}$, $(h_i=1,\cdots,m_i, k_i=0,1,2,\cdots) for i=1,\cdots,v$ (1.7)

The A-function is defined and represented in the following manner.

$$A(z_1,\cdots,z_r) = A_{p,q:p_1,q_1;\cdots;p_r,q_r}^{m,n:m_1,n_1;\cdots;m_r,n_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;A_j^{(1)},\cdots,A_j^{(r)})_{1,p}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;B_j^{(1)},\cdots,B_j^{(r)})_{1,q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},C_j^{(r)})_{1,p_r} \\ \\ (\mathrm{d}_j^{(1)},D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},D_j^{(r)})_{1,q_r} \end{array}\right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.9}$$

where $\phi(s_1,\cdots,s_r),\theta_i(s_i),i=1,\cdots,r$ are given by :

$$\phi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\sum_{i=1}^{r}B_j^{(i)}s_i)\prod_{j=1}^{n}\Gamma(1-a_j+\sum_{i=1}^{r}A_j^{(i)}s_j)}{\prod_{j=n+1}^{p}\Gamma(a_j-\sum_{i=1}^{r}A_j^{(i)}s_j)\prod_{j=m+1}^{q}\Gamma(1-b_j+\sum_{i=1}^{r}B_j^{(i)}s_j)} \tag{1.10}$$

$$\theta_i(s_i)=\frac{\prod_{j=1}^{n_i}\Gamma(1-c_j^{(i)}+C_j^{(i)}s_i)\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-D_j^{(i)}s_i)}{\prod_{j=n_i+1}^{p_i}\Gamma(c_j^{(i)}-C_j^{(i)}s_i)\prod_{j=m_i+1}^{q_i}\Gamma(1-d_j^{(i)}+D_j^{(i)}s_i)} \tag{1.11}$$

Here $m,n,p,m_i,n_i,p_i,c_i\in\mathbb{N}^*;i=1,\cdots,r\,;a_j,b_j,c_j^{(i)},d_j^{(i)},A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega_i)z_k|<\frac{1}{2}\eta_k\pi,\xi^*=0,\eta_i>0 \tag{1.12}$$

$$\Omega_i=\prod_{j=1}^{p}\{A_j^{(i)}\}^{A_j^{(i)}}\prod_{j=1}^{q}\{B_j^{(i)}\}^{-B_j^{(i)}}\prod_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}}\prod_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}};i=1,\cdots,r \tag{1.13}$$

$$\xi_i^*=Im(\sum_{j=1}^{p}A_j^{(i)}-\sum_{j=1}^{q}B_j^{(i)}+\sum_{j=1}^{q_i}D_j^{(i)}-\sum_{j=1}^{p_i}C_j^{(i)});i=1,\cdots,r \tag{1.14}$$

$$\eta_i=Re\left(\sum_{j=1}^{n}A_j^{(i)}-\sum_{j=n+1}^{p}A_j^{(i)}+\sum_{j=1}^{m}B_j^{(i)}-\sum_{j=m+1}^{q}B_j^{(i)}+\sum_{j=1}^{m_i}D_j^{(i)}-\sum_{j=m_i+1}^{q_i}D_j^{(i)}+\sum_{j=1}^{n_i}C_j^{(i)}-\sum_{j=n_i+1}^{p_i}C_j^{(i)}\right)$$

$$i=1,\cdots,r \tag{1.15}$$

Consider the second multivariable A-function.

$$A(z_1',\cdots,z_s')=A_{p',q':p_1',q_1';\cdots;p_r',q_r'}^{m',n':m_1',n_1';\cdots;m_r',n_r'}\left(\begin{array}{c|c}\mathrm{z}_1 & (\mathrm{a'}_j;A_j'^{(1)},\cdots,A_j'^{(s)})_{1,p'}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b'}_j;B_j'^{(1)},\cdots,B_j'^{(s)})_{1,q'}:\end{array}\right.$$

$$\left.\begin{array}{l}(\mathrm{c'}_j^{(1)},C_j'^{(1)})_{1,p_1'};\cdots;(c_j'^{(s)},C_j'^{(s)})_{1,p_s'} \\ \\ (\mathrm{d'}_j^{(1)},D_j'^{(1)})_{1,q_1'};\cdots;(d_j'^{(s)},D_j'^{(s)})_{1,q_s'}\end{array}\right) \tag{1.16}$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1'}\cdots\int_{L_s'}\phi'(t_1,\cdots,t_s)\prod_{i=1}^{s}\theta_i'(t_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.17}$$

where $\phi'(t_1,\cdots,t_s),\theta_i'(t_i),i=1,\cdots,s$ are given by :

$$\phi'(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{m'}\Gamma(b_j'-\sum_{i=1}^{s}B_j'^{(i)}t_i)\prod_{j=1}^{n'}\Gamma(1-a_j'+\sum_{i=1}^{s}A_j'^{(i)}t_j)}{\prod_{j=n'+1}^{p'}\Gamma(a_j'-\sum_{i=1}^{s}A_j'^{(i)}t_j)\prod_{j=m'+1}^{q'}\Gamma\left(1-b_j'+\sum_{i=1}^{s}B_j'^{(i)}t_j\right)} \tag{1.18}$$

$$\theta_i'(t_i) = \frac{\prod_{j=1}^{n_i'}\Gamma(1-c_j'^{(i)}+C_j'^{(i)}t_i)\prod_{j=1}^{m_i'}\Gamma(d_j'^{(i)}-D_j'^{(i)}t_i)}{\prod_{j=n_i'+1}^{p_i'}\Gamma(c_j'^{(i)}-C_j'^{(i)}t_i)\prod_{j=m_i'+1}^{q_i'}\Gamma(1-d_j'^{(i)}+D_j'^{(i)}t_i)} \tag{1.19}$$

Here $m', n', p', m_i', n_i', p_i', c_i' \in \mathbb{N}^*; i=1,\cdots,r\,; a_j', b_j', c_j'^{(i)}, d_j'^{(i)}, A_j'^{(i)}, B_j'^{(i)}, C_j'^{(i)}, D_j'^{(i)} \in \mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega_i')z_k'| < \frac{1}{2}\eta_k'\pi, \xi'^* = 0, \eta_i' > 0 \tag{1.20}$$

$$\Omega_i' = \prod_{j=1}^{p'}\{A_j'^{(i)}\}^{A_j'^{(i)}}\prod_{j=1}^{q'}\{B_j'^{(i)}\}^{-B_j'^{(i)}}\prod_{j=1}^{q_i'}\{D_j'^{(i)}\}^{D_j'^{(i)}}\prod_{j=1}^{p_i'}\{C_j'^{(i)}\}^{-C_j'^{(i)}}; i=1,\cdots,s \tag{1.21}$$

$$\xi_i'^* = Im(\sum_{j=1}^{p'}A_j'^{(i)} - \sum_{j=1}^{q'}B_j'^{(i)} + \sum_{j=1}^{q_i'}D_j'^{(i)} - \sum_{j=1}^{p_i'}C_j'^{(i)}); i=1,\cdots,s \tag{1.22}$$

$$\eta_i' = Re\left(\sum_{j=1}^{n'}A_j'^{(i)} - \sum_{j=n'+1}^{p'}A_j'^{(i)} + \sum_{j=1}^{m'}B_j'^{(i)} - \sum_{j=m'+1}^{q'}B_j'^{(i)} + \sum_{j=1}^{m_i'}D_j'^{(i)} - \sum_{j=m_i'+1}^{q_i'}D_j'^{(i)} + \sum_{j=1}^{n_i'}C_j'^{(i)} - \sum_{j=n_i'+1}^{p_i'}C_j'^{(i)}\right)$$

$$i = 1,\cdots,s \tag{1.23}$$

Srivastava and Garg [5] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u] = \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \tag{1.24}$$

The coefficients are $B[E;R_1,\ldots,R_v]$ arbitrary constants, real or complex.

2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [6 ,page 39 eq .30]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^{P}\Gamma(A_j+s_1+\cdots+s_r)}{\prod_{j=1}^{Q}\Gamma(B_j+s_1+\cdots+s_r)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j+s_1+\cdots+s_r)$ are separated from those of $\Gamma(-s_j), j=1,\cdots,r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j=1,\cdots,r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}$$

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right) \quad (2.2)$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i,\tau_j,h_j\in\mathbb{C},\lambda_j\in\mathbb{R}^+(i=1,\cdots,k;j=1,\cdots,l)$

$$min(Re(\alpha),Re(\beta))>0,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1,\ \max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,$$

and $F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust[3,page 454] given by :

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\prod_{j=1}^{l}\Gamma(\lambda_j)\prod_{j=1}^{k}\Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}}\int_{L_1}\cdots\int_{L_{l+k}}\frac{\Gamma\left(\alpha+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}{\Gamma\left(\alpha+\beta+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}\prod_{j=1}^{l}\Gamma(\lambda_j+s_j)\prod_{j=1}^{k}\Gamma(-\sigma_j+s_{l+j})$$

$$\prod_{j=1}^{l+k}\Gamma(-s_j)z_1^{s_1}\cdots z_l^{s_l}z_{l+1}^{s_{l+1}}\cdots,z_{l+k}^{s_{l+k}}\,\mathrm{d}s_1\cdots\mathrm{d}s_{l+k} \quad (2.3)$$

Here the contour $L_j's$ are defined by $L_j=L_{w\zeta_j\infty}(Re(\zeta_j)=v_j'')$ starting at the point $v_j''-\omega\infty$ and terminating at the point $v_j''+\omega\infty$ with $v_j''\in\mathbb{R}(j=1,\cdots,l)$ and each of the remaining contour $L_{l+1},\cdots,L_{l+k}$ run from $-\omega\infty$ to $\omega\infty$

(2.2) can be easily established by expanding $\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \quad (2.4)$$

integrating term by term with the help of the integral given by Saigo and Saxena [3, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [5, page 454].

3. Eulerian integral

In this section , we note :

$$\theta_i = \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}} , \zeta_j^{(i)} > 0 (i=1,\cdots,r) \; ; \; \theta'_i = \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'^{(i)}} , \zeta_j'^{(i)} > 0 (i=1,\cdots,s)$$

$$\theta''_i = \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j''^{(i)}} , \zeta_j''^{(i)} > 0 (i=1,\cdots,u)$$

$$\theta'''_i = \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'''^{(i)}} , \zeta_j'''^{(i)} > 0 (i=1,\cdots,v) \tag{3.1}$$

$$X = m_1, n_1; \cdots ; m_r, n_r; m'_1, n'_1; \cdots ; m'_s, n'_s; 1,0; \cdots ; 1,0; 1,0; \cdots ; 1,0 \tag{3.2}$$

$$Y = p_1, q_1; \cdots ; p_r, q_r; p'_1, q'_1; \cdots ; p'_s, q'_s; 0,1; \cdots ; 0,1; 0,1; \cdots ; 0,1 \tag{3.3}$$

$$A = (a_j; A_j^{(1)}, \cdots , A_j^{(r)}, 0, \cdots , 0, 0, \cdots , 0, 0, \cdots , 0)_{1,p} \tag{3.4}$$

$$B = (b_j; B_j^{(1)}, \cdots , B_j^{(r)}, 0 \cdots , 0, 0 \cdots , 0, 0 \cdots , 0)_{1,q} \tag{3.5}$$

$$A' = (a'_j; 0, \cdots , 0, A_j'^{(1)}, \cdots , A_j'^{(s)}, 0, \cdots , 0, 0, \cdots , 0)_{1,p'} \tag{3.6}$$

$$B' = (b'_j; 0, \cdots , 0, B_j'^{(1)}, \cdots , B_j'^{(s)}, 0, \cdots , 0, 0, \cdots , 0)_{1,q'} \tag{3.7}$$

$$\mathrm{C} = (\mathrm{c}_j^{(1)}, C_j^{(1)})_{1,p_1}; \cdots ; (c_j^{(r)}, C_j^{(r)})_{1,p_r}; (c_j'^{(1)}, C_j'^{(1)})_{1,p'_1}; \cdots ; (c_j'^{(r)}, C_j'^{(s)})_{1,p'_s}$$

$$(1,0); \cdots ; (1,0); (1,0); \cdots ; (1,0) \tag{3.8}$$

$$D = (\mathrm{d}_j^{(1)}, D_j^{(1)})_{1,q_1}; \cdots ; (d_j^{(r)}, D_j^{(r)})_{1,q_r}; (\mathrm{d'}_j^{(1)}, D_j'^{(1)})_{1,q'_1}; \cdots ; (d_j'^{(s)}, D_j'^{(s)})_{1,q'_s};$$

$$(0,1); \cdots ; (0,1); (0,1); \cdots ; (0,1) \tag{3.9}$$

$$K_1 = (1 - \alpha - \sum_{i=1}^{u} R_i a_i - \sum_{i=1}^{v} \eta_{G_i,g_i} a'_i; \mu_1, \cdots , \mu_r, \mu'_1, \cdots , \mu'_s, h_1, \cdots , h_l, 1, \cdots , 1) \tag{3.10}$$

$$K_2 = (1 - \beta - \sum_{i=1}^{u} R_i b_i - \sum_{i=1}^{v} \eta_{G_i,g_i} b'_i; \rho_1, \cdots , \rho_r, \rho'_1, \cdots , \rho'_s, 0, \cdots , 0, 0 \cdots , 0) \tag{3.11}$$

$$K_j = [1 - \lambda_j - \sum_{i=1}^{u} R_i \zeta_j''^{(i)} - \sum_{i=1}^{v} \eta_{G_i,g_i} \zeta_j'''^{(i)}; \zeta_j^{(1)}, \cdots , \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots , \zeta_j'^{(s)},$$

$$0, \cdots , \underset{\mathrm{j}}{1}, \cdots , 0, 0 \cdots , 0]_{1,l} \tag{3.12}$$

$$K'_j = [1+\sigma_j - \sum_{i=1}^{u} R_i \lambda_j''^{(i)} - \sum_{i=1}^{v} \eta_{G_i,g_i} \lambda_j'''^{(i)}; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)},$$

$$0, \cdots, 0, 0 \cdots, \underset{\text{j}}{1}, \cdots, 0]_{1,k} \tag{3.13}$$

$$L_1 = (1-\alpha-\beta - \sum_{i=1}^{u} R_i(a_i+b_i) - \sum_{i=1}^{v}(a'_i+b'_i)\eta_{G_i,g_i}; \mu_1+\rho_1, \cdots, \mu_r+\rho_r, \mu'_1+\rho'_1, \cdots, \mu'_r+\rho'_r,$$

$$h_1, \cdots, h_l, 1, \cdots, 1) \tag{3.14}$$

$$L_j = [1-\lambda_j - \sum_{i=1}^{u} R_i \zeta_j''^{(i)} - \sum_{i=1}^{s} \zeta_j'''^{(i)} \eta_{G_i,g_i}; \zeta_j^{(1)}, \cdots, \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots, \zeta_j'^{(s)}, 0, \cdots, 0, 0 \cdots, 0]_{1,l} \tag{3.15}$$

$$L'_j = [1+\sigma_j - \sum_{i=1}^{u} R_i \lambda_j''^{(i)} - \sum_{i=1}^{v} \lambda_j'''^{(i)} \eta_{G_i,g_i}; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)}, 0, \cdots, 0, 0, \cdots, 0]_{1,k} \tag{3.16}$$

$$\mathfrak{A} = A, A'; \mathfrak{B} = B, B' \tag{3.17}$$

$$P_1 = (b-a)^{\alpha+\beta-1} \left\{ \prod_{j=1}^{h} (af_j+g_j)^{\sigma_j} \right\} \tag{3.18}$$

$$B_{u,v} = (b-a)^{\sum_{i=1}^{v}(a'_i+b'_i)\eta_{G_i,g_i} + \sum_{i=1}^{u}(a_i+b_i)R_i} \left\{ \prod_{j=1}^{h} (af_j+g_j)^{-\sum_{i=1}^{v} \lambda_i''' \eta_{g_i,h_i} - \sum_{i=1}^{u} \lambda_i'' R_i} \right\} \tag{3.19}$$

$$B_u = \frac{(-L)_{h_1R_1+\cdots+h_uR_u} B(E; R_1, \cdots, R_u)}{R_1! \cdots R_u!} \tag{3.20}$$

We have the general Eulerian integral.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1} \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j} \prod_{j=1}^{k} (f_jt+g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_u} \begin{pmatrix} \mathrm{z}_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1} \prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u} \prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(u)}} \end{pmatrix}$$

$$\bar{I}\begin{pmatrix} z_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ . \\ . \\ . \\ z_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{pmatrix}$$

$$A\begin{pmatrix} z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ . \\ . \\ . \\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{pmatrix}$$

$$A\begin{pmatrix} z_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ . \\ . \\ . \\ z_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{pmatrix}\mathrm{d}t$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{R_k}B_uB_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$A_{p+p'+l+k+2,q+q'+l+k+1;Y}^{m+m',n+n'+l+k+2;X}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \\ \cdots & \mathfrak{A}\ ,\mathrm{K}_1,K_2,K_j,K_j':C \\ \cdots & \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & . \\ \frac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z_s'(b-a)^{\mu_s'+\rho_s'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & . \\ \tau_1(b-a)^{h_1} & . \\ \cdots & . \\ \cdots & \\ \tau_l(b-a)^{h_l} & \\ \frac{(b-a)f_1}{af_1+g_1} & \mathfrak{B}\ ,\mathrm{L}_1,L_j,L_j':D \\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (3.21)$$

where $\mathfrak{A}, \mathfrak{B}, C, D, X, K_1, K_2, K_j, K'_j, L_1, L_j, L'_j, P_1, P_u, B_u$ and $\mathfrak{B}_1$ are defined above.

Provided that

(A) $a, b \in \mathbb{R}(a < b); \mu_i, \mu'_u, \rho_i, \rho'_u, \lambda_j^{(i)}, \lambda_j'^{(u)}, h_v \in \mathbb{R}^+, f_i, g_j, \tau_v, \sigma_j, \lambda_v \in \mathbb{C}\ (i = 1, \cdots, r; j = 1, \cdots; k;$

$u = 1, \cdots, s; v = 1, \cdots, l), a_i, b_i, \lambda_j''^{(i)}, \zeta_j''^{(i)} \in \mathbb{R}^+, (i = 1, \cdots, u; j = 1, \cdots, k)$

$a'_i, b'_i, \lambda_j'''^{(i)}, \zeta_j'''^{(i)} \in \mathbb{R}^+, (i = 1, \cdots, v; j = 1, \cdots, k)$

(B) $m, n, p, m_i, n_i, p_i, c_i \in \mathbb{N}^*; i = 1, \cdots, r$ **;** $a_j, b_j, c_j^{(i)}, d_j^{(i)}, A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{C}$

$m', n', p', m'_i, n'_i, p'_i, c'_i \in \mathbb{N}^*; i = 1, \cdots, r\ ; a'_j, b'_j, c_j'^{(i)}, d_j'^{(i)}, A_j'^{(i)}, B_j'^{(i)}, C_j'^{(i)}, D_j'^{(i)} \in \mathbb{C}$

(C) $\max_{1 \leqslant j \leqslant k} \left\{ \left| \frac{(b-a)f_i}{af_i + g_i} \right| \right\} < 1$

(D) $Re\big[\alpha + \sum_{j=1}^{v} a'_j \min_{1 \leqslant k \leqslant M_i} \frac{\bar{d}_k^{(j)}}{\bar{\delta}_k^{(j)}} + \sum_{j=1}^{r} \mu_j \min_{1 \leqslant k \leqslant m_i} \frac{d_k^{(j)}}{D_k^{(j)}} + \sum_{j=1}^{s} \mu'_i \min_{1 \leqslant k \leqslant m'_i} \frac{d_k'^{(j)}}{D_k'^{(j)}}\big] > 0$

$$Re\big[\beta + \sum_{j=1}^{v} b'_j \min_{1 \leqslant k \leqslant M_i} \frac{\bar{d}_k^{(j)}}{\bar{\delta}_k^{(j)}} + \sum_{j=1}^{r} \rho_j \min_{1 \leqslant k \leqslant m_i} \frac{D_k^{(j)}}{D_k^{(j)}} + \sum_{j=1}^{s} \rho'_j \min_{1 \leqslant k \leqslant m'_i} \frac{d_k'^{(j)}}{D_k'^{(j)}}\big] > 0$$

(E) $Re\left(\alpha + \sum_{i=1}^{v} \eta_{G_i,g_i} a'_i + \sum_{i=1}^{u} R_i a_i + \sum_{i=1}^{r} \mu_i s_i + \sum_{i=1}^{s} t_i \mu'_i\right) > 0$

$$Re\left(\beta + \sum_{i=1}^{v} \eta_{G_i,g_i} b'_i + \sum_{i=1}^{u} R_i b_i + \sum_{i=1}^{r} v_i s_i + \sum_{i=1}^{s} t_i \rho'_i\right) > 0$$

$$Re\left(\lambda_j + \sum_{i=1}^{v} \eta_{G_i,g_i} \lambda_j'''^{(i)} + \sum_{i=1}^{u} R_i \lambda_j''^{(i)} + \sum_{i=1}^{r} s_i \zeta_j^{(i)} + \sum_{i=1}^{s} t_i \zeta_j'^{(i)}\right) > 0 (j = 1, \cdots, l);$$

$$Re\left(-\sigma_j + \sum_{i=1}^{v} \eta_{G_i,g_i} \lambda'''^{(i)} + \sum_{i=1}^{u} R_i \lambda_j''^{(i)} + \sum_{i=1}^{r} s_i \lambda_j^{(i)} + \sum_{i=1}^{s} t_i \lambda_j'^{(i)}\right) > 0 (j = 1, \cdots, k);$$

(F) $|arg(\Omega_i) z_k| < \frac{1}{2}\eta_i \pi, \xi^* = 0, \eta_i > 0$

$$\Omega_i = \prod_{j=1}^{p} \{A_j^{(i)}\}^{A_j^{(i)}} \prod_{j=1}^{q} \{B_j^{(i)}\}^{-B_j^{(i)}} \prod_{j=1}^{q_i} \{D_j^{(i)}\}^{D_j^{(i)}} \prod_{j=1}^{p_i} \{C_j^{(i)}\}^{-C_j^{(i)}}; i = 1, \cdots, r$$

$$\xi_i^* = Im\Big(\sum_{j=1}^{p} A_j^{(i)} - \sum_{j=1}^{q} B_j^{(i)} + \sum_{j=1}^{q_i} D_j^{(i)} - \sum_{j=1}^{p_i} C_j^{(i)}\Big); i = 1, \cdots, r$$

$$\eta_i = Re\left(\sum_{j=1}^{n} A_j^{(i)} - \sum_{j=n+1}^{p} A_j^{(i)} + \sum_{j=1}^{m} B_j^{(i)} - \sum_{j=m+1}^{q} B_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\right)$$

$$-\mu_i - \rho_i - \sum_{l=1}^{k} \lambda_j^{(i)} - \sum_{l=1}^{l} \zeta_j^{(i)} > 0\,;\ i = 1, \cdots, r$$

$|arg(\Omega'_i) z'_k| < \frac{1}{2}\eta'_i \pi, \xi'^* = 0, \eta'_i > 0$

$$\Omega_i' = \prod_{j=1}^{p'}\{A_j'^{(i)}\}^{A_j'^{(i)}} \prod_{j=1}^{q'}\{B_j'^{(i)}\}^{-B_j'^{(i)}} \prod_{j=1}^{q_i'}\{D_j'^{(i)}\}^{D_j'^{(i)}} \prod_{j=1}^{p_i'}\{C_j'^{(i)}\}^{-C_j'^{(i)}} ; i = 1, \cdots, s$$

$$\xi_i'^* = Im(\sum_{j=1}^{p'} A_j'^{(i)} - \sum_{j=1}^{q'} B_j'^{(i)} + \sum_{j=1}^{q_i'} D_j'^{(i)} - \sum_{j=1}^{p_i'} C_j'^{(i)}); i = 1, \cdots, s$$

$$\eta_i' = Re\left(\sum_{j=1}^{n'} A_j'^{(i)} - \sum_{j=n'+1}^{p'} A_j'^{(i)} + \sum_{j=1}^{m'} B_j'^{(i)} - \sum_{j=m'+1}^{q'} B_j'^{(i)} + \sum_{j=1}^{m_i'} D_j'^{(i)} - \sum_{j=m_i'+1}^{q_i'} D_j'^{(i)} + \sum_{j=1}^{n_i'} C_j'^{(i)} - \sum_{j=n_i'+1}^{p_i'} C_j'^{(i)}\right)$$

$$-\mu_i' - \rho_i' - \sum_{l=1}^{k} \lambda_j'^{(i)} - \sum_{l=1}^{l} \zeta_j'^{(i)} > 0 ; i = 1, \cdots, s$$

(H) $$\left| arg\left(z_i \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j^{(i)}} \right) \right| < \frac{1}{2}\eta_i \pi \quad (a \leqslant t \leqslant b; i = 1, \cdots, r)$$

$$\left| arg\left(z_i' \prod_{j=1}^{l} \left[1-\tau_j'(t-a)^{h_i'}\right]^{-\zeta_j'^{(i)}} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'^{(i)}} \right) \right| < \frac{1}{2}\eta_i' \pi \ (a \leqslant t \leqslant b; i = 1, \cdots, s)$$

(I) The multiple series occuring on the right-hand side of (3.21) is absolutely and uniformly convergent.

Proof

To prove (3.21), first, we express in serie the multivariable I-function defined by et Nambisan et al [2] with the help of (1.5), a class of multivariable polynomials defined by Srivastava $S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[.]$ in serie with the help of (1.24), the I-functions of r-variables and s-variables defined by Gautam et al [1] in terms of Mellin-Barnes type contour integral with the help of (1.9) and (1.17) respectively. Now collect the power of $\left[1-\tau_j(t-a)^{h_i}\right]$ with $(i = 1, \cdots, r; j = 1, \cdots, l)$ and collect the power of $(f_j t + g_j)$ with $j = 1, \cdots, k$. Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the $(r+s+k+l)$ dimensional Mellin-Barnes integral in multivariable I-function defined by Prasad [1], we obtain the equation (3.20).

Remarks

If a) $\rho_1 = \cdots, \rho_r = \rho_1' = \cdots, \rho_s' = 0$; b) $\mu_1 = \cdots, \mu_r = \mu_1' = \cdots, \mu_s' = 0$, we obtain the similar formulas that (3.21) with the corresponding simplifications.

4. Particular cases

a)If $A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{R}$,$m = 0$ and $A_j'^{(i)}, B_j'^{(i)}, C_j'^{(i)}, D_j'^{(i)} \in \mathbb{R}$ and $m' = 0$, the multivariable A-functions reduces to multivariable H-functions defined by Srivastava et al [7], we obtain the following result.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1} \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j} \prod_{j=1}^{k} (f_j t + g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_u}\begin{pmatrix} z_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_j t+g_j)^{-\lambda_j''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_j t+g_j)^{-\lambda_j''^{(u)}} \end{pmatrix}$$

$$\bar{I}\left(\begin{array}{c} z_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ . \\ . \\ . \\ z_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{array}\right)$$

$$H\left(\begin{array}{c} z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ . \\ . \\ . \\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array}\right)$$

$$H\left(\begin{array}{c} z_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ . \\ . \\ . \\ z_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\prod_{i=1}^{v}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{R_k}A_uB_{u,v}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$H^{0,n+n'+l+k+2;X}_{p+p'+l+k+2,q+q'+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathfrak{A}\ ,\mathrm{K}_1,K_2,K_j,K_j':C \\ \cdot\cdot\cdot & . \\ \cdot\cdot\cdot & . \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & . \\ \frac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & . \\ \cdot\cdot\cdot & . \\ \cdot\cdot\cdot & . \\ \frac{z_s'(b-a)^{\mu_s'+\rho_s'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & . \\ \tau_1(b-a)^{h_1} & \\ \cdot\cdot\cdot & \\ \cdot\cdot\cdot & \\ \tau_l(b-a)^{h_l} & \\ \frac{(b-a)f_1}{af_1+g_1} & \mathfrak{B}\ ,\mathrm{L}_1,L_j,L_j':D \\ \cdot\cdot\cdot & \\ \cdot\cdot\cdot & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (4.1)$$

under the same notations and validity conditions that (3.21) with $A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{R}$,$m=0$ and

$A'^{(i)}_j, B'^{(i)}_j, C'^{(i)}_j, D'^{(i)}_j \in \mathbb{R}$ and $m' = 0$

b) If $B(L;R_1,\cdots,R_u) = \dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_u\theta_j^{(u)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi'_j+\cdots+m_u\psi_j^{(u)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}}$ (4.2)

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [4]. We have

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}}\left(\begin{array}{c} z''_1\theta''_1(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda''^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ z''_u\theta''_u(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda''^{(u)}_j} \end{array}\right.$$

$$\left.\begin{array}{c}[(\text{-L});\text{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}]:[(b');\phi'];\cdots;[(b^{(u)});\phi^{(u)}] \\ [(\text{c});\psi',\cdots,\psi^{(u)}]:[(d');\delta'];\cdots;[(d^{(u)});\delta^{(u)}]\end{array}\right)$$

$$\bar{I}\left(\begin{array}{c} z'''_1\theta'''_1(t-a)^{a'_1}(b-t)^{b'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'''^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ z'''_v\theta'''_v(t-a)^{a'_v}(b-t)^{b'_v}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'''^{(v)}_j} \end{array}\right)$$

$$A\left(\begin{array}{c} z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda^{(r)}_j} \end{array}\right)$$

$$A\left(\begin{array}{c} z'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ z'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'^{(s)}_j} \end{array}\right)\text{d}t$$

$$= P_1 \sum_{h_1=1}^{M_1} \cdots \sum_{h_v=1}^{M_v} \sum_{k_1=0}^{\infty} \cdots \sum_{k_v=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \prod_{i=1}^{v} \frac{(-)^{k_i}}{\delta h_i^{(i)} k_i!} z_i'''^{\eta_{h_i,k_i}} \prod_{k=1}^{u} z''^{R_k} B_u' B_{u,v} [\phi_1 (\eta_{h_1,k_1}, \cdots, \eta_{h_r,k_r})]_{j \neq h_i}$$

$$A^{m+m',n+n'+l+k+2;X}_{p+p'+l+k+2,q+q'+l+k+1;Y} \left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^k (af_j+g_j)^{\lambda_j^{(1)}}} & \mathfrak{A}, \mathrm{K}_1, K_2, K_j, K_j' : C \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^k (af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^k (af_j+g_j)^{\lambda_j'^{(1)}}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z_s'(b-a)^{\mu_s'+\rho_s'}}{\prod_{j=1}^k (af_j+g_j)^{\lambda_j'^{(s)}}} & \cdot \\ \tau_1(b-a)^{h_1} & \\ \cdots & \\ \cdots & \\ \tau_l(b-a)^{h_l} & \\ \frac{(b-a)f_1}{af_1+g_1} & \mathfrak{B}, \mathrm{L}_1, L_j, L_j' : D \\ \cdots & \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array} \right) \quad (4.3)$$

under the same conditions and notations that (3.21)

where $B_u' = \dfrac{(-L)_{h_1R_1+\cdots+h_uR_u} B(E; R_1, \cdots, R_u)}{R_1! \cdots R_u!}$, $B[E; R_1, \ldots, R_v]$ is defined by (4.2)

Remark:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable A-functions and a class of multivariable polynomials defined by Srivastava [5].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable A-functions defined by Gautam et al [1],a expansion of multivariable I-function defined by Nambisan et al [2] and a class of multivariable polynomials defined by Srivastava et al [5] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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