Complex Composition Cordial Labeling Of Path and Star Graphs

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Abstract

*Let <A, *> be any abelian group. A graph G= (V(G), E(G)) is said to be A-cordial if there is a mapping f:V(G)→A which satisfies the following two conditions with each edge e = uv is labeled as f*(uv) = f(u)*f(v)*

- $|v_f(a) - v_f(b)| \leq 1 \ \forall\, a, b \in A$
- $|e_f(a) - e_f(b)| \leq 1 \ \forall\, a, b \in A$

where $v_f(a)$ *= the number of vertices with label a*
$v_f(b)$ *= the number of vertices with label b*
$e_f(a)$ *= the number of edges with label a*
$e_f(b)$ *= the number of edges with label b*

In [3], V_4*-cordial labeling is defined. It motivates me an idea to define CCCL as follows.*

We have defined a set $\mathbb{C}=\{f_1, f_2, f_3, f_4\}$ *where* $f_1=z$, $f_2=-z$, $f_3=1/z$, $f_4=-1/z$ $\forall\, z \in C-\{0\}$ *is an abelian group and under binary operation * is defined as* $f_1 * f_2 = f_1 \circ f_2 = f_1(f_2)$.

*We note that if <ℂ, *> is an abelian group. Then the labeling is known as Complex Composition Cordial Labeling, and in short denoted as CCCL. A graph which admits CCCL is called as Complex Composition Cordial Graph, which is denoted as CCCG.*

In this paper, it is proved that Path P_n *and Star* S_n *are Complex Composition Cordial graphs.*

Keywords: *Complex Composition Cordial Graph, Complex Composition Cordial Labeling.*
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1. Introduction:

A graph G is a finite non empty set of objects called vertices together with a set of pairs of distinct vertices of G which is called edges. Each e= {uv} of vertices in E is called an edge or a line of G. For Graph Theoretical Terminology, [2].

A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge *uv* a label depending on the vertex labels of *u* and *v*.

A graph *G* is said to be labeled if the *n* vertices are distinguished from one another by symbols such as $v_1, v_2, \ldots, v_n$.

In this paper, it is proved that Path P_n and Star S_n are Complex Composition Cordial graphs.

2. Preliminaries:

Definition:2.1

Let G = (V, E) be a simple graph. Let *f* : V(G) → {0, 1} and for each edge *uv*, assign the label |*f*(*u*) − *f*(*v*)|. *f* is called a **Cordial labeling** if the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the number of edges labeled 1 differ by atmost 1. A graph is called **Cordial** if it has a cordial labeling.

Definition:2.2

Let <*A*, *> be any abelian group. A graph *G= (V(G), E(G))* is said to be *A*-cordial if there is a mapping *f:V(G)→A* which satisfies the following two conditions with each edge *e* = *uv* is labeled as *f*(uv) = f(u)*f(v)*

- $|v_f(a) - v_f(b)| \leq 1 \ \forall\, a, b \in A$
- $|e_f(a) - e_f(b)| \leq 1 \ \forall\, a, b \in A$

where $v_f(a)$ = the number of vertices with label *a*

$v_f(b)$ = the number of vertices with label b
$e_f(a)$ = the number of edges with label a
$e_f(b)$ = the number of edges with label b

We have defined a set $\mathbb{C}=\{f_1, f_2, f_3, f_4\}$ where $f_1=z$, $f_2=-z$, $f_3=1/z$, $f_4=-1/z$ $\forall$ $z \in C-\{0\}$ is an abelian group and under binary operation * is defined as $f_1 * f_2 = f_1 \circ f_2 = f_1(f_2)$.

We note that if $<\mathbb{C}, *>$ is an abelian group. Then the labeling is known as **Complex Composition Cordial Labeling**, and in short denoted as **CCCL**. A graph which admits CCCL is called as **Complex Composition Cordial Graph**, which is denoted as **CCCG**.

Definition:2.3

A **Walk** of a graph G is an alternating sequence of vertices and edges $v_1, e_1, v_2, e_2, \ldots, v_{n-1}, e_n, v_n$ beginning and ending with vertices such that each edge e_i is incident with vertices v_{i-1} and v_i.

Definition:2.4

In a walk, if all the vertices are distinct, then it is called a **Path** and a path of length n is denoted by $\mathbf{P_{n+1}}$.

Definition:2.5

A **Bipartite Graph** is a graph whose vertex set V(G) can be partitioned into two subsets V_1 and V_2 such that every edge of G has one end in V_1 and the other end in V_2. (V_1, V_2) is called a bipartition of G. Further, if every vertex of V_1 is joined to all the vertices of V_2, then G is called a **Complete Bipartite Graph**. The complete bipartite graph with bipartition (V_1,V_2) such that $|V_1|$ = m and $|V_2|$ = n is denoted by $\mathbf{K_{m,n}}$. A complete bipartite graph $\mathbf{K_{1,n}}$ or $\mathbf{K_{n,1}}$ or $\mathbf{S_n}$ is called as **Star**.

3. Main Results:

Theorem:3.1

Path P_n is Complex Composition Cordial graphs.

Proof:

Let $V[P_n] = \{ u_i : 1 \le i \le n \}$ and $E[P_n] = \{ (u_iu_{i+1}): 1 \le i \le n-1 \}$.

Let f: $V[P_n] \to \{f_1, f_2, f_3, f_4\}$

The vertices are labeled under 3 cases

Case a:

Let P_n be the graph when $n \equiv 0 \ (mod\ 4)$ and $n \neq 4$.

$$f(u_i) = \begin{cases} f_1 & i \equiv 0, 7\ mod\ 8 \\ f_2 & i \equiv 1, 5\ mod\ 8 \\ f_3 & i \equiv 2, 6\ mod\ 8 \\ f_4 & i \equiv 3, 4\ mod\ 8 \end{cases}, \ 1 \le i \le 8.$$

$$f(u_i) = \begin{cases} f_1 & i \equiv 0,1\ mod\ 8 \\ f_2 & i \equiv 2, 6\ mod\ 8 \\ f_3 & i \equiv 3, 7\ mod\ 8 \\ f_4 & i \equiv 4, 5\ mod\ 8 \end{cases}, \ 9 \le i \le n.$$

Then the induced edge labeling are

$$f^*(u_iu_{i+1}) = \begin{cases} f_1 & i \equiv 3,7\ mod\ 8 \\ f_2 & i \equiv 2\ mod\ 8 \\ f_3 & i \equiv 4, 6\ mod\ 8 \\ f_4 & i \equiv 1, 5\ mod\ 8 \end{cases}, \ 1 \le i \le 7.$$

$$f^*(u_iu_{i+1}) = \begin{cases} f_1 & i \equiv 0, 4\ mod\ 8 \\ f_2 & i \equiv 1, 3\ mod\ 8 \\ f_3 & i \equiv 5, 7\ mod\ 8 \\ f_4 & i \equiv 2, 6\ mod\ 8 \end{cases}, \ 8 \le i \le n-1.$$

Case b:

Let P_n be the graph when $n \equiv 1 \ (mod\ 4)$ and $n \neq 5$.

$$f(u_i) = \begin{cases} f_1 & i \equiv 1 \; mod \; 8 \\ f_2 & i \equiv 2,6 \; mod \; 8 \\ f_3 & i \equiv 0,7 \; mod \; 8 \\ f_4 & i \equiv 4,5 \; mod \; 8 \end{cases}, \; 1 \le i \le 9.$$

$$f(u_i) = \begin{cases} f_1 & i \equiv 1,5 \; mod \; 8 \\ f_2 & i \equiv 2,3 \; mod \; 8 \\ f_3 & i \equiv 6,7 \; mod \; 8 \\ f_4 & i \equiv 0,4 \; mod \; 8 \end{cases}, \; 10 \le i \le n.$$

Then the induced edge labeling are

$$f^*(u_i u_{i+1}) = \begin{cases} f_1 & i \equiv 4,7 \; mod \; 8 \\ f_2 & i \equiv 1,3 \; mod \; 8 \\ f_3 & i \equiv 5,0 \; mod \; 8 \\ f_4 & i \equiv 2,6 \; mod \; 8 \end{cases}, \; 1 \le i \le 8.$$

$$f^*(u_i u_{i+1}) = \begin{cases} f_1 & i \equiv 2,6 \; mod \; 8 \\ f_2 & i \equiv 1,7 \; mod \; 8 \\ f_3 & i \equiv 3,5 mod \; 8 \\ f_4 & i \equiv 0,4 \; mod \; 8 \end{cases}, \; 9 \le i \le n-1.$$

Case c:

Let P_n be the graph when $n \equiv 2,3 \; (mod \; 4)$.

$$f(u_i) = \begin{cases} f_1 & i \equiv 1,0 \; mod \; 8 \\ f_2 & i \equiv 2,6 \; mod \; 8 \\ f_3 & i \equiv 3,7 \; mod \; 8 \\ f_4 & i \equiv 4,5 \; mod \; 8 \end{cases}, \; 1 \le i \le n.$$

Then the induced edge labeling are

$$f^*(u_i u_{i+1}) = \begin{cases} f_1 & i \equiv 0,4 \; mod \; 8 \\ f_2 & i \equiv 1,3 \; mod \; 8 \\ f_3 & i \equiv 5,7 \; mod \; 8 \\ f_4 & i \equiv 2,6 \; mod \; 8 \end{cases}, \; 1 \le i \le n-1.$$

When $n = 4$ and 5, it has been enumerated for all possible labeling and noted, it is not CCCL.

Vertex condition and Edge condition:

Case 1:

When $\boldsymbol{n \equiv 0 \; (mod \; 8)}$

$v_f(f_i) = \frac{n}{4}, 1 \le i \le 4$ $\qquad$ $e_f(f_i) = \frac{n}{4}$, $i = 1,3,4$

$e_f(f_2) = \frac{n}{4} - 1$

Case 2:

When $\boldsymbol{n \equiv 1 \; (mod \; 8)}$

$v_f(f_i) = \frac{n-1}{4}$, $i = 1,2,4$ $\qquad$ $e_f(f_i) = \frac{n-1}{4}, 1 \le i \le 4$

$v_f(f_3) = \frac{n+3}{4}$

Case 3:

When $\boldsymbol{n \equiv 2 \; (mod \; 8)}$

$v_f(f_i) = \frac{n+2}{4}$, $i = 1,2.$ $\qquad$ $e_f(f_i) = \frac{n-2}{4}$, $i = 1,3,4$

$v_f(f_i) = \frac{n-2}{4}$, $i = 3,4$ $\qquad$ $e_f(f_2) = \frac{n+2}{4}$

Case 4:

When $\boldsymbol{n \equiv 3 \; (mod \; 8)}$

$v_f(f_i) = \frac{n+1}{4}, 1 \le i \le 3$ $\qquad$ $e_f(f_i) = \frac{n-3}{4}$, $i = 1,3.$

$v_f(f_3) = \frac{n-3}{4}$ $\qquad$ $e_f(f_i) = \frac{n+1}{4}$, $i = 2,4.$

Case 5:

When $\boldsymbol{n \equiv 4 \; (mod \; 8)}$

$v_f(f_i) = \frac{n}{4}$, $1 \le i \le 4$ $\quad$ $e_f(f_i) = \frac{n}{4}$, $i = 1, 2, 4$

$e_f(f_3) = \frac{n}{4} - 1$

Case 6:

When $n \equiv 5 \ (mod\ 8)$

$v_f(f_i) = \frac{n-1}{4}$, $i = 1, 3, 4$ $\quad$ $e_f(f_i) = \frac{n-1}{4}$, $1 \le i \le 4$

$v_f(f_2) = \frac{n+3}{4}$

Case 7:

When $n \equiv 6 \ (mod\ 8)$

$v_f(f_i) = \frac{n-2}{4}$, $i = 1, 3$ $\quad$ $e_f(f_i) = \frac{n-2}{4}$, $i = 1, 3, 4$

$v_f(f_i) = \frac{n+2}{4}$, $i = 2, 4.$ $\quad$ $e_f(f_2) = \frac{n+2}{4}$

Case 8:

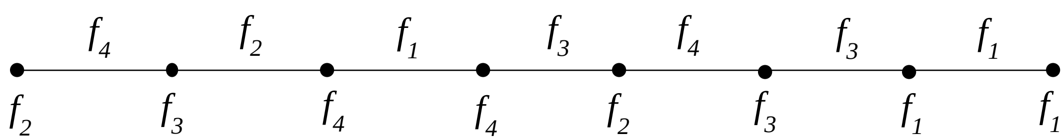
When $n \equiv 7 \ (mod\ 8)$

$v_f(f_1) = \frac{n-3}{4}$ $\quad$ $e_f(f_i) = \frac{n-3}{4}$, $i = 1, 3.$

$v_f(f_i) = \frac{n+1}{4}$, $2 \le i \le 3$ $\quad$ $e_f(f_i) = \frac{n+1}{4}$, $i = 2, 4.$

Example:

Consider the example P_8 for case 1 and labeling follows case a

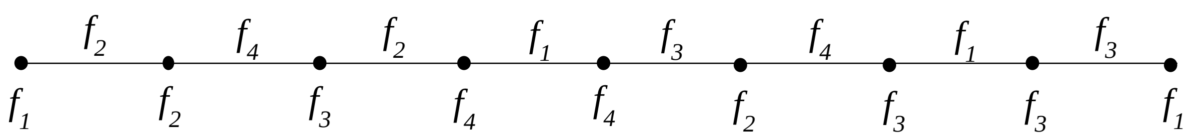


$v_f(f_i) = 2$, $i = 1, 3, 4$ $\quad$ $e_f(f_i) = 2$, $i = 1, 3, 4$

$v_f(f_2) = 2$ $\quad$ $e_f(f_2) = 1$

Here $|v_f(f_i) - v_f(f_j)| = 0$ and $|e_f(f_i) - e_f(f_j)| \le 1$

P_9



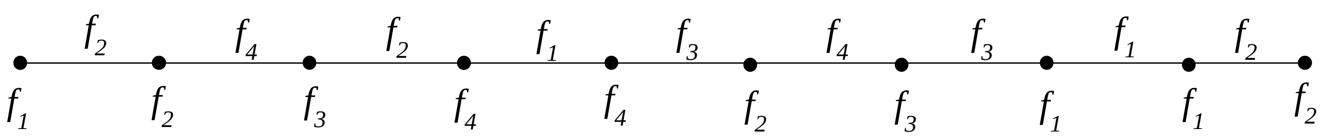
Case 2 and case b

$v_f(f_i) = 2$, $i = 1, 2, 4$ $\quad$ $e_f(f_i) = 2$, $i = 1, 2, 4$

$v_f(f_3) = 3$ $\quad$ $e_f(f_3) = 2$

Here $|v_f(f_i) - v_f(f_j)| \le 1$ and $|e_f(f_i) - e_f(f_j)| = 0$

P_{10}

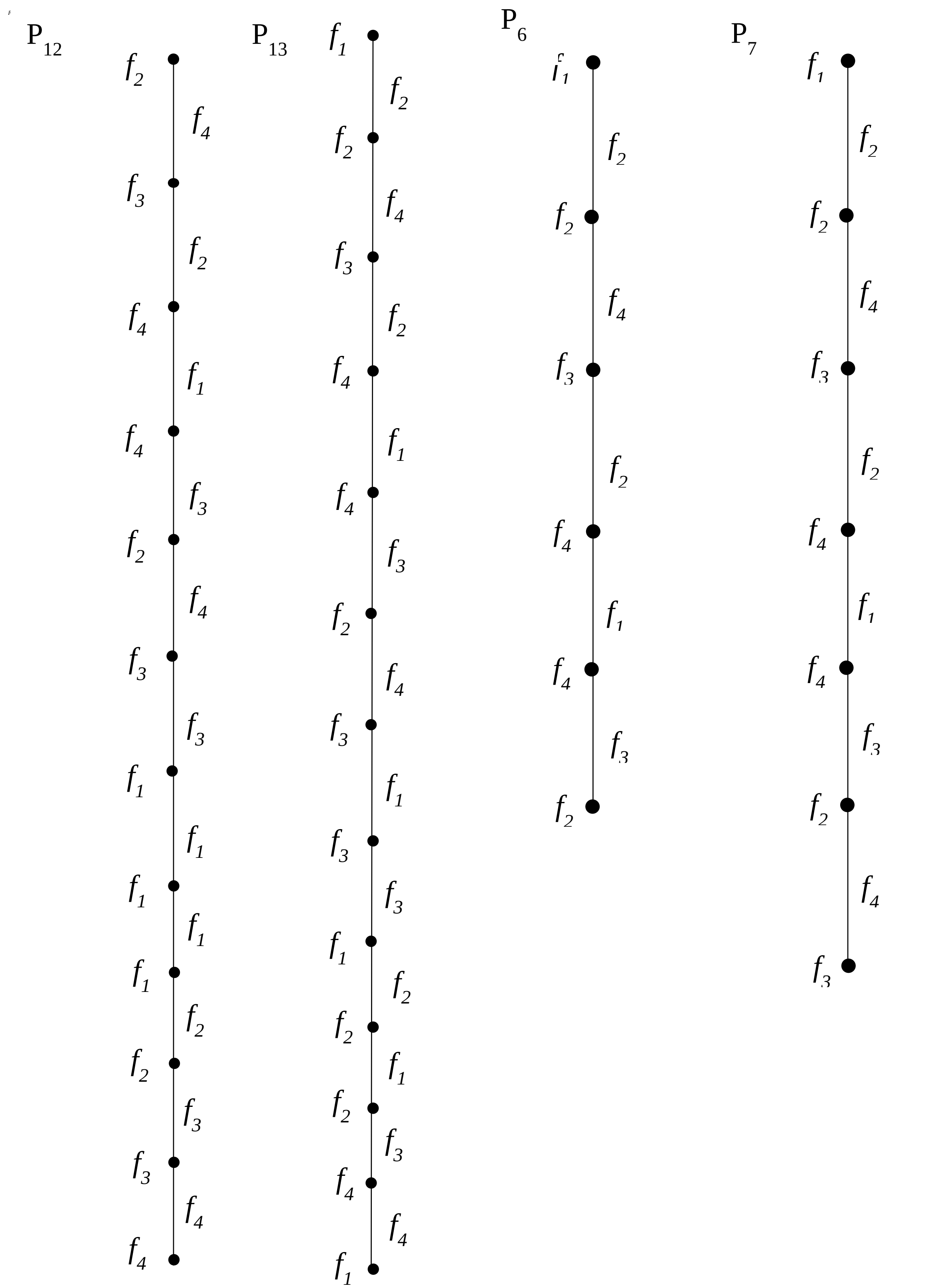


Case 3 and case c

$v_f(f_i) = 3$, $i = 1, 2$ $\quad$ $e_f(f_i) = 2$, $i = 1, 3, 4$

$v_f(f_i) = 2$, $i = 3, 4$ $\quad$ $e_f(f_2) = 3$

Here $|v_f(f_i) - v_f(f_j)| \le 1$ and $|e_f(f_i) - e_f(f_j)| \le 1$



P_{12}
f_2 f_4 f_3 f_2 f_4 f_1 f_4 f_3 f_2 f_4 f_3 f_3 f_1 f_1 f_1 f_1 f_1 f_2 f_2 f_3 f_3 f_4 f_4
P_{13}
f_1 f_2 f_2 f_4 f_3 f_2 f_4 f_1 f_4 f_3 f_2 f_4 f_3 f_1 f_3 f_3 f_1 f_2 f_2 f_1 f_2 f_3 f_4 f_4 f_1
P_6
f_1 f_2 f_2 f_4 f_3 f_2 f_4 f_1 f_4 f_3 f_2
P_7
f_1 f_2 f_2 f_4 f_3 f_2 f_4 f_1 f_4 f_3 f_2 f_4 f_3

case 5 and case a case 6 and case b case 7 and case c case 8 and case c

For P_{12}

$v_f(f_i) = 3, 1 \le i \le 4$ $e_f(f_i) = 3, i = 1, 2, 4$

$e_f(f_3) = 2$

Here $|v_f(f_i) - v_f(f_j)| = 0$ and $|e_f(f_i) - e_f(f_j)| \le 1$

For P_{13}

$v_f(f_i) = 3, i = 1, 3, 4$ $e_f(f_i) = 3, 1 \le i \le 4$

$v_f(f_2) = 4$

Here $|v_f(f_i) - v_f(f_j)| \le 1$ and $|e_f(f_i) - e_f(f_j)| = 0$

For P_6

$v_f(f_i) = 1, i = 1, 3$ $e_f(f_i) = 1, i = 1, 3, 4$

$v_f(f_i) = 2, i = 2, 4$ $e_f(f_2) = 2$

Here $|v_f(f_i) - v_f(f_j)| \le 1$ and $|e_f(f_i) - e_f(f_j)| \le 1$

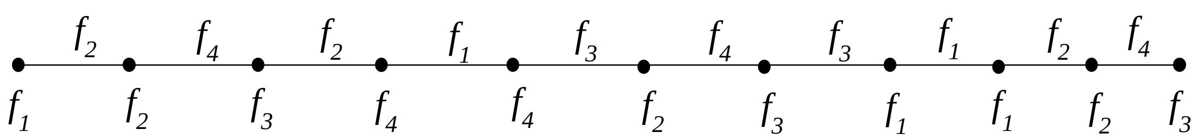
For P_6

$v_f(f_1) = 1$ $e_f(f_i) = 1, i = 1, 4$

$v_f(f_i) = 2, i = 2, 3, 4$ $e_f(f_i) = 2, i = 2, 3$

Here $|v_f(f_i) - v_f(f_j)| \le 1$ and $|e_f(f_i) - e_f(f_j)| \le 1$

P_{11}



Case 4 and case c

$v_f(f_i) = 3, 1 \le i \le 3$ $e_f(f_i) = 2, i = 1, 3$

$v_f(f_4) = 2$ $e_f(f_i) = 3, i = 2, 4$

Here $|v_f(f_i) - v_f(f_j)| \le 1$ and $|e_f(f_i) - e_f(f_j)| \le 1$

Thus P_n is a Complex Composition Cordial Graph.

Theorem 2:
Star S_n is Complex Composition Cordial graphs.
Proof:
Let $V[S_n] = \{u_i : 1 \le i \le n+1\}$
$E[S_n] = \{(u_1 u_i): 1 \le i \le n\}$.
Let f: $V[S_n] \to \{f_1, f_2, f_3, f_4\}$ defined by

$$f(u_i) = \begin{cases} f_1 & i \equiv 1 \bmod 4 \\ f_2 & i \equiv 2 \bmod 4 \\ f_3 & i \equiv 3 \bmod 4 \\ f_4 & i \equiv 0 \bmod 4 \end{cases}, \quad 1 \le i \le n+1.$$

Then the induced edge labeling are

$$f^*(u_1 u_i) = \begin{cases} f_1 & i \equiv 1 \bmod 4 \\ f_2 & i \equiv 2 \bmod 4 \\ f_3 & i \equiv 3 \bmod 4 \\ f_4 & i \equiv 0 \bmod 4 \end{cases}, \quad 2 \le i \le n+1.$$

Vertex condition and Edge condition:

Case 1:
When $n \equiv 0 \pmod 4$
$v_f(f_i) = \frac{n}{4} + 1, i = 1$ $e_f(f_i) = \frac{n}{4}, 1 \le i \le 4$

$v_f(f_i) = \frac{n}{4}, 2 \le i \le 4$

Case 2:

When $n \equiv 1 \ (mod\ 4)$

$v_f(f_i) = \frac{n+3}{4}, i = 1, 2$ $\qquad$ $e_f(f_i) = \frac{n-1}{4}, i = 1, 3, 4$

$v_f(f_i) = \frac{n-1}{4}, i = 3, 4$ $\qquad$ $e_f(f_i) = \frac{n+3}{4}, i = 2$

Case 3:

When $n \equiv 2 \ (mod\ 4)$

$v_f(f_i) = \frac{n+2}{4}, 1 \le i \le 3$ $\qquad$ $e_f(f_i) = \frac{n-2}{4}, i = 1, 4$

$v_f(f_i) = \frac{n-2}{4}, i = 4$ $\qquad$ $e_f(f_i) = \frac{n+2}{4}, i = 2, 3$

Case 4:

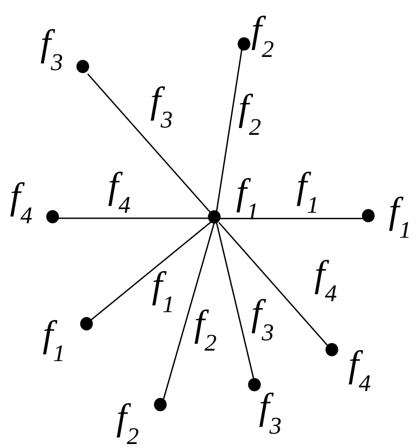
When $n \equiv 3 \ (mod\ 4)$

$v_f(f_i) = \frac{n+1}{4}, 1 \le i \le 4$ $\qquad$ $e_f(f_i) = \frac{n-3}{4}, i = 1.$

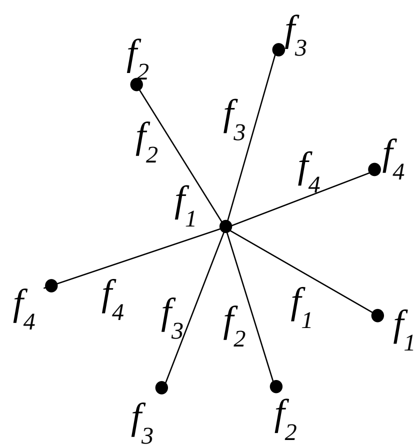
$e_f(f_i) = \frac{n+1}{4}, 2 \le i \le 4.$

Example:

S_8 $\qquad$ S_7



Case 1



Case 2

For S_8

$v_f(f_1) = 3$ $\qquad$ $e_f(f_i) = 2, 1 \le i \le 4$

$v_f(f_i) = 2, i = 2, 3, 4$

Here $|v_f(f_i) - v_f(f_j)| \le 1$ and $|e_f(f_i) - e_f(f_j)| = 0$

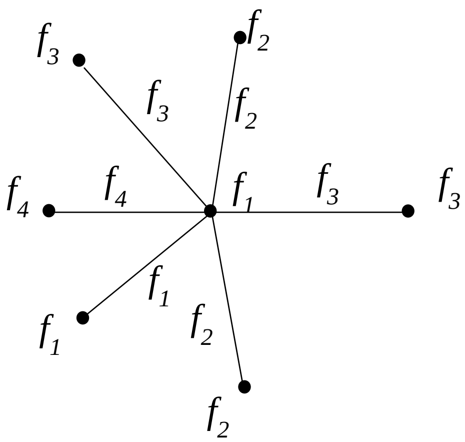
For S_7

$v_f(f_i) = 2, 1 \le i \le 4$ $\qquad$ $e_f(f_1) = 1$

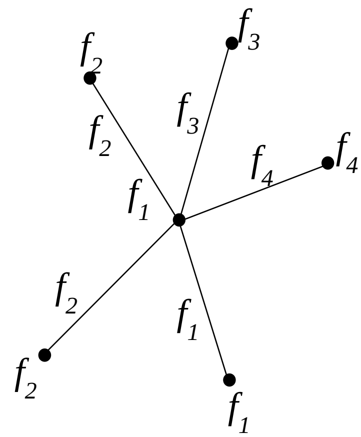
$e_f(f_i) = 2, i = 2, 3, 4$

Here $|v_f(f_i) - v_f(f_j)| \le 1$ and $|e_f(f_i) - e_f(f_j)| \le 1$

S_6 $\qquad$ S_5



Case 3



Case 4

For S_6

$v_f(f_i) = 2, 1 \leq i \leq 3$ $e_f(f_i) = 1, i = 1, 4$

$v_f(f_4) = 1$ $e_f(f_i) = 2, i = 2, 3$

Here $|v_f(f_i) - v_f(f_j)| \leq 1$ and $|e_f(f_i) - e_f(f_j)| \leq 1$

For S_5

$v_f(f_i) = 2, i = 1, 2$ $e_f(f_i) = 1, i = 1, 3, 4$

$v_f(f_i) = 1, i = 3, 4$ $e_f(f_2) = 2$

Here $|v_f(f_i) - v_f(f_j)| \leq 1$ and $|e_f(f_i) - e_f(f_j)| \leq 1$

Thus S_n is a Complex Composition Cordial Graph.

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