

Stability Criteria of Streaming Conducting Fluids through Porous Media under the Influence of a Uniform Normal Magnetic Field

Yasser Gamiel^{#1}, Waheed Zahra^{#2}, Marwa El-Behairy^{#3}

^{1, 2, 3}Department of Engineering Mathematics and Physics,
Faculty of Engineering, Tanta University, Egypt.

²Department of Mathematics, School of Basic and Applied Sciences

Egypt-Japan University of Science and Technology (E-JUST). New Bor El-Arab City, 21934, Alexandria, Egypt.

Abstract

The present work deals with temporal stability properties of streaming superposed conducting fluids through porous media under the influence of a uniform normal magnetic field. The considered system is composed of two semi-infinite fluids, a middle fluid sheet of finite thickness through porous media embedded between them. The linear stability criteria of the model discussed analytically and stability diagrams obtained for both the general, and the Rayleigh–Taylor cases. Such configuration displays a variety of fascinating dynamical behaviour, further the stability of this model is of practical significance in many chemical and nuclear engineering applications. The influence of different parameters governing the flow on the stability behaviour of the system discussed in detail.

Keywords: Conducting fluids, Magnetic field, Instability, Porous media, Rayleigh–Taylor.

1. INTRODUCTION

Flow through a porous media is of considerable interest particularly among geophysical fluid dynamists. Many technical processes involve the parallel flow of fluids of different density and viscosity through porous media. Such parallel flows exist in petroleum-production engineering, in counter current flow of liquid and vapor, and in many other processes as well. Magneto-hydrodynamics (MHD) is the macroscopic theory of electrically conducting fluids move in a magnetic field, providing practical theoretical framework for describing both laboratory and astrophysical plasma. Chandrasekhar [1] discussed the linear theory governing the Kelvin - Helmholtz instability of a plane interface separating two superposed streaming fluids, under varying assumptions of hydrodynamics and hydromagnetics in a treatise. The effect of streaming is destabilized through the linear approach in the Kelvin-Helmholtz model. The influence of electric field on the Kelvin-Helmholtz

incompressible flow in the presence of the surface tension effect discussed later by Melcher[2]. Rosensweig [3] has demonstrated the Kelvin-Helmholtz instability for continuum magnetic fluids. In all the works cited above, the medium is assumed nonporous. Sharma and Spanos [4] investigated the instability of the plane interface between two uniform superposed fluids streaming through a porous medium. Raghaven and Mardsen [5] investigated the Kelvin–Helmholtz instability for flow in porous media for Darcy type flow. They obtained a characteristic equation for the growth rate of the disturbance using linear stability analysis, and then solved this equation numerically.

Bau [6] introduced a linear theory of Kelvin–Helmholtz instability for parallel flow in porous media for Darcian and non-Darcian flows. In both cases, Bau found that the velocities should exceed some critical value for the instability to manifest itself. Gheorghitza [7], and Georgescu and Gheorghitza [8] have initiated a series of studies for Kelvin–Helmholtz instability, where uniform motions of inviscid, incompressible fluids and heterogeneous porous media are considered in several simple cases. For excellent reviews about porous media, see refs. [9], [10]. In recent years, Flow through a porous medium has been of considerable interest particularly among geophysical fluid dynamists. Several applications of the problems of flow through a porous medium in geophysics found in the recovery of crude oil from the pores of reservoir rocks, packed-bed reactors in the chemical industry, in petroleum production engineering. Zakaria *et. al.* [11] have analyzed the effect of an externally applied electric field on the stability of a thin fluid film over an inclined porous plane, using linear and non-linear stability analysis in the long wave limit. Kumar and Singh [12] have investigated the stability of a plane interface separating two viscoelastic-superposed fluids in the presence of suspended particles.

Khan and Bhatia [13] studied the stability of two non-streaming, superposed, viscoelastic fluids in a

horizontal magnetic field. El-Dib and Matoog [14] have studied the Kelvin–Helmholtz instability for a Maxwellian fluid sheet. They discussed the linear instability for the influence with the periodic electric field. El-Dib [15] investigated the Rayleigh–Taylor problem for hydromagnetic Darcian flow in the presence of a uniform horizontal magnetic field. Sunil *et al.* [16] investigated the instability of the plane interface between two uniform superposed fluids streaming through a porous medium. They used linear stability analysis to obtain a characteristic equation for the growth rate of the disturbance and then solved this equation numerically.

The aim of the present work is to study the instability for three conducting layers of fluids under the effect of a uniform normal magnetic field with uniform velocity through porous media. The organization of this paper is as follows: In Section 2, the description of the problem including the basic equations of the fluid mechanics, Maxwell’s equations governing the motion of our model according to the boundary conditions and the linear stability analysis. Section 3 is devoted to the derivation of the characteristic equations and numerical estimation for stability configuration in the presence of a uniform normal magnetic field, some stability diagrams discussed. While, the final section is concerned with the concluding remarks.

2. THE PROBLEM STATEMENT

2.1 MATHEMATICAL FORMULATION

In this work, we consider the motion of two semi-infinite superposed fluids separated by a middle layer of depth $2a$ through porous media. The fluids assumed incompressible and perfectly conducting and there are weak viscous stresses on the interfaces. The interface planes are expressed by $z = \xi_l(x, y, z)$ where $l = 1, 2$ at $z = \pm a$ respectively. A Cartesian co-ordinate system introduced such that the $x - y$ plane is the mid-plans intermediate layer and the positive z -axis measured vertically upwards. The fluids influenced by the gravity force in the negative z -direction and by an external magnetic field. The unit vectors e_x, e_y and e_z are in the x, y and z -directions respectively, the geometry of the flow is depicted in Fig. 1. Fluids are moving with uniform velocity and the system is subject to a uniform normal magnetic field permeates the fluids disturbance, such that

$$U^{(m)} = U_1^{(m)} e_x + U_2^{(m)} e_y, m = 1, 2, 3, \quad (1)$$

$$H = H e_z \quad (2)$$

The superscripts $m = 1, 2, 3$ refer to quantities in the upper fluid, intermediate layer and lower fluid respectively.

The dynamics of such kind of problems obtained by the simultaneous solution of three field equations: Maxwell’s electromagnetic equations, Navier-Stokes

equation, and the continuity equation. [17]. In order to understand hydrodynamic stability better, we shall often use dimensionless variables. The variables that are associated with a superscript “*” stand for a dimensional quantities, and omit the asterisk for the dimensionless from where it is desirable to use the forms of the same physical quantities.

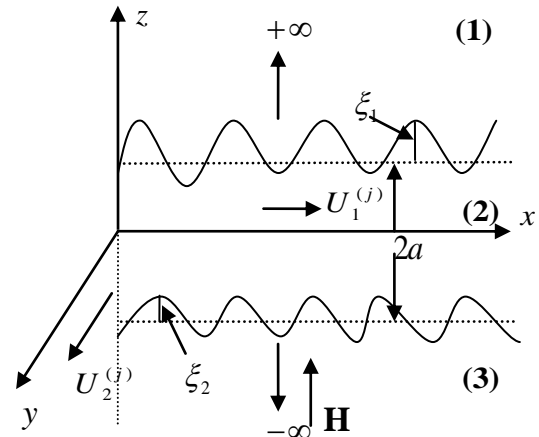


Fig.1 Schematic representation of the model.

Before solving the problem, we want to rewrite the problem precisely in dimensionless form. Sometimes we shall need only the dimensional from and so shall not use the asterisks. Thus, we may henceforth write $u^*(x^*, t^*)$ for the dimensional and $u(x, t)$ for the dimensionless total velocity of a disturbed flow. The surface disturbance $\xi^* = a\xi$, the stream velocity $U^* = \sqrt{ag}U$, the time $t^* = \sqrt{a/g}t$, the pressure $P^* = ag\rho_2 P$, the magnetic field $H^* = \sqrt{a\rho_2 g/\mu_2} H$, the porosity $\tilde{\sigma}^* = \rho_2 \sqrt{g/a} \tilde{\sigma}$, the viscosity $\eta_2^* = \rho_2 \sqrt{a^3 g} \eta$, and $(x^*, y^*, z^*) = a(x, y, z)$. The symbols $\hat{\rho}_m = \rho_m/\rho_2$, $m = 1, 2, 3$ and $W_l = T_l/a^2 g \rho_2$, $l = 1, 2$ in the equations of motion, where W_l is the Weber number and T_l is the surface tension coefficient. Under the present circumstances, these equations formulated as follows:

The hydro-magnetic vector equation of motion that governs the motion of conducting fluid through porous media is

$$\rho_m \frac{D\mathbf{u}^{(m)}}{Dt} = -\nabla P^{(m)} - \rho_m \mathbf{g} - \tilde{\sigma}_m \mathbf{u}^{(m)} + \mu_m (\nabla \wedge \mathbf{H}^{(m)}) \wedge \mathbf{H}^{(m)}, m = 1, 2, 3, \quad (3)$$

where, $\frac{D}{Dt} = \partial/\partial t + (\mathbf{u} \cdot \nabla)$ stand for the convective derivative, $\partial/\partial t$ is the partial derivative with respect to time, $\nabla \equiv (\partial/\partial x, \partial/\partial y, \partial/\partial z)$ is the gradient operator, P refers to the fluid pressure, μ is the magnetic permeability, g is the gravitational

acceleration, and \mathbf{u} is the total fluid velocity given by:

$$\mathbf{u}^{(m)} = \mathbf{U}^{(m)} + \mathbf{V}^{(m)}(x, y, z, t) \quad (4)$$

Where, $\mathbf{V}^{(m)} = (u^{(m)}, v^{(m)}, w^{(m)})$ is the perturbed velocity and \mathbf{h} is the total magnetic field vector given by

$$\mathbf{h}^{(m)} = \mathbf{H} + \mathbf{h}_1^{(m)}(x, y, z, t) \quad (5)$$

Where, $\mathbf{h}_1^{(m)} = (h_x^{(m)}, h_y^{(m)}, h_z^{(m)})$ is the perturbed field. The equation of continuity expressing the conservation of mass appropriate for incompressible fluid is

$$\nabla \cdot \mathbf{u}^{(m)} = 0 \quad (6)$$

The equation expressing the conservation of flux is

$$\nabla \cdot \mathbf{H}^{(m)} = 0 \quad (7)$$

This is identically satisfied for any magnetic field of any intensity. The equation of motion of magnetic field is

$$\frac{\partial \mathbf{h}^{(m)}}{\partial t} = (\mathbf{h}^{(m)} \cdot \nabla) \mathbf{u}^{(m)} - (\mathbf{u}^{(m)} \cdot \nabla) \mathbf{h}^{(m)} \quad (8)$$

In addition, we use the assumption $\mathbf{h} = \sqrt{\mu/\rho} \mathbf{H}$ (units of velocity).

$$\frac{\partial \mathbf{u}^{(m)}}{\partial t} + (\mathbf{u}^{(m)} \cdot \nabla) \mathbf{u}^{(m)} = -\nabla \pi^{(m)} - \frac{\tilde{\sigma}_m}{\tilde{\rho}_m} \mathbf{u}^{(m)} + (\mathbf{h}^{(m)} \cdot \nabla) \mathbf{h}^{(m)}, m = 1, 2, 3 \quad (9)$$

Where, the total pressure on the interfaces is,

$$\pi^{(m)} = \frac{p^{(m)}}{\tilde{\rho}_m} + z + \frac{1}{2} h^{(m)2} \quad (10)$$

The interfaces between the fluids assumed to be perturbed about its equilibrium location, to investigate the stabilization of the present problem. Analyzing the perturbations into normal modes, we assume that the perturbed quantities have a space and time dependence of the form:

$$f(z) \exp(ik_1x + ik_2y + \omega t) \quad (11)$$

Where, k_1, k_2 are the wave numbers along the x and y direction respectively (horizontal wave numbers). $K = \sqrt{k_1^2 + k_2^2}$ is the resultant wave number of disturbance, ($K = 2\pi/\lambda$, where λ is the wavelength of the disturbance). The growth rate of the harmonic disturbance which is, in general, a complex constant $\omega = \omega_r + i\omega_i$ (where ω_r represents the rate of growth of the disturbance, ω_i is 2π times the disturbance frequency), and $f(z)$ is some function of z.

The deformation in the planes $z = \pm 1$ are due to the perturbation about the equilibrium values for all the other variables. The form of horizontal variation for all the other perturbed variables will be the same as the displacement description (11). Perturbation bulk variables are functions of both the horizontal and vertical co-ordinates as well as time.

In accordance with the interface deflecting given by (11) and in view of a standard Fourier decomposition, we may similarly assume that the bulk solutions are periodic functions in x, y and exponential functions in t, which regarded as:

$$\begin{aligned} \xi_l(x, y, t) &= \xi_l \exp(ik_1x + ik_2y + \omega t) + c.c., \\ \mathbf{h}^{(m)}(x, y, z, t) &= \hat{\mathbf{h}}^{(m)}(z) \exp(ik_1x + ik_2y + \omega t) \\ &\quad + c.c., \\ \mathbf{u}^{(m)}(x, y, z, t) &= \hat{\mathbf{u}}^{(m)}(z) \exp(ik_1x + ik_2y + \omega t) \\ &\quad + c.c. \end{aligned}$$

(12)

Where, ξ_l is the initial amplitude of the disturbance $\xi_l \ll a$, the symbol i denotes $\sqrt{-1}$, the imaginary number, and $c.c$ stands for the complex conjugate of the preceding terms.

2.2. BOUNDARY CONDITIONS AND SOLUTIONS

The flow field solutions of the above governing equations have to satisfy the kinematic and dynamic boundary conditions at the two interfaces. The fluids and the magnetic stresses balanced at the boundaries among fluids. The components of these stresses consist of the hydrodynamics pressure, surface tension, porosity effects and magnetic stresses [18], [19]. As the interfaces are deformed, all variables are slightly perturbed from their equilibrium values. Because the interfacial displacement is small, the boundary conditions on perturbation interfacial variables evaluated at the equilibrium position rather than at the interface $z = \xi_l(x, y, t)$. Therefore, it is necessary to express all the physical quantities involved in terms of Taylor series about $z = \pm 1$.

(i) The kinematical condition: We first specify a kinematic boundary condition: fluid particles can only move tangentially to the fluid interface i.e. the normal velocity at the interface vanished. The function that defines the perturbed interface is $F_l = z - \xi_l(x, y, t)$, \mathbf{n}_l is the outward normal unit vector to the interfaces which given from the relation, $\mathbf{n}_l = \frac{\nabla F_l}{|\nabla F_l|}$. This implies that

$$\mathbf{n}_l \cdot \mathbf{u}^{(l)} = \mathbf{n}_l \cdot \mathbf{u}^{(l+1)}, z = (-1)^{l+1}, l = 1, 2 \quad (13)$$

$$w^{(l),(l+1)} = \frac{\partial \xi_l}{\partial t} + U_1^{(l),(l+1)} \frac{\partial \xi_l}{\partial x} + U_2^{(l),(l+1)} \frac{\partial \xi_l}{\partial y} \quad (14)$$

(ii) The continuity of the normal component of the magnetic displacement at the interface:

$$\mathbf{n}_l \cdot \mathbf{h}^{(l)} = \mathbf{n}_l \cdot \mathbf{h}^{(l+1)}, z = (-1)^{l+1}, l = 1, 2 \quad (15)$$

(iii) Normal condition: The problem that we focus on takes the effect of surface tension of the perturbed surface into consideration [19]. So, we can write the dynamical condition as,

$$[[\mathbf{n}_l \cdot \boldsymbol{\tau} \cdot \mathbf{n}_l]]_l^{l+1} = -W_{l(l+1)} \nabla \cdot \mathbf{n}_l, z = \pm 1 + \xi_l(x, y, t), l = 1, 2 \quad (16)$$

Where the Weber number $W_{l(l+1)} = T_{l(l+1)}/a^2 \rho_2 g$, $T_{l(l+1)}$ is the surface tension through the surfaces separating fluid l from fluid $l + 1$, and τ is the viscous stress tensor at the interfaces given from

$$\tau_{mn} = -\pi \delta_{mn} + \eta \left(\frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m} \right) \quad (17)$$

δ_{mn} , is the kronecker's delta, η is viscosity coefficient. To determine the perturbed pressure $\pi_1^{(m)}$, we take the divergence of Eq. (9). Thus, we obtain Laplace's equation (up to first order) such that:

$$\nabla^2 \pi_1^{(m)} = 0 \quad (18)$$

Since the boundary conditions require that, the disturbances vanish as $z \rightarrow \pm 1$. Thus the solution of Eq. (18) is

$$\begin{aligned} \pi_1^{(1)} &= A_1^{(1)} e^{-kz} \exp(ik_1x + ik_2y + \omega t) + c.c, \quad z > 1 \\ \pi_1^{(2)} &= (A_1^{(2)} e^{-kz} + A_2^{(2)} e^{kz}) \exp(ik_1x + ik_2y + \omega t) + c.c, \quad -1 < z < 1 \\ \pi_1^{(3)} &= A_2^{(3)} e^{kz} \exp(ik_1x + ik_2y + \omega t) + c.c, \quad z < -1 \end{aligned} \quad (19)$$

Where $A_1^{(m)}$ and $A_2^{(m)}$ are integration constants, to be determined from the above conditions and given in Appendix A. Solving the system of Eqs. (6) - (10), we get:

$$u^{(m)} = \frac{-ik_x(A_1^{(m)} e^{-kz} + A_2^{(m)} e^{kz}) \exp(ik_1x + ik_2y + \omega t)}{F^{(m)} \hat{\rho}_m^{-1} \tilde{\sigma}_m + F^{(m)-1} (k_1 H_1^{(m)} + k_2 H_2^{(m)})^2} \quad (20)$$

$$v^{(m)} = \frac{-ik_y(A_1^{(m)} e^{-kz} + A_2^{(m)} e^{kz}) \exp(ik_1x + ik_2y + \omega t)}{F^{(m)} \hat{\rho}_m^{-1} \tilde{\sigma}_m + F^{(m)-1} (k_1 H_1^{(m)} + k_2 H_2^{(m)})^2} \quad (21)$$

$$w^{(m)} = \frac{(kA_1^{(m)} e^{-kz} - kA_2^{(m)} e^{kz}) \exp(ik_1x + ik_2y + \omega t)}{F^{(m)} \hat{\rho}_m^{-1} \tilde{\sigma}_m + F^{(m)-1} (k_1 H_1^{(m)} + k_2 H_2^{(m)})^2} \quad (22)$$

$$(h_x^{(m)}, h_y^{(m)}, h_z^{(m)}) = F^{(m)-1} i (k_1 H_1^{(m)} + k_2 H_2^{(m)}) (u^{(m)}, v^{(m)}, w^{(m)}) \quad (23)$$

Where, $F^{(m)} = (\omega + ik_1 U_1^{(m)} + ik_2 U_2^{(m)})$.

3. DERIVATION OF THE CHARACTERISTIC EQUATIONS AND THEIR STABILITY

3.1. DISPERSION EQUATION

Inserting Eqs. (19)-(23) into the dynamical conditions (16), to determine the boundary-value problem, which constitutes a homogeneous system of equations and boundary conditions. After careful mathematical calculation, the characteristic equation obtained and presented as:

$$\omega^4 + (\delta_1 + i\delta_2)\omega^3 + (\delta_3 + i\delta_4)\omega^2 + (\delta_5 + i\delta_6)\omega + \delta_7 + i\delta_8 = 0 \quad (24)$$

Equation (24) is the desired dispersion relation of the stability of streaming conducting fluids through

porous media under the influence of a uniform normal magnetic field with two perturbed interfaces, the coefficients δ 's defined in Appendix B.

3.2 NEUTRAL (MARGINAL) STATIONARY STABILITY

To discuss the marginal stationary state (the critical state), we set $\omega = 0$ in Eqn. (24) i.e., $\omega_r = 0$ and $\omega_i = 0$, then we get;

$$\delta_7 + i\delta_8 = 0, \quad (25)$$

which represents a stationary state. Equation (25) represents the transition curves that separate between the stable and unstable regions as shown in Figs. 2 and 3. The influence of magnetic field H on the variation of wave number k_1 is presented in Fig. 2 for a system having the dimensionless physical parameters $\hat{\rho}_1 = 0.6, \hat{\rho}_3 = 1.2, \sigma_1 = 2, \sigma_2 = 3, \sigma_3 = 1, \eta_1 = 1, \eta_3 = 0.2, U_{22} = 8.7, U_{12} = 5, U_{21} = 2, U_{11} = 1, U_{13} = 3, U_{23} = 2, \eta_2 = 4, W_1 = 10, W_2 = 15, k_2 = 0.1$.

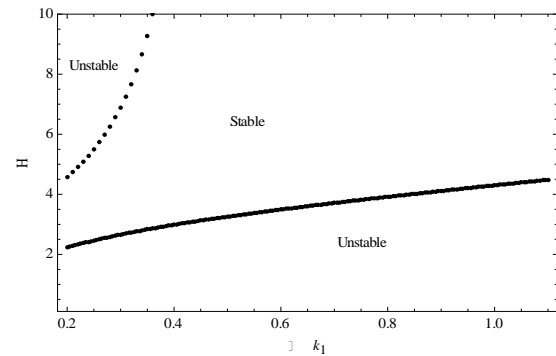


Fig.2 Sample graph of the magnetic field H versus k_1 .

The stable region lies between the two transition curves of Eqn. (25). Regarding this figure, we notice that at low wave number $k_1 < 0.4$ the stability region is bounded with a minimum and a maximum values of the magnetic field. On growing the value of k_1 the stability is achieved without any upper limiting values of the magnetic field. This implies that the magnetic field plays a stabilizing role in the movement of the waves. This influence physically interpreted as the magnetic field works as a damper of the kinetic energy of the waves of perturbation.

The variation of the magnetic field H with the density number of the lower layer $\hat{\rho}_3$, for the following dimensionless parameters; $\hat{\rho}_1 = 0.1, k_1 = 0.2, W_1 = 8, W_2 = 10, \sigma_1 = 8, \sigma_2 = 3, \sigma_3 = 0.5, U_{11} = 8.7, U_{12} = 0.5, \eta_2 = 0.4, \eta_1 = 5, U_{23} = 0.5, U_{13} = 0.85, U_{21} = 5, U_{22} = 8, k_2 = 0.5$ for various values of the viscosity of the lower layer $\eta_3 = 0.5, 2.5, 4$ respectively is depicted in Fig. 3.

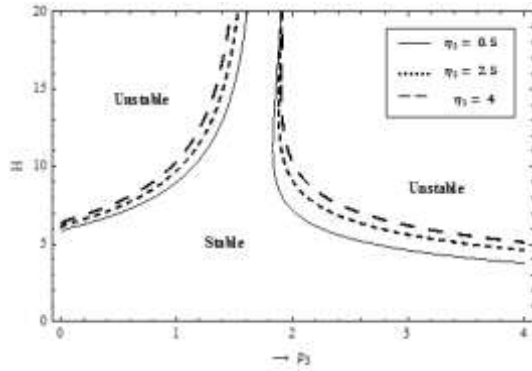


Fig. 3 Neutral curves in the H versus $\hat{\rho}_3$ plane for various viscosities η_3 .

One observes from this figure that the stable area increases in a monotonic fashion with the increase in the viscosity of the lower layer η_3 , especially for medium values of the density number of the lower layer $\hat{\rho}_3$. This implies that the increase in the values of the viscosity changes the unstable waves into stable waves. Hence, we deduce that the viscosity coefficient η_3 plays a stabilizing role in the movement of the waves in the lower fluid. This result agrees with the study of Kadry *et.al.* Fig. 2(c) [20]. This regular influence may be physically interpreted as a part of the kinetic energy of the waves has been absorbed, which leads to damping in the frequency of the waves.

3.3 GENERALIZED INSTABILITY

Based on normal mode method Eqn. (11), ω is a complex number we can rearrange the exponential to give:

$$e^{\omega t} = e^{\omega_r t} e^{i\omega_i t} \tag{26}$$

The first term, $e^{\omega_r t}$ is a monotonic function of time, whereas the second term, $e^{i\omega_i t}$ is just a sinusoidal function. If ω_r is less than zero, this term becomes a damping function and we have stability; if ω_r is equal to zero, we have neutral stability (the disturbances neither grow nor decline); and, if ω_r is greater than zero, the final term becomes a growth function and we have instability. Generally, the stability examination performed by fixing the value of all physical parameters except for one parameter having varying values for comparison. To have a stable state, all the possible real roots of Eqn. (24) must be negative otherwise the system is unstable. We investigate the behavior of stability criteria versus various parameters like magnetic field, viscosity and density.

Figure 4 represents the temporal growth rate at $\hat{\rho}_1 = 0.2, \hat{\rho}_3 = 5, W_1 = 8, W_2 = 10, \sigma_1 = 2, \sigma_2 = 3, \sigma_3 = 1, k_1 = 0.2, k_2 = 0.5, \eta_1 = 0.9, \eta_2 =$

$1.2, \eta_3 = 1, U_{12} = 6, U_{23} = 4, U_{13} = 2, U_{21} = 2, U_{11} = 1, U_{22} = 8$. As regarding this figure, for small and intermediate values of H ($H \leq 10.7$) the system is stable, on growing the value of H the system becomes unstable.

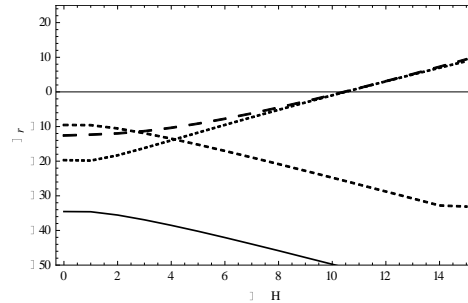


Fig. 4 Amplitude growth rate of disturbed waves against the magnetic field H .

3.4 RAYLEIGH TAYLOR INSTABILITY

As a limiting case of the model, the case of the non-streaming fluids, the stability analysis classified according to the initial state i.e. according to the streaming velocities of the fluids. The linear stability of the Rayleigh–Taylor problem for general surface deflections,

($U^{(m)} = 0$) the dispersion relation (24) reduces to

$$\omega^4 + \hat{\delta}_1 \omega^3 + \hat{\delta}_3 \omega^2 + \hat{\delta}_5 \omega + \hat{\delta}_7 = 0 \tag{27}$$

where, the coefficients $\hat{\delta}_r = \delta_r$ ($r = 1, 3, 5, 7$) as $U^{(m)} = 0$.

First, we study the marginal stationary state for Rayleigh Taylor case by setting $\omega = 0$ in Eqn. (27), then $\hat{\delta}_7 = 0$, which represents a stationary state.

The influence of the density ratio $\hat{\rho}_3 = \frac{\rho_3}{\rho_2}$ is displayed in Fig. 5 at $\hat{\rho}_1 = 0.6, W_1 = 10, W_2 = 15$ with $\hat{\rho}_1 = 1.2, 1.5$ and 1.8 respectively. We remark that for a given value of the density ratio $\hat{\rho}_3 = 1.2$ the unstable area grows swiftly with increasing the value of wave number K . On the other hand, for any value of the wave number, increasing the density ratio $\hat{\rho}_3$ increases the instability especially, at small values of H . This implies that the density ratio $\hat{\rho}_3$ plays a destabilizing role especially for the short waves of perturbation. In other words, this result can be physically interpreted that there is an increasing of the inertia of the particles of the lower fluid. This is due to the increasing of the density, which leads to an increasing in the perturbed motions. It is worthwhile to notice that the effect of the stability is independent on the viscosity as the three fluids are stationary.

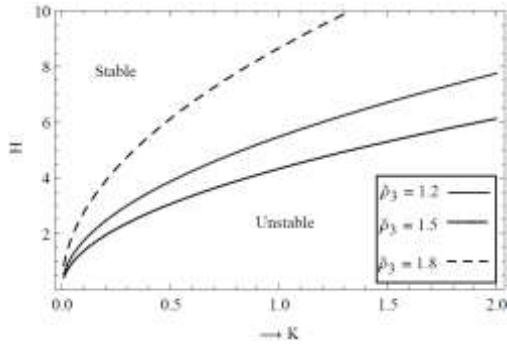


Fig. 5 The influence of the density ratio $\hat{\rho}_3$.

Second, we study the general case for Rayleigh Taylor case based on normal mode method Eqn. (11) then examining the real roots of the dispersion relation Eqn. (27)

Figure 6 represents the temporal growth rate at $\hat{\rho}_1 = 0.2, \hat{\rho}_3 = 5, W_1 = 8, W_2 = 10, \sigma_1 = 2, \sigma_2 = 3, \sigma_3 = 1, k_1 = 0.2, k_2 = 0.5, \eta_1 = 0.9, \eta_2 = 1.2, \eta_3 = 1$. It has been observed that, for small values of H ($H \leq 5.7$) the system is stable, on growing the value of H the system becomes unstable. Comparing Fig. 6 with Fig. 4, it has been observed that if the disturbance leads to energy accumulation, the rate of motion will increase, hence the system turns to the unstable mode as shown in the figure.

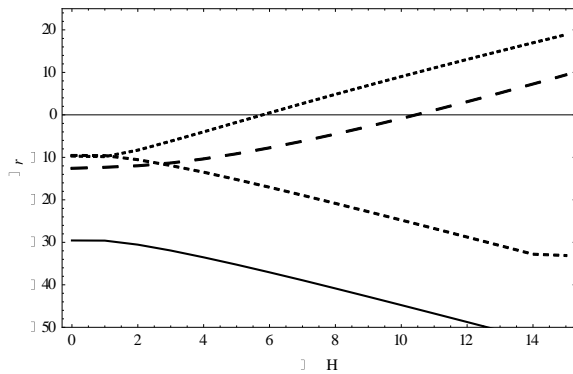


Fig. 6 Graph of the growth rate ω_r against H for the case of Rayleigh Taylor

4. CONCLUDING REMARKS

In this work, the stability of three conducting layers of fluids under the effect of a uniform normal magnetic field with uniform velocity through porous media is investigated. The linear stability dynamics is analyzed by using the normal mode method to derive a dispersion relation for the perturbed waves. Such configuration displays a variety of fascinating dynamical behavior, further the stability of this model is of practical significance in many chemical and nuclear engineering applications. The solution

of the system leads to fourth degree dispersion equation with complex coefficients, from which we could analytically conclude stability conditions. The behavior of this model discussed through plotting some stability diagrams for the general and Rayleigh Taylor instabilities. The stability examination yields the following results:

- 1- The magnetic field plays a stabilizing role in the movement of the waves in marginal state. This influence physically interpreted as the magnetic field works as a damper of the kinetic energy of the waves of perturbation.
- 2- In the stationary case, the viscosity coefficient η_3 plays a stabilizing role in the movement of the waves in the lower fluid. This regular influence may be physically interpreted as a part of the kinetic energy of the waves has been absorbed, which leads to damping in the frequency of the waves, whereas, in the case of Rayleigh Taylor. We conclude that the stability is independent on the viscosity as the three fluids are stationary.
- 3- For the general case of Rayleigh Taylor based on normal mode method. It has been observed that if the disturbance leads to energy accumulation, the rate of motion will increase.

APPENDIX A

$$A_1^{(1)} = K^{-1} (e^K(\omega + ik_1U_{11} + ik_2U_{21})\xi_1(\omega + ik_1U_{11} + ik_2U_{21} - H^2K^2(\omega + ik_1U_{11} + ik_2U_{21})^{-1} + \tilde{\sigma}_1\hat{\rho}_1^{-1}))$$

$$A_1^{(2)} = (e^{4K} - 1)^{-1}K^{-1}e^K(e^{2K}\xi_1 - \xi_2)(H^2K^2 - \omega^2 + k_1^2U_{12}^2 + k_2^2U_{22}^2 - \omega\tilde{\sigma}_2 - ik_2U_{22}(2\omega + \tilde{\sigma}_2) - ik_1U_{12}(2\omega + 2ik_2U_{22} + \tilde{\sigma}_2))$$

$$A_2^{(2)} = (1 - e^{4K})^{-1}K^{-1}e^K(e^{2K}\xi_2 - \xi_1)(H^2K^2 - \omega^2 + k_1^2U_{12}^2 + k_2^2U_{22}^2 - \omega\tilde{\sigma}_2 - ik_2U_{22}(2\omega + \tilde{\sigma}_2) - ik_1U_{12}(2\omega + 2ik_2U_{22} + \tilde{\sigma}_2))$$

$$A_2^{(3)} = -K^{-1}e^K(\omega + ik_1U_{13} + ik_2U_{23})\xi_2(\omega + ik_1U_{13} + ik_2U_{23} - H^2K^2(\omega + ik_1U_{11} + ik_2U_{21})^{-1} + \tilde{\sigma}_3\hat{\rho}_3^{-1})$$

$$Q_1 = K^2H^2(\omega + ik_1U_{11} + ik_2U_{21})^{-1},$$

$$Q_2 = K^2H^2(\omega + ik_1U_{12} + ik_2U_{22})^{-1},$$

$$Q_3 = K^2H^2(\omega + ik_1U_{13} + ik_2U_{23})^{-1}$$

$$R_1 = (\omega + ik_1U_{11} + ik_2U_{21} + \sigma_1\hat{\rho}_1^{-1}),$$

$$R_2 = (\omega + ik_1U_{12} + ik_2U_{22} + \sigma_2),$$

$$R_3 = (\omega + ik_1U_{13} + ik_2U_{23} + \sigma_3\hat{\rho}_3^{-1})$$

Appendix B

$$s = \hat{\rho}_3 - 1$$

$$\delta_1 = (2K^2\eta_1 + \sigma_1 + (2K^2\eta_2 + \sigma_2)\text{Coth}2K + [2K^2(\eta_3(\text{Coth}2K + \hat{\rho}_1) + \eta_2(1 + \text{Coth}2K\hat{\rho}_1))]s^{-1})(\text{Coth}2K + \hat{\rho}_1)^{-1} + \sigma_3\hat{\rho}_3^{-1}$$

$$\delta_2 = (2(k_1(U_{12} + U_{13}) + k_2(U_{22} + U_{23})\text{Cosh}2k) + 2(k_1(U_{11} + U_{13}) + k_2(U_{21} + U_{23}))\hat{\rho}_1\text{Sinh}2k)(\text{Cosh}2k + \hat{\rho}_1\text{Sinh}2k)^{-1}$$

$$\delta_3 = ((\text{Coth}2k + \hat{\rho}_1)\hat{\rho}_3s)^{-1}(\sigma_1\sigma_3 + \sigma_2\sigma_3\text{Coth}2k - H^2K^2\hat{\rho}_3 - 3H^2K^2\hat{\rho}_3\text{Coth}2k - k_1^2U_{12}^2\hat{\rho}_3\text{Coth}2k - 4k_1^2U_{12}U_{13}\hat{\rho}_3\text{Coth}2k - k_1^2U_{13}^2\hat{\rho}_3\text{Coth}2k - 2k_1k_2U_{12}U_{22}\hat{\rho}_3\text{Coth}2k - 4k_1k_2U_{13}U_{22}\hat{\rho}_3\text{Coth}2k - k_2^2U_{22}^2\hat{\rho}_3\text{Coth}2k - 4k_1k_2U_{12}U_{23}\hat{\rho}_3\text{Coth}2k - 2k_1k_2U_{13}U_{23}\hat{\rho}_3\text{Coth}2k - 4k_2^2U_{22}U_{23}\hat{\rho}_3\text{Coth}2k - k_2^2U_{23}^2\hat{\rho}_3\text{Coth}2k - Kk_1^2W_1\hat{\rho}_3 - Kk_2^2W_1\hat{\rho}_3 + Kk_1^2W_2\hat{\rho}_3\text{Coth}2k + Kk_2^2W_2\hat{\rho}_3\text{Coth}2k - 4K^4\eta_2^2\hat{\rho}_3 - 2K^2\eta_3\sigma_1\hat{\rho}_3 - 2K^2\eta_3\sigma_2\hat{\rho}_3\text{Coth}2k - \sigma_1\sigma_3\hat{\rho}_3 - \sigma_2\sigma_3\hat{\rho}_3\text{Coth}2k - 3H^2K^2\hat{\rho}_1\hat{\rho}_3 - k_1^2U_{11}^2\hat{\rho}_1\hat{\rho}_3 - 4k_1^2U_{11}U_{13}\hat{\rho}_1\hat{\rho}_3 - k_1^2U_{13}^2\hat{\rho}_1\hat{\rho}_3 - 2k_1k_2U_{11}U_{21}\hat{\rho}_1\hat{\rho}_3 - 4k_1k_2U_{13}U_{21}\hat{\rho}_1\hat{\rho}_3 - k_2^2U_{21}^2\hat{\rho}_1\hat{\rho}_3 - 4k_1k_2U_{11}U_{23}\hat{\rho}_1\hat{\rho}_3 - 2k_1k_2U_{13}U_{23}\hat{\rho}_1\hat{\rho}_3 - 4k_2^2U_{21}U_{23}\hat{\rho}_1\hat{\rho}_3 - k_2^2U_{23}^2\hat{\rho}_1\hat{\rho}_3 + Kk_1^2W_2\hat{\rho}_1\hat{\rho}_3 + Kk_2^2W_2\hat{\rho}_1\hat{\rho}_3 + H^2K^2\hat{\rho}_3^2 + 2H^2K^2\hat{\rho}_3^2\text{Coth}2k + k_1^2U_{12}^2\hat{\rho}_3^2\text{Coth}2k + 4k_1^2U_{12}U_{13}\hat{\rho}_3^2\text{Coth}2k + k_1^2U_{13}^2\hat{\rho}_3^2\text{Coth}2k + 2k_1k_2U_{12}U_{22}\hat{\rho}_3^2\text{Coth}2k + 4k_1k_2U_{13}U_{22}\hat{\rho}_3^2\text{Coth}2k + k_2^2U_{22}^2\hat{\rho}_3^2\text{Coth}2k + 4k_1k_2U_{12}U_{23}\hat{\rho}_3^2\text{Coth}2k + 2k_1k_2U_{13}U_{23}\hat{\rho}_3^2\text{Coth}2k + 4k_2^2U_{22}U_{23}\hat{\rho}_3^2\text{Coth}2k + Kk_1^2W_1\hat{\rho}_3^2 + Kk_2^2W_1\hat{\rho}_3^2 + 2H^2K^2\hat{\rho}_1\hat{\rho}_3^2 + k_1^2U_{11}^2\hat{\rho}_1\hat{\rho}_3^2 + 4k_1^2U_{11}U_{13}\hat{\rho}_1\hat{\rho}_3^2 + k_1^2U_{13}^2\hat{\rho}_1\hat{\rho}_3^2 + 2k_1k_2U_{11}U_{21}\hat{\rho}_1\hat{\rho}_3^2 + 4k_1k_2U_{13}U_{21}\hat{\rho}_1\hat{\rho}_3^2 + k_2^2U_{21}^2\hat{\rho}_1\hat{\rho}_3^2 + 4k_1k_2U_{11}U_{23}\hat{\rho}_1\hat{\rho}_3^2 + 2k_1k_2U_{13}U_{23}\hat{\rho}_1\hat{\rho}_3^2 + 4k_2^2U_{21}U_{23}\hat{\rho}_1\hat{\rho}_3^2 + k_2^2U_{23}^2\hat{\rho}_1\hat{\rho}_3^2 - 2K^2\eta_1(\sigma_3 - s + 2K^2(\eta_2\text{Coth}2k + \eta_3)\hat{\rho}_3) - 2K^2\eta_2(\sigma_3\text{Coth}2k - s + (2K^2\eta_3\text{Coth}2k + \sigma_1\text{Coth}2k + \sigma_2)\hat{\rho}_3))$$

$$\delta_4 = (((\text{Coth}2k + \hat{\rho}_1)\hat{\rho}_3s)^{-1}(k_1(U_{11}(2\sigma_3\hat{\rho}_1s + (4K^2(\eta_2\text{Coth}2k + \eta_3)\hat{\rho}_1 + 2K^2\eta_1s + \sigma_1s)\hat{\rho}_3) + U_{13}(\sigma_3(\text{Coth}2k + \hat{\rho}_1)s + 2\hat{\rho}_3(K^2\eta_3\text{Coth}2k - \sigma_1 - \sigma_2\text{Coth}2k + K^2\eta_3\hat{\rho}_1 + 2K^2\eta_1s + 2sK^2\eta_2\text{Coth}2k + \sigma_1\hat{\rho}_3 + \sigma_2\hat{\rho}_3\text{Coth}2k)) + U_{12}(2s\sigma_3\text{Coth}2k + \hat{\rho}_3((4K^2\eta_3 + \sigma_2s)\text{Coth}2k + 2K^2\eta_2(3 - \text{Coth}2k + \hat{\rho}_1\text{Coth}2k + \hat{\rho}_3\text{Coth}2k)))) + k_2(U_{21}(2\sigma_3\hat{\rho}_1s + (4K^2(\eta_2\text{Coth}2k + \eta_3)\hat{\rho}_1 + 2K^2\eta_1s + \sigma_1s)\hat{\rho}_3) + U_{23}(\sigma_3(\text{Coth}2k + \hat{\rho}_1)s + 2\hat{\rho}_3(K^2\eta_3\text{Coth}2k - \sigma_1 - \sigma_2\text{Coth}2k + K^2\eta_3\hat{\rho}_1 + 2K^2\eta_1s + 2sK^2\eta_2\text{Coth}2k + \sigma_1\hat{\rho}_3 + \sigma_2\hat{\rho}_3\text{Coth}2k)) + U_{22}(2s\sigma_3\text{Coth}2k + \hat{\rho}_3((4K^2\eta_3 + \sigma_2s)\text{Coth}2k + 2K^2\eta_2(3 - \text{Coth}2k + \hat{\rho}_1\text{Coth}2k + \hat{\rho}_3\text{Coth}2k))))))$$

$$\delta_5 = (-(\text{Coth}2k + \hat{\rho}_1)\hat{\rho}_3s)^{-1}(\sigma_3(H^2K^2(1 + \text{Coth}2k + \hat{\rho}_1) + k_1^2(U_{12}^2\text{Coth}2k + 2U_{12}U_{13}\text{Coth}2k + KW_1 + U_{11}(U_{11} + 2U_{13})\hat{\rho}_1) + k_2^2(U_{22}^2\text{Coth}2k + 2U_{22}U_{23}\text{Coth}2k + KW_1 + U_{21}(U_{21} + 2U_{23})\hat{\rho}_1) + 2k_1k_2(U_{12}(U_{22} + U_{23})\text{Coth}2k + U_{11}(U_{21} + U_{23})\hat{\rho}_1 + U_{13}(U_{22}\text{Coth}2k + U_{21}\hat{\rho}_1)))s + \hat{\rho}_3(2H^2K^4\eta_3 + 2H^2K^4\eta_3\text{Coth}2k + 2K^2k_1^2U_{12}^2\eta_3\text{Coth}2k + 4K^2k_1^2U_{12}U_{13}\eta_3\text{Coth}2k + 4K^2k_1k_2U_{12}U_{22}\eta_3\text{Coth}2k + 4K^2k_1k_2U_{13}U_{22}\eta_3\text{Coth}2k + 2K^2k_2^2U_{22}^2\eta_3\text{Coth}2k + 4K^2k_1k_2U_{12}U_{23}\eta_3\text{Coth}2k + 4K^2k_2^2U_{22}U_{23}\eta_3\text{Coth}2k + 2K^3k_1^2W_1\eta_3 + 2K^3k_2^2W_1\eta_3 - 2H^2K^2\sigma_1 - 2k_1^2U_{11}U_{13}\sigma_1 - k_1^2U_{13}^2\sigma_1 - 2k_1k_2U_{13}U_{21}\sigma_1 - 2k_1k_2U_{11}U_{23}\sigma_1 - 2k_1k_2U_{13}U_{23}\sigma_1 - 2k_2^2U_{21}U_{23}\sigma_1 - k_2^2U_{23}^2\sigma_1 + Kk_1^2W_2\sigma_1 + Kk_2^2W_2\sigma_1 - 2H^2K^2\sigma_2\text{Coth}2k - 2k_1^2U_{12}U_{13}\sigma_2\text{Coth}2k - k_1^2U_{13}^2\sigma_2\text{Coth}2k - 2k_1k_2U_{13}U_{22}\sigma_2 - 2k_1k_2U_{12}U_{23}\sigma_2 - 2k_2^2U_{22}U_{23}\sigma_2 - k_2^2U_{23}^2\sigma_2 + Kk_1^2W_2\sigma_2 + Kk_2^2W_2\sigma_2\text{Coth}2k + 2H^2K^4\eta_3\hat{\rho}_1 + 2K^2k_1^2U_{11}^2\eta_3\hat{\rho}_1 + 4K^2k_1^2U_{11}U_{13}\hat{\rho}_1K^2k_1k_2U_{11}U_{21}\eta_3\hat{\rho}_1 + 4K^2k_1k_2U_{13}U_{21}\eta_3\hat{\rho}_1 + 2K^2k_2^2U_{21}^2\eta_3\hat{\rho}_1 + 4K^2k_1k_2U_{11}U_{23}\eta_3\hat{\rho}_1 + 4K^2k_2^2U_{21}U_{23}\eta_3\hat{\rho}_1 + 2K^2\eta_1(KW_2 + 2U_{11}U_{13}s + U_{13}s) + k_2^2(KW_2 + 2U_{21}U_{23}s + U_{23}s) + H^2K^2(s - 1) + 2k_1k_2(U_{11}U_{23} + U_{13}(U_{21} + U_{23}))s) + H^2K^2\sigma_1\hat{\rho}_3 + 2k_1^2U_{11}U_{13}\sigma_1\hat{\rho}_3 + k_1^2U_{13}^2\sigma_1\hat{\rho}_3 + 2k_1k_2U_{13}U_{21}\sigma_1\hat{\rho}_3 + 2k_1k_2U_{11}U_{23}\sigma_1\hat{\rho}_3 + 2k_2^2U_{21}U_{23}\sigma_1\hat{\rho}_3 + k_2^2U_{23}^2\sigma_1\hat{\rho}_3 + H^2K^2\sigma_2\hat{\rho}_3\text{Coth}2k + 2k_1^2U_{12}U_{13}\sigma_2\hat{\rho}_3\text{Coth}2k + k_1^2U_{13}^2\sigma_2\hat{\rho}_3\text{Coth}2k + 2k_1k_2U_{13}U_{22}\sigma_2\hat{\rho}_3\text{Coth}2k + 2k_1k_2U_{12}U_{23}\sigma_2\hat{\rho}_3\text{Coth}2k + 2k_1k_2U_{13}U_{23}\sigma_2\hat{\rho}_3\text{Coth}2k + 2k_2^2U_{22}U_{23}\sigma_2\hat{\rho}_3\text{Coth}2k + k_2^2U_{23}^2\sigma_2\hat{\rho}_3\text{Coth}2k + 2K^2\eta_2(k_1^2(3U_{12}^2 + 2U_{12}(U_{11}\hat{\rho}_1 + U_{13}s)\text{Coth}2k + (KW_1 + KW_2 + U_{11}^2\hat{\rho}_1 + U_{13}^2s)\text{Coth}2k) + k_2^2(3U_{22}^2 + 2U_{22}(U_{21}\hat{\rho}_1 + sU_{23})\text{Coth}2k + (KW_1 + KW_2 + U_{21}^2\hat{\rho}_1 + sU_{23})\text{Coth}2k) + 2k_1k_2(U_{12}(3U_{22} + (U_{21}\hat{\rho}_1 + sU_{23})\text{Coth}2k) + (U_{11}(U_{21} + U_{22})\hat{\rho}_1 + U_{13}(U_{22} + U_{23}s)\text{Coth}2k) + H^2K^2(1 - \text{Coth}2k + \hat{\rho}_1\text{Coth}2k + \hat{\rho}_3\text{Coth}2k))))$$

$$\delta_6 = (((\text{Coth}2K + \hat{\rho}_1)\hat{\rho}_3s)^{-1}(-2K_1^3(KU_{11}W_2\hat{\rho}_1 + U_{12}(KW_2 + U_{13}s)\text{Coth}2K + U_{12}^2U_{13}s\text{Coth}2K + U_{11}U_{13}^2\hat{\rho}_1s + U_{13}(KW_1 + U_{11}\hat{\rho}_1)s)\hat{\rho}_3 - 2K_2^2k_2\hat{\rho}_3(-KU_{23}W_1 + KU_{22}W_2\text{Coth}2K - U_{11}^2U_{23}\hat{\rho}_1 + KU_{21}W_2\hat{\rho}_1 + U_{12}^2U_{23}s\text{Coth}2K + 2U_{12}U_{13}(U_{22} + U_{23})s\text{Coth}2K + 2U_{11}U_{13}(U_{21} + U_{23})\hat{\rho}_1s + U_{13}^2(U_{22}\text{Coth}2K + U_{21}\hat{\rho}_1)s + KU_{23}W_1\hat{\rho}_3 + U_{11}^2U_{23}\hat{\rho}_1\hat{\rho}_3) + k_2(-2K_2^2U_{22}^2U_{23}^2s\hat{\rho}_3\text{Coth}2K - 2K_2^2U_{21}^2U_{23}\hat{\rho}_1s\hat{\rho}_3 + U_{23}(-\sigma_1\sigma_3 - \sigma_2\sigma_3\text{Coth}2K + 2H^2K^2\hat{\rho}_3 + 2H^2K^2\hat{\rho}_3\text{Coth}2K + 2K_2^2U_{22}^2\hat{\rho}_3\text{Coth}2K + 2Kk_2^2W_1\hat{\rho}_3 + 2K^2\eta_3\sigma_1\hat{\rho}_3 + 2K^2\eta_3\sigma_2\hat{\rho}_3\text{Coth}2K + \sigma_1\sigma_3\hat{\rho}_3 + \sigma_2\sigma_3\hat{\rho}_3\text{Coth}2K + 2H^2K^2\hat{\rho}_1\hat{\rho}_3 - 2H^2K^2\hat{\rho}_3^2 - 2H^2K^2\text{Coth}2K\hat{\rho}_3^2 - 2K_2^2U_{22}^2\hat{\rho}_3^2\text{Coth}2K - 2Kk_2^2W_1\hat{\rho}_3^2 - 2H^2K^2\hat{\rho}_1\hat{\rho}_3^2 + 2K^2\eta_1(\sigma_3s + 2K^2\eta_3\hat{\rho}_3) + 2K^2\eta_2(\sigma_3s + 2K^2\eta_3\hat{\rho}_3)\text{Coth}2K) + U_{21}(-2\hat{\rho}_1(k_2^2(KW_2 + U_{23}s) + H^2K^2(s - 1))\hat{\rho}_3 + 2K^2\eta_1(\sigma_3s + 2K^2(\eta_2\text{Coth}2K + \eta_3)\hat{\rho}_3)) + \sigma_1(\sigma_3s + 2K^2(\eta_2\text{Coth}2K + \eta_3)\hat{\rho}_3)) + U_{22}(8K^4\eta_2^2\hat{\rho}_3 + 2K^2\eta_2(\sigma_3s\text{Coth}2K + (2K^2\eta_1\text{Coth}2K + 2K^2\eta_3\text{Coth}2K + \sigma_1\text{Coth}2K + 2\sigma_2)\hat{\rho}_3) + \text{Coth}2K(-2K(k_2^2W_2 + H^2K(s - 1))\hat{\rho}_3 + \sigma_2(\sigma_3s + 2K^2\eta_3\hat{\rho}_3))) + k_1(U_{13}(2H^2K^2\hat{\rho}_3 + 2H^2K^2\hat{\rho}_3\text{Coth}2K - \sigma_1\sigma_3 - \sigma_2\sigma_3\text{Coth}2K + 2K_2^2U_{22}^2\hat{\rho}_3\text{Coth}2K + 4k_2^2U_{22}U_{23}\hat{\rho}_3\text{Coth}2K + 2Kk_2^2W_1\hat{\rho}_3 + 2K^2\eta_3\sigma_1\hat{\rho}_3 + 2K^2\eta_3\sigma_2\hat{\rho}_3\text{Coth}2K + \sigma_1\sigma_3\hat{\rho}_3 + \text{Coth}2K\sigma_2\sigma_3\hat{\rho}_3 + 2H^2K^2\hat{\rho}_1\hat{\rho}_3 + 2K_2^2U_{21}^2\hat{\rho}_1\hat{\rho}_3 + 4k_2^2U_{21}U_{23}\hat{\rho}_1\hat{\rho}_3 - 2H^2K^2\hat{\rho}_3^2 - 2H^2K^2\text{Coth}2K\hat{\rho}_3^2 - 2K_2^2U_{22}^2\hat{\rho}_3^2\text{Coth}2K - 4k_2^2U_{22}U_{23}\hat{\rho}_3^2\text{Coth}2K - 2Kk_2^2W_1\hat{\rho}_3^2 - 2H^2K^2\hat{\rho}_1\hat{\rho}_3^2 - 2K_2^2U_{21}^2\hat{\rho}_1\hat{\rho}_3^2 - 4k_2^2U_{21}U_{23}\hat{\rho}_1\hat{\rho}_3^2 + 2K^2\eta_1(\sigma_3s + 2K^2\eta_3\hat{\rho}_3) + 2K^2\eta_2(\sigma_3s + 2K^2\eta_3\hat{\rho}_3)\text{Coth}2K) + U_{11}(-2\hat{\rho}_1(k_2^2(KW_2 + 2U_{21}U_{23}s + U_{23}s) + H^2K^2(s - 1))\hat{\rho}_3 + 2K^2\eta_1(\sigma_3s + 2K^2(\eta_2\text{Coth}2K + \eta_3)\hat{\rho}_3) + \sigma_1(\sigma_3s + 2K^2(\eta_2\text{Coth}2K + \eta_3)\hat{\rho}_3)) + U_{12}(8K^4\eta_2^2\hat{\rho}_3 + 2K^2\eta_2(\sigma_3s\text{Coth}2K + (2K^2\eta_1\text{Coth}2K + (2K^2\eta_3\text{Coth}2K + \sigma_1\text{Coth}2K + 2\sigma_2)\hat{\rho}_3) + \text{Coth}2K(-2K(k_2^2W_2 + H^2K(s - 1))\hat{\rho}_3 + \sigma_2(\sigma_3s + 2K^2\eta_3\hat{\rho}_3)))$$

$$+ (\sigma_2(\sigma_3s + 2K^2\eta_3\hat{\rho}_3) - 2(k_2^2(KW_2 + 2U_{22}U_{23}s + U_{23}s) + H^2K^2s)\hat{\rho}_3)\text{Coth}2K)))$$

$$\delta_7 = (((\text{Coth}2K + \hat{\rho}_1)\hat{\rho}_3s)^{-1}(k_1^4(U_{12}^2\text{Coth}2K + KW_1 + U_{11}\hat{\rho}_1)(KW_2 + U_{13}^2s)\hat{\rho}_3 + k_2^4(U_{22}^2\text{Coth}2K + KW_1 + U_{21}\hat{\rho}_1)(KW_2 + U_{23}^2s)\hat{\rho}_3 + 2k_1^3k_2(KU_{11}U_{21}W_2\hat{\rho}_1 + U_{12}U_{22}(KW_2 + U_{13}^2s)\text{Coth}2K + U_{12}^2U_{13}U_{23}s\text{Coth}2K) + U_{11}U_{13}^2U_{21}\hat{\rho}_1s + U_{13}U_{23}^2(KW_1 + U_{11}\hat{\rho}_1)s)\hat{\rho}_3 + H^4K^4(1 + \text{Coth}2K + \hat{\rho}_1)(s - 1)\hat{\rho}_3 + k_2^2(-K^2U_{22}^2(4K^2\eta_2^2 + 2\eta_2\sigma_2 - H^2(s - 1))\text{Coth}2K)\hat{\rho}_3 + H^2K^2(K(W_2(1 + \text{Coth}2K + \hat{\rho}_1) + W_1(s - 1)) + U_{23}(1 + \text{Coth}2K + \hat{\rho}_1)s)\hat{\rho}_3 + H^2K^2U_{21}\hat{\rho}_1(s - 1)\hat{\rho}_3 - U_{22}U_{23}(2K^2\eta_2 + \sigma_2)(\sigma_3s + 2K^2\eta_3\hat{\rho}_3)\text{Coth}2K - U_{21}(2K^2\eta_1 + \sigma_1)(2K^2U_{22}\eta_2\hat{\rho}_3\text{Coth}2K + U_{23}(\sigma_3s + 2K^2\eta_3\hat{\rho}_3))) + k_1k_2(U_{23}(2K^2\eta_1 + \sigma_1)(\sigma_3s + 2K^2\eta_3\hat{\rho}_3) - U_{11}(2K(KU_{22}\eta_2(2K^2\eta_1 + \sigma_1)\text{Coth}2K - U_{21}\hat{\rho}_1(k_2^2W_2 + H^2K(s - 1)))\hat{\rho}_3 - 2k_2^2U_{21}U_{23}\hat{\rho}_1s\hat{\rho}_3) - U_{13}(-2k_2^2U_{22}^2U_{23}s\hat{\rho}_3\text{Coth}2K - 2k_2^2U_{21}^2U_{23}\hat{\rho}_1s\hat{\rho}_3 - 2KU_{23}(k_2^2W_1 + H^2K(1 + \text{Coth}2K + \hat{\rho}_1))s)\hat{\rho}_3 + U_{21}(2K^2\eta_1 + \sigma_1)(\sigma_3s + 2K^2\eta_3\hat{\rho}_3) + U_{22}(2K^2\eta_2 + \sigma_2)(\sigma_3s + 2K^2\eta_3\hat{\rho}_3)\text{Coth}2K) - U_{12}(2K(KU_{21}\eta_2(2K^2\eta_1 + \sigma_1)\text{Coth}2K + U_{22}(K(4K^2\eta_2^2 + 2\eta_2\sigma_2 - H^2(s - 1))\text{Coth}2K) - k_2^2W_2\text{Coth}2K)\hat{\rho}_3 - 2k_2^2U_{22}U_{23}s\hat{\rho}_3\text{Coth}2K + U_{23}(2K^2\eta_2 + \sigma_2)(\sigma_3s + 2K^2\eta_3\hat{\rho}_3)\text{Coth}2K) - U_{11}(2K(KU_{21}\eta_2(2K^2\eta_1 + \sigma_1)\text{Coth}2K + U_{22}(K(4K^2\eta_2^2 + 2\eta_2\sigma_2 - H^2(s - 1))\text{Coth}2K) - k_2^2W_2\text{Coth}2K)\hat{\rho}_3 - 2k_2^2U_{22}U_{23}s\hat{\rho}_3\text{Coth}2K + U_{23}(2K^2\eta_2 + \sigma_2)(\sigma_3s + 2K^2\eta_3\hat{\rho}_3)\text{Coth}2K) + k_1^2(U_{12}^2(K^2(H^2(s - 1))\text{Coth}2K - 4K^2\eta_2^2 - 2\eta_2\sigma_2) + k_2^2(KW_2 + U_{23}s)\text{Coth}2K)\hat{\rho}_3 + U_{11}\hat{\rho}_1(k_2^2(KW_2 + U_{23}s) + H^2K^2(s - 1))\hat{\rho}_3 + (K(W_2(H^2K^2(1 + \text{Coth}2K + \hat{\rho}_1) + k_2^2(U_{22}^2\text{Coth}2K + U_{21}\hat{\rho}_1)) + W_1(k_2^2(2KW_2 + U_{23}s) + H^2K^2(s - 1))) + U_{13}(H^2K^2(1 + \text{Coth}2K + \hat{\rho}_1) + k_2^2(U_{22}^2\text{Coth}2K + KW_1 + U_{21}\hat{\rho}_1)s)\hat{\rho}_3 - U_{12}U_{13}(2K^2\eta_2(\sigma_3s + 2K^2\eta_3\hat{\rho}_3) - 4sk_2^2U_{22}U_{23}\hat{\rho}_3\text{Coth}2K + \sigma_2(\sigma_3s + 2K^2\eta_3\hat{\rho}_3)) - U_{11}(2K^2U_{12}\eta_2(2K^2\eta_1 + \sigma_1)\hat{\rho}_3\text{Coth}2K + U_{13}(2K^2\eta_1(\sigma_3s + 2K^2\eta_3\hat{\rho}_3) - 4sk_2^2U_{22}U_{23}\hat{\rho}_3 + \sigma_1(\sigma_3s + 2K^2\eta_3\hat{\rho}_3))))))$$

$$\delta_8 = ((\text{Coth}2K + \hat{\rho}_1)\hat{\rho}_3s)^{-1}(k_1^3(2K^2U_{12}^3\eta_2\hat{\rho}_3 + KU_{11}W_2(2K^2\eta_1 + \sigma_1)\hat{\rho}_3 + U_{12}(K(2K^2W_1\eta_2 + W_2(2K^2\eta_2 + \sigma_2)) + 2KU_{11}^2\eta_2\hat{\rho}_1) - sU_{13}^2(2K^2\eta_2 + \sigma_2))\hat{\rho}_3\text{Coth}2K + U_{11}U_{13}^2(2K^2\eta_1 + \sigma_1)s\hat{\rho}_3 + U_{12}^2U_{13}(s\sigma_3 + 2K^2\eta_3\hat{\rho}_3)\text{Coth}2K + U_{13}(KW_1 + U_{11}^2\hat{\rho}_1)(2K^2\eta_3\hat{\rho}_3 - s\sigma_3)) + k_2^2k_2(2K^3U_{21}W_2\eta_1\hat{\rho}_3 - 2U_{11}U_{13}U_{21}\sigma_3\hat{\rho}_1 - 2K^2U_{13}^2U_{21}\eta_1\hat{\rho}_3 - 2K^2U_{13}^2U_{22}\eta_2\hat{\rho}_3\text{Coth}2K + 2K^3U_{22}W_1\eta_2\hat{\rho}_3\text{Coth}2K + 2K^3U_{22}W_2\eta_2\hat{\rho}_3\text{Coth}2K - U_{13}^2U_{21}\sigma_1\hat{\rho}_3 + KU_{21}W_2\sigma_1\hat{\rho}_3 - U_{13}^2U_{22}\sigma_2\hat{\rho}_3\text{Coth}2K + KU_{22}W_2\sigma_2\hat{\rho}_3\text{Coth}2K + 2K^2U_{11}^2U_{22}\eta_2\hat{\rho}_1\hat{\rho}_3\text{Coth}2K + 4K^2U_{11}U_{13}U_{21}\eta_3\hat{\rho}_1\hat{\rho}_3 + 2U_{11}U_{13}U_{21}\sigma_3\hat{\rho}_1\hat{\rho}_3 + 2K^2U_{13}^2U_{21}\eta_1\hat{\rho}_3^2 + 2K^2U_{13}^2U_{22}\eta_2\hat{\rho}_3^2\text{Coth}2K + U_{13}^2U_{21}\sigma_1\hat{\rho}_3^2 + U_{13}^2U_{22}\sigma_2\hat{\rho}_3^2\text{Coth}2K + U_{12}^2(6K^2U_{22}\eta_2\hat{\rho}_3 + U_{23}(2K^2\eta_3\hat{\rho}_3 - s\sigma_3)\text{Coth}2K) + 2U_{12}(2K^2U_{11}U_{21}\eta_2\hat{\rho}_1\hat{\rho}_3 + U_{13}(U_{22}(2K^2\eta_3\hat{\rho}_3 - s\sigma_3) - sU_{23}(2K^2\eta_2 + \sigma_2)\hat{\rho}_3))\text{Coth}2K + U_{23}(KW_1(2K^2\eta_3\hat{\rho}_3 - s\sigma_3) + U_{11}(-2U_{13}(2K^2\eta_1 + \sigma_1)s\hat{\rho}_3 + U_{11}\hat{\rho}_1(2K^2\eta_3\hat{\rho}_3 - s\sigma_3)))) + k_2(k_2^2U_{23}^2(U_{21}(2K^2\eta_1 + \sigma_1) + U_{22}(2K^2\eta_2 + \sigma_2)\text{Coth}2K)s\hat{\rho}_3 + U_{23}(H^2K^2(1 + \text{Coth}2K + \hat{\rho}_1) + k_2^2(U_{22}^2\text{Coth}2K + KW_1 + U_{21}^2\hat{\rho}_1))(2K^2\eta_3\hat{\rho}_3 - s\sigma_3) + K\rho_3(2Kk_2^2U_{21}^2U_{22}\eta_2\hat{\rho}_1\text{Coth}2K + U_{21}(2K^2\eta_1 + \sigma_1)(k_2^2W_2 + H^2K(s-1)) + U_{22}(\sigma_2(k_2^2W_2 + H^2K(s-1))\text{Coth}2K + 2K\eta_2(k_2^2(U_{22}^2 + K(W_1 + W_2)\text{Coth}2K) + H^2K^2(1 - \text{Coth}2K + \hat{\rho}_1\text{Coth}2K + \hat{\rho}_3\text{Coth}2K)))))) + k_1(U_{13}(\sigma_3(H^2K^2(1 + \text{Coth}2K + \hat{\rho}_1) + k_2^2(U_{22}^2\text{Coth}2K + KW_1 + U_{21}^2\hat{\rho}_1))s + 2(H^2K^4\eta_3(1 + \text{Coth}2K + \hat{\rho}_1) + k_2^2(K^2U_{22}^2\eta_3\text{Coth}2K + K^3W_1\eta_3 + K^2U_{21}^2\eta_3\hat{\rho}_1 + U_{21}U_{23}(2K^2\eta_1 + \sigma_1)s + sU_{22}U_{23}(2K^2\eta_2 + \sigma_2)\text{Coth}2K))\hat{\rho}_3) + H^2K^2\hat{\rho}_3(U_{11}(2K^2\eta_1 + \sigma_1)(s-1) + U_{12}(\sigma_2(s-1)\text{Coth}2K + 2K^2\eta_2(1 - \text{Coth}2K + \hat{\rho}_1\text{Coth}2K + \hat{\rho}_3\text{Coth}2K))) + k_2^2(U_{12}(6K^2U_{22}\eta_2\hat{\rho}_3 + \text{Coth}2K(K(2K^2W_1\eta_2 + W_2(2K^2\eta_2 + \sigma_2)) + 2KU_{21}^2\eta_2\hat{\rho}_1) + U_{23}^2(2K^2\eta_2 + \sigma_2)s)\hat{\rho}_3 + 2U_{22}U_{23}(\sigma_3s + 2K^2\eta_3\hat{\rho}_3)\text{Coth}2K) + U_{11}((2K^2\eta_1 + \sigma_1)(KW_2 + U_{23}^2s)\hat{\rho}_3 + 2U_{21}\hat{\rho}_1(2K^2U_{22}\eta_2\hat{\rho}_3\text{Coth}2K + U_{23}(\sigma_3s + 2K^2\eta_3\hat{\rho}_3))))))$$

REFERENCES

- [1] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Oxford Univ.Press, Oxford, 1961.
- [2] J.R. Melcher, Continuum Electromechanics, MIT Press, Cambridge, MA, 1981.
- [3] R.E. Rosensweig, Ferrohydrodynamics, Cambridge Univ. Press, Cambridge,UK, 1985.
- [4] R.C. Sharma, T.J.T. Spanos, Can. J. Phys. vol. 60, pp.1391, 1982.
- [5] R. Raghaven, S.S.Q. Mardsen, J. Mech. Appl. Math. vol. 26, pp. 205, 1973.
- [6] H.H. Bau, Phys. Fluids vol. 25, pp. 1719,1982.
- [7] St.I. Gheorghitza, J. Sci. Eng. Res. vol. 13, pp. 39,1969.
- [8] A. Georgescu, St.I. Gheorghitza, Rev. Roum. Math. Pure Appl. vol. 16, pp. 27,1971.
- [9] J. Bear and Y. Bachmat. Introduction to modeling of transport phenomena in porous media. Kluwer Academic, Dordrecht. 1991.
- [10] G.I. Baranblatt, V.M. Entov, and V.M.Ryzhik. Theory of fluid flows through natural rocks. Kluwer Academic, Dordrecht. 1990.
- [11] K. Zakaria, M.A.Sirwah, andS.A. Alkharashi, Non-Linear Analysis of Creeping Flow on the Inclined Permeable Substrate Plane Subjected to an Electric Field. International Journal of Non-Linear Mechanics, vol. 47, pp. 577-598, 2012.
- [12] P. Kumarand G.J. Singh. Stability of Two Superposed Rivlin-Ericksen Viscoelastic Fluids in the Presence of Suspended Particles. Romanian Journal of Physics. vol. 51, pp. 927-935, 2006.
- [13] Khan, A. and Bhatia, P.K., “Stability of Two Superposed Viscoelastic Fluids in a Horizontal Magnetic Field”, Ind. J. Pure and Appl. Maths, vol.32, pp.99-108, 2001.
- [14] Y.O. El-Dib, R.T. Matoog, J. Colloid Interface Sci. vol.229, pp. 29, 2000.
- [15] Y.O. El-Dib, J. Colloid Interface Sci. vol. 259, pp.309, 2003.
- [16] Sunil, R.C. Sharma, R.S. Chandel, On superposed coupled-stress fluids in porous medium in hydromagnetics, Z. Naturforsch. vol. 57a, pp.955, 2002.
- [17] H.H. Woodson, J.R. Melcher, Electromechanical Dynamics, John Wiley & Sons, 1968.
- [18] L.D. Landau, E.M. Lifshitz, Electrohydrodynamics of Continuous Media, Pergamon Press, Oxford, 1960.
- [19] P.M. Adler, Porous Media, Butterworth–Heinemann, London, 1992.
- [20] K. Zakaria, M.A. Sirwah, S. Alkharashi, Instability through porous media of three layers superposed conducting fluids, European Journal of Mechanics B/Fluids vol.28, pp.263, 2009.