Selberg integral involving the sequence of functions,a class of pol,ynomials , a multivariable I-function and a multivariable A-function

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ABSTRACT
In the present paper we evaluate the modified Selberg integral involving the product of a multivariable A-function defined by Gautam et al [4], a sequence of functions, the multivariable I-function defined by Nambisan et al [5] and a general class of polynomials of several variables. The importance of the result established in this paper lies in the fact they involve the A-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializating the parameters their in.We will study two particular cases.

Keywords:Multivariable A-function, general class of polynomials, modified Selberg integral, sequence of functions , multivariable I-function, multivariable H-function.

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1.Introduction

First time, we define the multivariable $\bar{I}$-function by : (see Nambisan et al [5])

$$\bar{I}(z_1,\cdots,z_r) = \bar{I}^{0,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1;\cdots;p_r,q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{n+1,p}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{m+1,q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};1)_{1,n_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{n_1+1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};1)_{1,n_r},(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{n_r+1,p_r} \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};1)_{1,m_1},(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{m_1+1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};1)_{1,m_r},(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{m_r+1,q_r} \end{array}\right) \quad (1.1)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi_1(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \quad (1.2)$$

where $\phi_1(s_1,\cdots,s_r),\theta_i(s_i), i=1,\cdots,r$ are given by :

$$\phi_1(s_1,\cdots,s_r) = \frac{1}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=m+1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \quad (1.3)$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)} \quad (1.4)$$

$i = 1, \cdots, r$

Serie representation

If $z_i \neq 0; i = 1, \cdots, r$

$\delta_{h_i}^{(i)}(d_j^{(i)} + k_i) \neq \delta_j^{(i)}(\delta_{h_i}^{(i)} + \eta_i) for j \neq h_i, j, h_i = 1, \cdots, m_i (i = 1, \cdots, r), k_i, \eta_i = 0, 1, 2, \cdots (i = 1, \cdots, r)$, then

$$\bar{I}(z_1, \cdots, z_r) = \sum_{h_1=1}^{m_1} \cdots \sum_{h_r=1}^{m_r} \sum_{k_1=0}^{\infty} \cdots \sum_{k_r=0}^{\infty} \left[\phi_1 \left(\frac{dh_1^{(1)} + k_1}{\delta h_1^{(1)}}, \cdots, \frac{dh_r^{(r)} + k_r}{\delta h_r^{(r)}} \right) \right]_{j \neq h_i} \prod_{i=1}^{r} \frac{(-)^{k_i}}{\delta h_i^{(i)} k_i!} z_i^{\frac{dh_i + k_i}{\delta h_i}} \tag{1.5}$$

This result can be proved on computing the residues at the poles :

$$s_i = \frac{dh_i^{(i)} + k_i}{\delta h_i^{(i)}}, (h_i = 1, \cdots, m_i, k_i = 0, 1, 2, \cdots) for i = 1, \cdots, r \tag{1.6}$$

We may establish the the asymptotic expansion in the following convenient form :

$$\bar{I}(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}), max(|z_1|, \cdots, |z_r|) \rightarrow 0$$

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}), min(|z_1|, \cdots, |z_r|) \rightarrow \infty$$

where $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will note $\eta_{h_i, k_i} = \dfrac{dh_i^{(i)} + k_i}{\delta h_i}$, $(h_i = 1, \cdots, m_i, k_i = 0, 1, 2, \cdots) for i = 1, \cdots, r$ (1.7)

The A-function is defined and represented in the following manner, (see Gautam et al [4]).

$$A(z'_1, \cdots, z'_s) = A_{p', q' : p'_1, q'_1; \cdots; p'_s, q'_s}^{m', n' : m'_1, n'_1; \cdots; m'_s, n'_s} \left(\begin{array}{c|l} z'_1 & (a'_j; A'^{(1)}_j, \cdots, A'^{(s)}_j)_{1,p'} : \\ \cdot & \\ \cdot & \\ \cdot & \\ z'_s & (b'_j; B'^{(1)}_j, \cdots, B'^{(s)}_j)_{1,q'} : \end{array} \right.$$

$$\left. \begin{array}{l} (c'^{(1)}_j, C'^{(1)}_j)_{1,p'_1}; \cdots; (c'^{(s)}_j, C'^{(s)}_j)_{1,p'_s} \\ \\ (d'^{(1)}_j, D'^{(1)}_j)_{1,q'_1}; \cdots; (d'^{(s)}_j, D'^{(s)}_j)_{1,q'_s} \end{array} \right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L'_1} \cdots \int_{L'_s} \phi(t_1, \cdots, t_s) \prod_{i=1}^{s} \theta_i(t_i) z_i'^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.9}$$

where $\phi(t_1, \cdots, t_s), \theta_i(t_i), i = 1, \cdots, s$ are given by :

$$\phi(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{m'}\Gamma(b'_j-\sum_{i=1}^{s}B'^{(i)}_j t_i)\prod_{j=1}^{n'}\Gamma(1-a'_j+\sum_{i=1}^{s}A'^{(i)}_j t_j)}{\prod_{j=n'+1}^{p'}\Gamma(a'_j-\sum_{i=1}^{s}A^{(i)}_j t_j)\prod_{j=m'+1}^{q'}\Gamma(1-b'_j+\sum_{i=1}^{s}B'^{(i)}_j t_j)} \tag{1.10}$$

and

$$\theta_i(t_i)=\frac{\prod_{j=1}^{n'_i}\Gamma(1-c'^{(i)}_j+C'^{(i)}_j t_i)\prod_{j=1}^{m'_i}\Gamma(d'^{(i)}_j-D'^{(i)}_j t_i)}{\prod_{j=n'_i+1}^{p'_i}\Gamma(c'^{(i)}_j-C'^{(i)}_j t_i)\prod_{j=m'_i+1}^{q'_i}\Gamma(1-d'^{(i)}_j+D'^{(i)}_j t_i)} \tag{1.11}$$

Here $m',n',p',m'_i,n'_i,p'_i,c'_i\in\mathbb{N}^*;i=1,\cdots,s\,;a'_j,b'_j,c'^{(i)}_j,d'^{(i)}_j,A'^{(i)}_j,B'^{(i)}_j,C'^{(i)}_j,D'^{(i)}_j\in\mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega_i)z'_k|<\frac{1}{2}\eta_k\pi,\xi^*=0,\eta_i>0 \tag{1.12}$$

$$\Omega_i=\prod_{j=1}^{p'}\{A'^{(i)}_j\}^{A'^{(i)}_j}\prod_{j=1}^{q'}\{B'^{(i)}_j\}^{-B'^{(i)}_j}\prod_{j=1}^{q'_i}\{D'^{(i)}_j\}^{D'^{(i)}_j}\prod_{j=1}^{p'_i}\{C'^{(i)}_j\}^{-C'^{(i)}_j};i=1,\cdots,s \tag{1.13}$$

$$\xi^*_i=Im(\sum_{j=1}^{p'}A'^{(i)}_j-\sum_{j=1}^{q'}B'^{(i)}_j+\sum_{j=1}^{q'_i}D'^{(i)}_j-\sum_{j=1}^{p'_i}C'^{(i)}_j);i=1,\cdots,s \tag{1.14}$$

$$\eta_i=Re\left(\sum_{j=1}^{n'}A'^{(i)}_j-\sum_{j=n'+1}^{p'}A'^{(i)}_j+\sum_{j=1}^{m'}B'^{(i)}_j-\sum_{j=m'+1}^{q'}B'^{(i)}_j+\sum_{j=1}^{m'_i}D'^{(i)}_j-\sum_{j=m'_i+1}^{q'_i}D'^{(i)}_j+\sum_{j=1}^{n'_i}C'^{(i)}_j-\sum_{j=n'_i+1}^{p_i}C'^{(i)}_j\right)$$

$$i=1,\cdots,s \tag{1.15}$$

We note

$$X=m'_1,n'_1;\cdots;m'_s,n'_s \tag{1.16}$$

$$Y=p'_1,q'_1;\cdots;p'_s,q'_s \tag{1.17}$$

$$A=(a'_j;A'^{(1)}_j,\cdots,A'^{(s)}_j)_{1,p'}\,;\ \mathrm{C}=(\mathrm{c}'^{(1)}_j,C'^{(1)}_j)_{1,p'_1};\cdots;(c'^{(s)}_j,C'^{(s)}_j)_{1,p'_s} \tag{1.18}$$

$$B=(b'_j;B'^{(1)}_j,\cdots,B'^{(s)}_j)_{1,q'}\,;\quad D=\quad(\mathrm{d}'^{(1)}_j,D'^{(1)}_j)_{1,q'_1};\cdots;(d'^{(s)}_j,D'^{(s)}_j)_{1,q'_s}; \tag{1.19}$$

The contracted form is :

$$A(z_1,\cdots,z_s)=\ A^{m',n';X}_{p',q';Y}\left(\begin{array}{c|c} \mathrm{z}'_1 & \mathrm{A}:\mathrm{C} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}'_s & \mathrm{B}:\mathrm{D}\end{array}\right) \tag{1.20}$$

Srivastava and Garg [9] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_t}[z_1,\cdots,z_t] = \sum_{R_1,\cdots,R_t=0}^{h_1R_1+\cdots h_tR_t\leqslant L} (-L)_{h_1R_1+\cdots+h_tR_t} B(E;R_1,\cdots,R_t)\frac{z_1^{R_1}\cdots z_t^{R_t}}{R_1!\cdots R_t!} \tag{1.21}$$

the coefficients $B(E;R_1,\cdots,R_t)$ are arbitrary constants, real or complex.

2. Sequence of functions

Agarwal and Chaubey [1], Salim [7] and several others have studied a general sequence of functions. In the present document we shall study the following useful series formula for a general sequence of functions.

$$R_n^{\alpha,\beta}[x;E,F,g,h;p,q;\gamma;\delta;e^{-sx^r}] = \sum_{w,v,u,t,e,k_1,k_2,} \psi(w,v,u,t,e,k_1,k_2)x^R \tag{2.1}$$

where $$\sum_{w,v,u,t,e,k_1,k_2} = \sum_{w=0}^{\infty}\sum_{v=0}^{n}\sum_{u=0}^{v}\sum_{t=0}^{n}\sum_{c=0}^{t}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} \tag{2.2}$$

and the infinite series on the right side (2.1) is absolutely convergent, $R = ln + qv + pt + rw + k_1r + k_2q$

and $$\psi(w,v,u,t,e,k_1,k_2) = \frac{(-)^{t+w+k_2}(-v)_u(-t)_e(\alpha)_t l^n}{w!v!u!t!e!K_n k_1!k_2!}\,\frac{s^{w+k_1}F^{\gamma n-t}}{(1-\alpha-t)_e}(\alpha-\gamma n)_e$$

$$(-\beta-\delta n)_v g^{v+k_2} h^{\delta n-v-k_2}(v-\delta n)_{k_2} E^t \left(\frac{pe+rw+\lambda+qn}{l}\right)_n \tag{2.3}$$

where K_n is a sequence of constants.

By suitably specializing the parameters involving in (2.1), a general sequence of function reduced to generalized polynomials set studied by Raizada [6], a class of polynomials introduced by Fujiwara [3] and several others authors.

3. Required integral

We note $S(a,b,c)$, the Selberg integral , see Askey et al ([2], page 402) by :

$$S(a,b,c) = \int_0^1\cdots\int_0^1 \prod_{i=1}^{n} x_i^{a-1}(1-x_i)^{b-1} \prod_{1\leqslant j<k\leqslant n} |x_j-x_k|^{2c}\,\mathrm{d}x_1\cdots\mathrm{d}x_n =$$

$$= \prod_{j=0}^{n-1} \frac{\Gamma(a+jc)\Gamma(b+jc)\Gamma(1+(j+1)c)}{\Gamma(a+b+(n-1+j)c)\Gamma(1+c)} \tag{3.1}$$

with $Re(a)>0, Re(b)>0, Re(c)> Max\left\{-\frac{1}{n}, -\frac{Re(a)}{n-1}, -\frac{Re(b)}{n-1}\right\}$

We consider the new integral , see Askey et al ([2], page 402) defined by :

Lemme

$$\int_0^1\cdots\int_0^1\prod_{i=1}^{k}x_i\prod_{i=1}^{n}x_i^{a-1}(1-x_i)^{b-1}\prod_{1\leqslant j<k\leqslant n}|x_j-x_k|^{2c}\,\mathrm{d}x_1\cdots\mathrm{d}x_n=$$

$$=\prod_{i=1}^{k}\frac{(a+(n-i)c)}{(a+b+(2n-i-1)c)}\ S(a,b,c) \tag{3.2}$$

with $Re(a)>0,Re(b)>0,Re(c)>Max\left\{-\frac{1}{n},-\frac{Re(a)}{n-1},-\frac{Re(b)}{n-1}\right\}$ and $k\leqslant n$

where $S(a,b,c)$ is defined by (3.1). In this paper, we will denote the modified Selberg integral

4. Main integral

Let $X_{u,v,w}=\prod_{i=1}^{n}x_i^u(1-x_i)^v\prod_{1\leqslant j<k\leqslant n}|x_j-x_k|^{2w}$ $\quad b_{n'}=\frac{\prod_{j=1}^{p}(\lambda_j)_{n'\rho_j}}{(a+n')^{\mathfrak{s}}\prod_{j=1}^{q}(\mu_j)_{n'\sigma_j}}$ and

$$B_t=\frac{(-L)_{h_1R_1+\cdots+h_tR_t}B(E;R_1,\cdots,R_t)}{R_1!\cdots R_t!}$$

we have the following formula

Theorem

$$\int_0^1\cdots\int_0^1\prod_{i=1}^{k}x_i\prod_{i=1}^{n}x_i^{a-1}(1-x_i)^{b-1}\prod_{1\leqslant j<k\leqslant n}|x_j-x_k|^{2c}S_L^{h_1,\cdots,h_t}\begin{pmatrix}\mathrm{y}_1X_{\alpha_1,\beta_1,\gamma_1}\\ \cdot\;\cdot\;\cdot\\ \mathrm{y}_tX_{\alpha_t,\beta_t,\gamma_t}\end{pmatrix}$$

$$\bar{I}\begin{pmatrix}\mathrm{z}_1X_{\delta_1,\psi_1,\phi_1}\\ \cdot\;\cdot\;\cdot\\ \mathrm{z}_rX_{\delta_r,\psi_r,\phi_r}\end{pmatrix}A_{p',q';W}^{m',n';X}\begin{pmatrix}\mathrm{Z}_1X_{\epsilon_1,\eta_1,\zeta_1}\\ \cdot\;\cdot\;\cdot\\ \mathrm{Z}_sX_{\epsilon_s,\eta_s,\zeta_s}\end{pmatrix}R_n^{\alpha',\beta'}[zX_{\alpha,\beta,\gamma};E,F,g,h;p,q;\gamma;\delta;e^{-\mathfrak{s}(zX_{\alpha,\beta,\gamma})^{\mathfrak{r}}}]$$

$$\mathrm{d}x_1\cdots\mathrm{d}x_n=\sum_{R_1,\cdots,R_t=0}^{h_1R_1+\cdots h_tR_t\leqslant L}\ \sum_{w,v,u,t,e,k_1,k_2}\sum_{h_1=1}^{m_1}\cdots\sum_{h_r=1}^{m_r}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}[\phi_1\left(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r}\right)]_{j\neq h_i}$$

$$\psi(w,v,u,t,e,k_1,k_2)\prod_{i=1}^{r}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i^{\eta_{h_i,k_i}}B_tz^R\quad y_1^{R_1}\cdots y_t^{R_t}A_{p'+3n+2k,q'+2n+2k;W}^{m',n'+3n+2k;X}\left(\begin{matrix}\mathrm{Z}_1\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{Z}_s\end{matrix}\right|\begin{matrix}\mathrm{A}\ ,\\ \\ \\ \mathrm{B},\end{matrix}$$

$$[\text{1-a-R}\alpha - \sum_{i=1}^{t} R_i\alpha_i - \sum_{i=1}^{r} \eta_{G_i,g_i}\delta_i - j(c+\gamma' R + \sum_{i=1}^{t}\gamma_i K_i + \sum_{i=1}^{r}\phi_i\eta_{G_i,g_i}); \epsilon_1 + j\zeta_1, \cdots, \epsilon_s + j\zeta_s]_{0,n-1}$$

$$\cdots$$

$$(\text{-c-R}\gamma - \sum_{i=1}^{t} R_i\gamma_i - \sum_{i=1}^{r}\phi_i\eta_{G_i,g_i}; \zeta_1, \cdots, \zeta_s), \cdots,$$

$$[\text{1-b-R}\beta - \sum_{i=1}^{t} R_i\beta_i - \sum_{i=1}^{r} \eta_{G_i,g_i}\psi_i - j(c+\gamma' R + \sum_{i=1}^{t}\gamma_i K_i + \sum_{i=1}^{r}\phi_i\eta_{G_i,g_i}); \eta_1 + j\zeta_1, \cdots, \eta_s + j\zeta_s]_{0,n-1}$$

$$\cdots$$

$$(\text{-c-R}\gamma - \sum_{i=1}^{t} \gamma_i R_i - \sum_{i=1}^{r}\phi_i\eta_{G_i,g_i}; \zeta_1, \cdots, \zeta_s), B_1, B_2, B_3 :$$

$$\left.\begin{matrix}[\text{ -(j+1)(c+n'}\gamma + \sum_{i=1}^{t}\gamma_i R_i + \sum_{i=1}^{r}\phi_i\eta_{G_i,g_i}); (j+1)\zeta_1, \cdots, (j+1)\zeta_s)]_{0,n-1}, A_2, A_3 : C \\ \cdots \\ \text{D}\end{matrix}\right) \quad (4.1)$$

where $B_1 = [1 - a - b - (\alpha+\beta)R - \sum_{i=1}^{t} R_i(\alpha_i+\beta_i) - \sum_{i=1}^{r}(\delta_i+\phi_i)\eta_{G_i,g_i} - (n-1+j)\times$

$$(c + n'\gamma + \sum_{i=1}^{t} R_i\gamma_i + \sum_{i=1}^{r}\phi_i\eta_{G_i,g_i}); \epsilon_1 + \eta_1 + j\zeta_1, \cdots, \epsilon_s + \eta_s + j\zeta_s]_{0,n-1} \quad (4.2)$$

$$A_2 = \big[-a - R\alpha - \sum_{i=1}^{t} R_i\alpha_i - \sum_{i=1}^{r}\eta_{G_i,g_i}\delta_i - (n-j)(c + m\gamma + \sum_{i=1}^{t}\gamma_i R_i + \sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});$$

$$\epsilon_1 + (n-j)\zeta_1, \cdots, \epsilon_s + (n-j)\zeta_s\big]_{1,k} \quad (4.3)$$

$$B_2 = \big[1 - a - R\alpha - \sum_{i=1}^{t} R_i\alpha_i - \sum_{i=1}^{r}\eta_{G_i,g_i}\delta_i - (n-j)(c + m\gamma + \sum_{i=1}^{t}\gamma_i K_i + \sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});$$

$$\epsilon_1 + (n-j)\zeta_1, \cdots, \epsilon_s + (n-j)\zeta_s\big]_{1,k} \quad (4.4)$$

$$B_3 = \big[-a - R\alpha - \sum_{i=1}^{t} R_i\alpha_i - \sum_{i=1}^{r}\eta_{G_i,g_i}\delta_i - b - m\beta - \sum_{i=1}^{t} R_i\beta_i - \sum_{i=1}^{r}\eta_{G_i,g_i}\psi_i$$

$$\epsilon_1 + \eta_1 + (2n-j-1)\zeta_1, \cdots - (2n-j-1)(c + m\gamma + \sum_{i=1}^{t}\gamma_i R_i + \sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});$$

$$\epsilon_s + \eta_s + (2n-j-1)\zeta_s\big]_{1,k} \quad (4.5)$$

$$A_3 = \big[1 - a - R\alpha - \sum_{i=1}^{t} R_i\alpha_i - \sum_{i=1}^{r}\eta_{G_i,g_i}\delta_i - b - m\beta - \sum_{i=1}^{t} R_i\beta_i - \sum_{i=1}^{r}\eta_{G_i,g_i}\psi_i$$

$$\epsilon_1+\eta_1+(2n-j-1)\zeta_1,\cdots-(2n-j-1)(c+m\gamma+\sum_{i=1}^{t}\gamma_iR_i+\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});$$

$$\epsilon_s+\eta_s+(2n-j-1)\zeta_s]_{1,k} \quad \textbf{(4.6)}$$

A, B, C and D are defined respectively by (1.18) and (1.19)

Provided that

a) $min\{\alpha,\beta,\gamma,\alpha_i,\beta_i,\gamma_i,\delta_j,\psi_j,\phi_j,\epsilon_l,\eta_l,\zeta_l\}>0, i=1,\cdots,t, j=1,\cdots,r, l=1,\cdots,s$,

b) $A=Re[a+R\alpha+\sum_{i=1}^{r}\delta_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^{s}\epsilon_i\min_{1\leqslant j\leqslant m'_i}\frac{d'^{(i)}_j}{D'^{(i)}_j}]>0$

c) $B=Re[b+R\beta+\sum_{i=1}^{r}\psi_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^{s}\eta_i\min_{1\leqslant j\leqslant m'_i}\frac{d'^{(i)}_j}{D'^{(i)}_j}]>0$

d) $C=Re[c+R\gamma+\sum_{i=1}^{r}\phi_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^{s}\zeta_i\min_{1\leqslant j\leqslant m'_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>Max\left\{-\frac{1}{n},-\frac{A}{n-1},-\frac{B}{n-1}\right\}$

e) $|arg(\Omega'_i)z''_k|<\frac{1}{2}\eta'_k\pi, \xi'^{*}=0, \eta'_i>0$

f) The series occuring on the right-hand side of (4.1) is absolutely and uniformly convergent.

Proof

first, expressing the sequence of functions in serie with the help of equation (2.1), the $\bar{I}$-function of r-variables in series with the help of equation (1.5), the general class of polynomial of several variables $S_L^{h_1,\cdots,h_t}[.]$ with the help of equation (1.21) and the A-function of s variables defined by Gautam et al [4] in Mellin-Barnes contour integral with the help of equation (1.9), changing the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process).
Now evaluating the resulting modified Selberg integral with the help of equation (3.2). Use the following relations $\Gamma(a)(a)_n=\Gamma(a+n)$ and $a=\frac{\Gamma(a+1)}{\Gamma(a)}$ several times with $Re(a)>0$. Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

5. Particular cases

1) If $A'^{(i)}_j, B'^{(i)}_j, C'^{(i)}_j, D'^{(i)}_j \in \mathbb{R}$ and $m'=0$, the multivariable A-function reduces to multivariable H-function defined by Srivastava et al [10]. We obtain the following formula

Corollary 1

$$\int_0^1\cdots\int_0^1\prod_{i=1}^{k}x_i\prod_{i=1}^{n}x_i^{a-1}(1-x_i)^{b-1}\prod_{1\leqslant j<k\leqslant n}|x_j-x_k|^{2c}S_L^{h_1,\cdots,h_t}\begin{pmatrix}\mathrm{y}_1X_{\alpha_1,\beta_1,\gamma_1}\\ \cdots\\ \mathrm{y}_tX_{\alpha_t,\beta_t,\gamma_t}\end{pmatrix}$$

$$R_n^{\alpha',\beta'}[zX_{\alpha,\beta,\gamma};E,F,g,h;p,q;\gamma;\delta;e^{-\mathfrak{s}(zX_{\alpha,\beta,\gamma})^{\mathfrak{r}}}]\bar{I}\left(\begin{array}{c}\mathrm{z}_1X_{\delta_1,\psi_1,\phi_1}\\ \cdots\\ \mathrm{z}_rX_{\delta_r,\psi_r,\phi_r}\end{array}\right)H^{0,n';X}_{p',q';W}\left(\begin{array}{c}\mathrm{Z}_1X_{\epsilon_1,\eta_1,\zeta_1}\\ \cdots\\ \mathrm{Z}_sX_{\epsilon_s,\eta_s,\zeta_s}\end{array}\right)$$

$$\mathrm{d}x_1\cdots\mathrm{d}x_n=\sum_{R_1,\cdots,R_t=0}^{h_1R_1+\cdots h_tR_t\leqslant L}\sum_{w,v,u,t,e,k_1,k_2}\sum_{h_1=1}^{m_1}\cdots\sum_{h_r=1}^{m_r}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}$$

$$\psi(w,v,u,t,e,k_1,k_2)\prod_{i=1}^{r}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i^{\eta_{h_i,k_i}}B_tz^R\quad y_1^{R_1}\cdots y_t^{R_t}H^{0,n'+3n+2k;X}_{p'+3n+2k,q'+2n+2k;W}\left(\begin{array}{c|l}\mathrm{Z}_1 & \mathrm{A}\ ,\\ \cdots & \\ \cdots & \\ \mathrm{Z}_s & \mathrm{B},\end{array}\right.$$

$$[1\text{-a-R}\alpha-\sum_{i=1}^{t}R_i\alpha_i-\sum_{i=1}^{r}\eta_{G_i,g_i}\delta_i-j(c+\gamma'R+\sum_{i=1}^{t}\gamma_iK_i+\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});\epsilon_1+j\zeta_1,\cdots,\epsilon_s+j\zeta_s]_{0,n-1}$$
$$\cdots$$
$$(\text{-c-R}\gamma-\sum_{i=1}^{t}R_i\gamma_i-\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i};\zeta_1,\cdots,\zeta_s),\cdots,$$

$$[1\text{-b-R}\beta-\sum_{i=1}^{t}R_i\beta_i-\sum_{i=1}^{r}\eta_{G_i,g_i}\psi_i-j(c+\gamma'R+\sum_{i=1}^{t}\gamma_iK_i+\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});\eta_1+j\zeta_1,\cdots,\eta_s+j\zeta_s]_{0,n-1}:$$
$$\cdots$$
$$(\text{-c-R}\gamma-\sum_{i=1}^{t}\gamma_iR_i-\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i};\zeta_1,\cdots,\zeta_s),B_1,B_2,B_3:$$

$$\left.\begin{array}{c}[\text{-(j+1)(c+n'}\gamma+\sum_{i=1}^{t}\gamma_iR_i+\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});(j+1)\zeta_1,\cdots,(j+1)\zeta_s)]_{0,n-1},A_2,A_3:C\\ \cdots\\ \mathrm{D}\end{array}\right)\quad(5.1)$$

under the same notations and conditions that (4.1) with $A'^{(i)}_j, B'^{(i)}_j, C'^{(i)}_j, D'^{(i)}_j\in\mathbb{R}$ and $m'=0$.

2) If $B(L;R_1,\cdots,R_t)=\dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_t\theta_j^{(t)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(t)}}(b_j^{(t)})_{R_t\phi_j^{(t)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi'_j+\cdots+m_t\psi_j^{(t)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(t)}}(d_j^{(t)})_{R_t\delta_j^{(t)}}}$ **(5.2)**

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_t}[z_1,\cdots,z_t]$ reduces to generalized Lauricella function defined by Srivastava et al [8].

$$F^{1+\bar{A}:B';\cdots;B^{(t)}}_{\bar{C}:D';\cdots;D^{(t)}}\left(\begin{array}{c|c}\mathrm{z}_1 & [(\text{-L});\mathrm{R}_1,\cdots,R_t][(a);\theta',\cdots,\theta^{(t)}]:[(b');\phi'];\cdots;[(b^{(t)});\phi^{(t)}]\\ \cdots & \\ \cdots & [(\mathrm{c});\psi',\cdots,\psi^{(t)}]:[(d');\delta'];\cdots;[(d^{(t)});\delta^{(t)}]\\ \mathrm{z}_t & \end{array}\right)\quad\textbf{(5.3)}$$

We have the following formula

$$\int_0^1\cdots\int_0^1\prod_{i=1}^{k}x_i\prod_{i=1}^{n}x_i^{a-1}(1-x_i)^{b-1}\prod_{1\leqslant j<k\leqslant n}|x_j-x_k|^{2c}R_n^{\alpha',\beta'}[zX_{\alpha,\beta,\gamma};E,F,g,h;p,q;\gamma;\delta;e^{-\mathfrak{s}(zX_{\alpha,\beta,\gamma})^{\mathfrak{r}}}]$$

$$F_{\bar{C}:D';\cdots;D^{(t)}}^{1+\bar{A}:B';\cdots;B^{(t)}}\left(\begin{array}{c}\mathrm{y}_1X_{\alpha_1,\beta_1,\gamma_1}\\ \cdots\\ \mathrm{y}_tX_{\alpha_t,\beta_t,\gamma_t}\end{array}\middle|\begin{array}{c}[(\text{-L});\mathrm{R}_1,\cdots,R_t][(a);\theta',\cdots,\theta^{(t)}]:[(b');\phi'];\cdots;[(b^{(t)});\phi^{(t)}]\\ [(\mathrm{c});\psi',\cdots,\psi^{(t)}]:[(d');\delta'];\cdots;[(d^{(t)});\delta^{(t)}]\end{array}\right)$$

$$\bar{I}\left(\begin{array}{c}\mathrm{z}_1X_{\delta_1,\psi_1,\phi_1}\\ \cdots\\ \mathrm{z}_rX_{\delta_r,\psi_r,\phi_r}\end{array}\right)A_{p',q';W}^{m',n';X}\left(\begin{array}{c}\mathrm{Z}_1X_{\epsilon_1,\eta_1,\zeta_1}\\ \cdots\\ \mathrm{Z}_sX_{\epsilon_s,\eta_s,\zeta_s}\end{array}\right)\mathrm{d}x_1\cdots\mathrm{d}x_n=\sum_{R_1,\cdots,R_t=0}^{h_1R_1+\cdots h_tR_t\leqslant L}\sum_{w,v,u,t,e,k_1,k_2}$$

$$\sum_{h_1=1}^{m_1}\cdots\sum_{h_r=1}^{m_r}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\left[\phi_1\left(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r}\right)\right]_{j\neq h_i}\psi(w,v,u,t,e,k_1,k_2)$$

$$\prod_{i=1}^{r}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i^{\eta_{h_i,k_i}}B_t'z^R\quad y_1^{R_1}\cdots y_t^{R_t}A_{p'+3n+2k,q'+2n+2k;W}^{m',n'+3n+2k;X}\left(\begin{array}{c}\mathrm{Z}_1\\ \cdots\\ \cdots\\ \mathrm{Z}_s\end{array}\middle|\begin{array}{c}\mathrm{A}\ ,\\ \\ \\ \mathrm{B},\end{array}\right.$$

$$[\text{1-a-R}\alpha-\textstyle\sum_{i=1}^{t}R_i\alpha_i-\sum_{i=1}^{r}\eta_{G_i,g_i}\delta_i-j(c+\gamma'R+\sum_{i=1}^{t}\gamma_iK_i+\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});\epsilon_1+j\zeta_1,\cdots,\epsilon_s+j\zeta_s]_{0,n-1}$$
$$\cdots$$
$$(\text{-c-R}\gamma-\textstyle\sum_{i=1}^{t}R_i\gamma_i-\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i};\zeta_1,\cdots,\zeta_s),\cdots,$$

$$[\text{1-b-R}\beta-\textstyle\sum_{i=1}^{t}R_i\beta_i-\sum_{i=1}^{r}\eta_{G_i,g_i}\psi_i-j(c+\gamma'R+\sum_{i=1}^{t}\gamma_iK_i+\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});\eta_1+j\zeta_1,\cdots,\eta_s+j\zeta_s]_{0,n-1}$$
$$\cdots$$
$$(\text{-c-R}\gamma-\textstyle\sum_{i=1}^{t}\gamma_iR_i-\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i};\zeta_1,\cdots,\zeta_s),B_1,B_2,B_3:$$

$$\left.[\ \text{-(j+1)(c+n'}\gamma+\textstyle\sum_{i=1}^{t}\gamma_iR_i+\sum_{i=1}^{r}\phi_i\eta_{G_i,g_i});(j+1)\zeta_1,\cdots,(j+1)\zeta_s)]_{0,n-1},A_2,A_3:C\atop{\cdots\atop \mathrm{D}}\right)\quad(5.4)$$

under the same conditions and notations that (4.1)

and $B_t'=\dfrac{(-L)_{h_1R_1+\cdots+h_tR_t}B(E;R_1,\cdots,R_t)}{R_1!\cdots R_t!}$; $B(L;R_1,\cdots,R_t)$ **is defined by (5.2)**

6. Conclusion

In this paper we have evaluated a modified Selberg integral involving the product of the multivariable A-function defined by Gautam et al [4], the multivariable $\bar{I}$-function defined by Nambisan et al [5], a class of polynomials of several variables and a sequence of functions.The integral established in this paper is of very general nature as it contains Multivariable A-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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