On Steiner Domination Number of Graphs

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Abstract -*For a connected graph G, a set of vertices W in G is called a Steiner dominating set if W is both a Steiner set and a dominating set. The minimum cardinality of a Steiner dominating set of G is its Steiner domination numberand is denoted by* $\gamma_s(G)$*. In this paper, it is studied that how the Steiner domination number is affected by adding a single edge to paths, complete graphs, cycles, star and wheel graph. Also, it is studied that how it is affected by deleting edges from complete graphs.*

Keywords- *Domination, Steiner number and Steiner domination number***.**

I. INTRODUCTION

The concept of domination in graphs was introduced by Ore and Berge [4]. Let$G$ = (V,E) be a finite undirected graph with neither loops nor multiple edges. Asubset D of $V(G)$ is a dominating set of G if every vertex in $V−D$ is adjacentto at least one vertex in D. The minimum cardinality of a dominating set of $G$is called the domination number of G and is denoted by $\gamma(G)$. The concept ofSteiner number of a graph was introduced by G. Chatrand and P. Zhang [1]. For anonempty set W of vertices in a connected graph G, the Steiner distance $d(W)$ of$W$ is the minimum size of a connected sub graph of G containing W. Necessarilyeach such subgraph is a tree and is called a Steiner tree with respect to W or aSteiner W-tree. The set of all vertices of G that lie on some Steiner W-tree isdenoted by $S(W)$. If $S(W)$=V, then W is called a Steiner set for G. A Steinerset of minimum cardinality is the Steiner number $s(G)$ of G.

The concept of Steiner domination number of a graph was introduced by J.John,G.Edwin and P. PaulSudhahar [3]. For a connected graph G, a set of vertices W in$G$ is called a Steiner dominating set if W is both a Steiner set and a dominating set.The minimum cardinality of a Steiner dominating set of G is its Steiner dominationnumber and is denoted by $\gamma_s(G)$. A Steiner dominating set of cardinality $\gamma_s(G)$ is said to be a γ_s- set. The concept of (G,D) number of a graph was introducedby K.Palani and A.Nagarajan[5]. They further studied the (G,D)-number of edgeadded and edge deleted graphs in [6,7]. Motivated by those results in this paperwe tried to find the Steiner domination number of edge added and edge deletedgraphs.

A clique in G is a complete subgraph of G. The complete bipartite graph $K_{1,n}$or $K_{n,1}$ is called a star. Let us recall certain existing results which are useful in thesequel of the paper.

Theorem 1.1.*[3] For any integer* $p \geq 2$, $\gamma_s(K_p) = p$.

Theorem1.2. *[8] For any integer* $n \geq 2$,

$$\gamma_s(P_n) = \begin{cases} \left\lceil \frac{n-4}{3} \right\rceil + 2 & if\ n \geq 5; \\ 2 & if\ n = 2,3 or 4 \end{cases}$$

Theorem 1.3. *[8] For any n > 5,* $\gamma_s(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

Theorem 1.4. *[9] For any* $n \geq 5$, $\gamma_s(W_{1,n}) = n - 2$.

II. STEINER DOMINATION NUMBER OF EDGE ADDED GRAPHS

Theorem 2.1. *Let G′ be the graph obtained from* P_{3k}*by adding a vertex v to oneof its vertices. Then,* $\gamma_s(G')$ *is either* $\gamma_s(P_{3k})$ *or* $\gamma_s(P_{3k})$ *+ 1.*

Proof. Let $P_{3k} = (v_1, v_2, ..., v_{3k})$.

Case (i):v is attached to v_1 or v_{3k}. Then $G' \cong P_{3k+1}$and so,

$$\gamma_s(G') = \gamma_s(P_{3k+1}) = \left\lceil \frac{3k+1-4}{3} \right\rceil + 2 = (k-1) + 2 =$$

$$\left\lceil \frac{3(k-1)}{3} - \frac{1}{3} \right\rceil + 2 = \left\lceil \frac{3k-3-1}{3} \right\rceil + 2 = \left(\left\lceil \frac{3k-4}{3} \right\rceil + 2 \right) = \gamma_s(P_{3k})$$

Therefore, $\gamma_s(G') = \gamma_s(P_{3k})$

Case (ii):v is attached to an internal vertex.

Therefore, v becomes an end vertex of G'. Therefore, $v \in$ every Steiner dominatingset of G'. Further, $W \cup \{v\}$ is a Steiner dominating set of G'if and only if W is aSteiner dominating set of P_{3k}.

Therefore, $\gamma_s(G') = \gamma_s(P_{3k}) + 1$.

By cases (i) and (ii), $\gamma_s(G')$ is either $\gamma_s(P_{3k})$ or $\gamma_s(P_{3k})$ + 1.

Theorem 2.2. *If G′ is the graph obtained from* P_{3k+1} *by adding a new vertex v to one of its vertices, then* $\gamma_s(G') = \gamma_s(P_{3k+1}) + 1$.

Proof. Let $P_{3k+1} = (v_1, v_2, ..., v_{3k+1})$

Case (i):v is attached to v_1 or v_{3k+1}.

Then, $G' \cong P_{3k+2}$.

Therefore,

$$\gamma_s(G')=\gamma_s(P_{3k+2})=\left\lceil\frac{3k+2-4}{3}\right\rceil+2=\left\lceil\frac{3k-2}{3}\right\rceil+2=$$
$$\left\lceil\frac{3(k-1)+1}{3}\right\rceil+2=\left\lceil(k-1)+\frac{1}{3}\right\rceil+2=k+2=(k+1)+1=$$
$$\left(\left\lceil\frac{3k+1-4}{3}\right\rceil+2\right)+1=\gamma_s(P_{3k+1})+1$$

Therefore, $\gamma_s(G')=\gamma_s(P_{3k+1})+1$.

Case (ii): v is attached to an internal vertex.
Therefore, v becomes an end vertex of G'. Therefore, $v\in$ every Steiner dominatingset of G' and clearly, $W\cup\{v\}$ is a unique Steiner dominating set of G' where W isthe unique Steiner dominating set of P_{3k+1}.
Therefore, $\gamma_s(G')=\gamma_s(P_{3k+1})+1$.
By cases (i) and (ii), $\gamma_s(G')=\gamma_s(P_{3k+1})+1$.

Theorem 2.3. *Let n=3k+2. If G′ is the graph obtained from P_n by adding anew vertex v to one of its vertices, then*

$$\gamma_s(G')=\begin{cases}\gamma_s(P_n) & \text{if } v \text{ is attached to } v_1,v_3,v_6,\ldots,v_{3k},v_n\\ \gamma_s(P_n)+1 & \text{otherwise}\end{cases}$$

Proof. Let $P_n=(v_1, v_2, \ldots, v_n)$.
When v is attached to v_1 or v_n, $G'\cong P_{n+1}=P_{3k+3}$.
Therefore,

$$\gamma_s(G')=\gamma_s(P_{3k+3})=\left\lceil\frac{3k+3-4}{3}\right\rceil+2=\left\lceil\frac{3k-1}{3}\right\rceil+2=$$
$$\left\lceil\frac{3k-1}{3}\right\rceil+2=\left\lceil k-\frac{1}{3}\right\rceil+2=k+2=\left\lceil\frac{3k-2}{3}\right\rceil+2=$$
$$\left\lceil\frac{3k+2-4}{3}\right\rceil+2=\gamma_s(P_{3k+2})=\gamma_s(P_n).$$

Therefore, $\gamma_s(G')=\gamma_s(P_n)$.
When v is attached to $v_3, v_6, \ldots, v_{3k}$, v becomes an end vertex and belongs toevery Steiner dominating set W' of G'. Let Wbe the minimum Steiner dominatingset of P_n, then $W-\{v_i\}\cup\{v\}$ is the minimum Steiner dominating set of G'.
Therefore, $|W'|=|W|$.
Hence, $\gamma_s(G')=\gamma_s(P_n)$.
When v is attached to other vertices, v becomes an end vertex of G'. Therefore,$v\in$ every Steiner dominating set of G'. Further, $W\cup\{v\}$ is a Steiner dominatingset of G' if and only if W is a Steiner dominating set of P_n.
Therefore, $\gamma_s(G')=\gamma_s(P_n)+1$.

Theorem 2.4. *If G′ is the graph obtained from the star graph $K_{1,n}$by adding a new vertex v′ to one of its vertices, then*

$$\gamma_s(G')=\begin{cases}\gamma_s(K_{1,n})+1 & \text{if } v' \text{ is added to the central vertex}\\ \gamma_s(K_{1,n}) & \text{otherwise.}\end{cases}$$

Proof. Let $V(K_{1,n})=\{v, v_1, v_2, \ldots, v_n\}$.
Case (i): $v'$is added to the central vertex.
Then, $v'$is an end vertex of G'. Therefore, $v'\in$ every Steiner dominating set ofG' and clearly, $W\cup\{v'\}$ is a unique Steiner dominating set of G' where W is theunique Steiner dominating set of $K_{1,n}$.
Therefore, $\gamma_s(G')=\gamma_s(K_{1,n})+1$.
Case (ii): $v'$is added to an end vertex.
Then, v' becomes an end vertex of G' and the end vertex in which $v'$is joinedbecomes an internal vertex of G', let it be v_i, $1\le i\le n$. Therefore, $v'\in$ everySteiner dominating set of G'. Let W be the unique Steiner dominating set of $K_{1,n}$.
Then, $W'=W-\{v_i\}\cup\{v'\}$ is the unique Steiner dominating set of G', since $W'$is the set of all end vertices of G'.
Therefore, $\gamma_s(G')=|W'|=|W|=\gamma_s(K_{1,n})$.
Hence, $\gamma_s(G')=\gamma_s(K_{1,n})$ or $\gamma_s(K_{1,n})+1$.

Theorem 2.5. *Let n = 3k and $k\ge 2$. If G′ is the graph obtained from the cycleC_n by adding a new vertex v to one of its vertices, then $\gamma_s(G')=\gamma_s(C_n)+1$.*

Proof. Let $V(C_n)=\{v_1, v_2, \ldots, v_n\}$.
When $k=2$, the cycle is C_6 and $V(C_6)=\{v_1,v_2,\ldots, v_6\}$.If v is added to any vertex of the cycle then, v becomes an end vertex of G'.
Therefore, v belongs to every Steiner dominating set of G′. Label the vertex towhich v is added as v_1. Then, $W'=\{v,v_1,v_4\}$ is the unique Steiner dominatingset of G'.

Therefore, $\gamma_s(G')=3=2+1=\left\lceil\frac{6}{3}\right\rceil+1=\gamma_s(C_6)+1$.

Hence, $\gamma_s(G')=\gamma_s(C_n)+1$, if $k=2$.
Let $k>2$. Suppose $V(C_{3k})=\{v_1, v_2, \ldots, v_{3k}\}$.
As before, label the vertex to which v is added as v_1.Then, $W_1=\{v, v_3, v_6, v_9, \ldots, v_{3k}\}$,$W_2=\{v, v_2, v_5, v_8, \ldots, v_{3k-1}\}$ and$W_3=\{v, v_1, v_4, v_7, \ldots, v_{3k-2}\}$ are the Steiner dominating sets of G'.
Further, $|W_1|=|W_2|=|W_3|=k+1$.

Therefore, $\gamma_s(G')=k+1=\left\lceil\frac{3k}{3}\right\rceil+1=\gamma_s(C_{3k})+1$.

Hence, $\gamma_s(G')=\gamma_s(C_n)+1$, if $k>2$.

Hence, $\gamma_s(G')=\gamma_s(C_n)+1$, whenever $n=3k$ and $k\ge 2$.

Remark 2.6. *In contrast to the fact obtained in Theorem 2.5, note that $\gamma_s(C_3)=3$.*

Theorem 2.7. *Let n = 3k+1. If G′ is the graph obtained from the cycle by addinga new vertex v to oneof its vertices, then*

$$\gamma_s(G')=\begin{cases}\gamma_s(C_n)+1 & \text{if } k=2\\ \gamma_s(C_n) & \text{otherwise}\end{cases}$$

Proof. Let $V(C_n)=\{v_1, v_2, \ldots, v_n\}$. Label the vertex to which v is added as v_1.
Case (i): $k=2$. Then, the cycle is C_7.

Then, W_1={v, v_1, v_4, v_7},W_2={v, v_2, v_4, v_6} andW_3={v,v_3,v_5,v_6} are the Steiner dominating sets ofG'.Therefore,

$$\gamma_s(G') = 4 = 3+1 = \left\lceil \frac{7}{3} \right\rceil + 1 = \gamma_s(C_7)+1.$$

Hence, $\gamma_s(G') = \gamma_s(C_n)+1$.

Case (ii):$k \neq 2$. Then, W' = {v, v_3, v_6, v_9, ...,v_{3k}} is the unique Steiner dominating set of G'.
Therefore,

$$\gamma_s(G') = k+1 = \left\lceil \frac{3k+1}{3} \right\rceil + 1 = \gamma_s(C_{3k+1}) = \gamma_s(C_n).$$

Theorem 2.8. *Let n = 3k + 2 and k ≥ 2. If G′ is the graph obtained from thecycle C_n by adding a new vertex v to one of its vertices, then* $\gamma_s(G') = \gamma_s(C_n)+1$.

Proof. Let C_n = (v_1, v_2, ...,v_n, v_1). Label the vertex to which v is added as v_1.
Let k = 2. The cycle is C_8. Then, W_1= {v,v_1, v_4,v_7},W_2 = {v, v_2, v_5, v_8}andW_3 = {v, v_3, v_5, v_7} are the Steiner dominating sets of G'.

Therefore, $\gamma_s(G') = 4 = 3+1 = \left\lceil \frac{8}{3} \right\rceil + 1 = \gamma_s(C_8)+1.$

Hence, $\gamma_s(G') = \gamma_s(C_n)+1$ if k = 2.

Let k> 2.
Then, W_1 = {v, v_3, v_6, v_9, ..., v_{3k}, v_{3k+2}},W_2 = {v, v_3, v_6, v_9, ..., v_{3k}, v_{3k+1}},W_3 = {v, v_2, v_5, v_8, ..., v_{3k-1}, v_{3k}},W_4 = {v, v_2, v_5, v_8, ..., v_{3k-1}, v_{3k+2}},W_5 = {v, v_1, v_4, v_7, ..., v_{3k-2}, v_{3k}} and W_6 = {v, v_1, v_4, v_7, ..., v_{3k-2}, v_{3k+1}} are theSteiner dominating sets of G'. Further, the cardinality of the above sets is same.Therefore,

$$\gamma_s(G') = k+2 = \left\lceil \frac{3k+2}{3} \right\rceil + 1 = \gamma_s(C_{3k+2})+1 = \gamma_s(C_n)+1.$$

Hence, $\gamma_s(G') = \gamma_s(C_n)+1$, if k> 2.

Hence, $\gamma_s(G') = \gamma_s(C_n)+1$, whenever n = 3k + 2 and k≥ 2.

Remark 2.9. *In contrast to Theorem 2.8 if k = 1, then the cycle is C_5 and* $\gamma_s(G') = \gamma_s(C_5)$.

Theorem 2.10. *If G′ is the graph obtained from the complete graph K_n by addinga new vertex v′ to one of its vertices then,* $\gamma_s(G') = \gamma_s(K_n)$.

Proof. Let $V(K_n)$ = {v_1, v_2, ...,v_n}.
If v is added to any vertex v_i, 1 ≤ i≤ n of K_n, then v becomes an end vertex.Therefore, v∈ every Steiner dominating set of G' and W−{v_i}∪{v} is the unique Steiner dominating set of $G'$where W is the unique Steiner dominating set of K_n.Therefore, $\gamma_s(G') = |W| = \gamma_s(K_n)$.

Theorem 2.11. *Let G′ be the graph obtained from the wheel graph $W_{1,p}$(p ≥ 4) byadding a new vertex v′ to one of its vertices.Then,*

$$\gamma_s(G') = \begin{cases} \gamma_s(W_{1,p})+3 & \text{if } v' \text{ is attached to the apex} \\ \gamma_s(W_{1,p}) & \text{otherwise} \end{cases}$$

Proof. Let $V(W_{1,p})$ = {v, v_1, v_2, ..., v_p}
Case (i):v' is attached to the apex. Then, v' becomes an end vertex of G' andhence belongs to every Steiner dominating set of G'. Now, v' along with the set ofall rim vertices forms a unique Steiner dominating set of G'.
Therefore, $\gamma_s(G')$ = p + 1 = p− 2 + 3 = $\gamma_s(W_{1,p})$ + 3.

Case (ii):v' is attached to one of the rim vertices of $W_{1,p}$. Here also, v' becomesan end vertex of G' and v'belongs to every Steiner dominating set of G'. Labelthe vertex to which $v'$is attached as v_1. Then,W' = {v', v_3, v_4, ...,v_{p-1}} forms aunique Steiner dominating set of G'.
Therefore, $\gamma_s(G')$ = (p−1− 2) + 1 = p− 2 = $\gamma_s(W_{1,p})$.

III. STEINER DOMINATION NUMBER OF EDGE DELETED GRAPHS

Theorem 3.1.*For a complete graph K_p,* $\gamma_s(K_p - \{e\}) = 2$ *for every edge e in K_p.*

Proof. Let e = uv∈ $E(K_p)$. Let W = {u, v}. Then, every vertex w of $V(K_p-e) - W$ lie on the Steiner W-treeuwv of K_p− e. Also u and v dominate all the vertices of $V(K_p - e) - W$. Hence W is a Steiner dominating set of K_p− {e}.Further, as |W| = 2, W is a minimum Steiner dominating set of K_p−e. Therefore, $\gamma_s(K_p - \{e\}) = 2$.

Theorem 3.2. *For p ≥ 4,*

$$\gamma_s(K_p - \{e_1, e_2\}) = \begin{cases} 2 & \text{if } e_1 \text{ and } e_2 \text{ are non adjacent} \\ 3 & \text{otherwise} \end{cases}$$

Proof. Let e_1 = uv and e_2 = $u'v'$.
Let G^*=K_p− {e_1, e_2}
Case (i):e_1 and e_2 are non-adjacent.
Let W = {u, v} or W = {u', v'}. In both the cases W is a minimum Steinerdominating set of $G^*$and hence $\gamma_s(G^*) = 2$.
Case (ii):e_1 and e_2 are adjacent.
Here, e_1 and e_2 have a common vertex, say v = u'. Let W = {u, v, v'}. Then, everyvertex w of $V(G^*)$−W lie on a Steiner W-tree and also dominated by the verticesof W. Therefore, 2≤ $\gamma_s(G^*)$≤ 3.

Claim: $\gamma_s(G^*) \neq 2$.

Suppose W={x, y} is a Steiner dominating set of G^*. Therefore, x and y are notadjacent in G^*. Then, W is either {u, v} or {v, v'}. In both the cases, there is avertex in {u, v, v'} − W, which does not lie in any Steiner W- tree. Therefore, notwo point set of $G^*$is a Steiner dominating set of G^*.
Hence, $\gamma_s(G^*) = 3$.

Theorem 3.3.*Let p > 3. Suppose e_1, e_2, e_3∈$E(K_p)$ such that they form a pathin K_p. If G^*=K_p−{e_1,e_2,e_3}, then*

$$\gamma_s(G^*) = \begin{cases} 2 & \text{if } p = 4 \\ 3 & \text{otherwise.} \end{cases}$$

Proof. Let $P = (u, v, w, x)$ where $e_1 = uv$, $e_2 = vw$ and $e_3 = wx$.

Case (i): $p = 4$.
It is obvious that, $W = \{v,w\}$ is a Steiner dominating set of G^* and so $\gamma_s(G^*) = 2$.

Case (ii): $p > 4$.
Let $W = \{u, v, w\}$. In G^*, every vertex of $V(G^*) - W$ lie in some Steiner W- tree and so W is a Steiner dominating set of G^*. Therefore, $2 \le \gamma_s(G^*) \le 3$.

Claim: $\gamma_s(G^*) \neq 2$.

Suppose $W = \{x, y\}$ is a Steiner dominating set of G^*. Therefore, x and y are not adjacent in G^*. Then, W is any one of the following three sets, they are $\{u, v\}$, $\{v,w\}$ and $\{w, x\}$. In all the three cases, there is a vertex in $\{u, v, w\} - W$, which does not lie in any Steiner W- tree. Therefore, no two point set of G^* is a Steiner dominating set of G^*.
Hence, $\gamma_s(G^*) = 3$.

Theorem 3.4. *Let $G = K_p$, $p > 4$. Suppose $e_1, e_2, \ldots, e_k$ are in $E(G)$, where $4 \le k < p - 1$, such that $\{e_1, e_2, \ldots, e_k\}$ forms a path of length k. Let $G^* = K_p - \{e_1, e_2, \ldots, e_k\}$. Then, $\gamma_s(G^*) = 3$.*

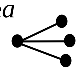
Proof. Let $V(K_p) = \{v_1, v_2, \ldots, v_p\}$.
Let $W = \{v_1, v_2, v_3\}$ and let $P = (v_1, v_2, \ldots, v_{k+1})$ with $e_i = (v_i v_{i+1})$, $1 \le i \le k$. Every vertex of $V(G^*) - W$ lies in some Steiner $W-$ tree and also dominated by W. Hence, W is a Steiner dominating set of G^*. Proceeding as in Theorem 3.3, no two element subset of $V(G^*)$ is a Steiner dominating set of G^*. Hence $\gamma_s(G^*) = 3$.

Theorem 3.5. *Let $p > 3$ and $3 \le k \le p$. Let $G^* = K_p - \{e_1, e_2, \ldots, e_k\}$, where $\{e_1, e_2, \ldots, e_k\}$ forms a cycle of length k in K_p. Then, the following are true.*
1. If $k = 3$, then $\gamma_s(G^*) = 3$.

2. If $k = 4$, then $\gamma_s(G^*) = 4$.

3. If $k \ge 5$, then $\gamma_s(G^*) = 3$.

Proof. Let C be a cycle of length k in K_p.
Case (i): $k = 3$.
Let $C = (a, b, c, a)$ where $e_1 = ab$, $e_2 = bc$ and $e_3 = ca$. Suppose $W = \{a, b, c\}$. Clearly, every vertex v' of $V(G^*) - W$ lie in the Steiner W-tree *a*
v' b
c

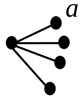


Also, v' is dominated by the vertices of W. Therefore, W is a Steiner dominating set of G^*.
Hence $2 \le \gamma_s(G^*) \le 3$.

Claim: $\gamma_s(G^*) \neq 2$.

Let $W^* = \{x, y\}$ be a minimum Steiner dominating set of G^*. As $p > 3$, W^* is a proper subset of $V(G^*)$. By definition of Steiner dominating set, x and y are non-adjacent. Therefore, W^* is precisely $\{a, b\}$ or $\{b, c\}$ or $\{c, a\}$. In all the three cases, there is a vertex of $\{a, b, c\} - W^*$ which does not lie on any Steiner W^*-tree. Hence, no two element subset of $V(G^*)$ is a Steiner dominating set of G^*.
Therefore, $\gamma_s(G^*) = 3$.

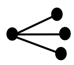
Case (ii): $k = 4$.
Let $C = (a, b, c, d, a)$ where $e_1 = ab$, $e_2 = bc$, $e_3 = cd$ and $e_4 = da$. If $W = \{a, b, c, d\}$, then every vertex v' in $V(G^*) - W$ lie in the Steiner W-tree *a*
v' b
c
d



Also, v' is dominated by the vertices of W. Therefore, W is a Steiner dominating set of G^*.
Hence, $2 \le \gamma_s(G^*) \le 4$.

Claim 1: There is no Steiner dominating set of G^* with 3 elements.
Let $W^* = \{x, y, z\}$ be a Steiner dominating set of G^*. If W^* is a clique in G^*, then W^* is not a Steiner dominating set of G^*. If two elements of W^*, say, x and y are non-adjacent, then $\{x, y\}$ is either $\{a, b\}$ or $\{b, c\}$ or $\{c, d\}$ or $\{d, a\}$.
Suppose $z \in \{a, b, c, d\}$. Then, there is a vertex in $\{a, b, c, d\} - W^*$ which does not lie in any Steiner W^*-tree.
If $z \notin \{a, b, c, d\}$. Then, xyz is the Steiner W^*-tree. Therefore, no vertex of $V(G^*) - W^*$ lie in any Steiner W^*-tree. Therefore, in both the cases W^* is not a Steiner set of G^* and hence not a Steiner dominating set of G^*.
Hence, there is no Steiner dominating set of G^* with 3 elements.
Claim 2: There is no Steiner dominating set of G^* with 2 elements.
Let $W^* = \{x, y\}$ be a Steiner dominating set of G^*. If x and y are adjacent, then obviously W^* is not a Steiner dominating set of G^*. If x and y are non-adjacent, then W^* is precisely $\{a, b\}$ or $\{b, c\}$ or $\{c, d\}$ or $\{d, a\}$. In all the cases there is a vertex of $\{a, b, c, d\} - W^*$ which does not lie on any Steiner W^*-tree. Therefore, W^* is not a Steiner set of G^* and hence not a Steiner dominating set of G^*.
Hence, there is no Steiner dominating set of G^* with 2 or 3 elements. Therefore, $\gamma_s(G^*) = 4$.

Case (iii): $k \ge 5$.
Let $C = (v_1, v_2, \ldots, v_k, v_1)$ where $e_i = v_i v_{i+1}$, $1 \le i \le k-1$ and $e_k = (v_k v_1)$. Let $W = \{v_1, v_2, v_3\}$.
Claim: W is a Steiner dominating set of G^*.
Let $v' \in V(G^*)$. If $v' \notin C$, then v' is adjacent to v_1, v_2 and v_3. Therefore, v_1
v' v_2
v_3



is the Steiner W-tree containing v'. Further, v' is dominated by the vertices of W.
Let $v' \in C$. Suppose $v' \notin \{v_k, v_4\}$. Again, v' is adjacent to v_1, v_2, v_3. Therefore, proceeding as above, v' is Steiner dominated by W.
Suppose $v' = v_4$ or v_k. Then, the path $v_2 v' v_1 v_3$ is the Steiner W-tree containing v'. Further, v' is dominated by the vertices of W.

Hence $2 \le \gamma_s(G^*) \le 3$.

Proceeding as in case (i), no two element subset of $V(G^*)$is a Steiner dominatingset of G^*.

Therefore, $\gamma_s(G^*) = 3$.

Theorem 3.6. Let G be a complete graph on $p(\ge 3)$ vertices. Let G^* be a graphobtained from G by removing the edges of a clique on $m(2 \le m \le p-1)$ vertices in$G$. Then, $\gamma_s(G^*) = m$.

Proof. Let H be a clique on m vertices in G. If $W=V(H)=\{v_1, v_2, ..., v_m\}$,then the subgraph induced by W in $G^*$is totally disconnected and every vertex in$V(G^*)-W$lies in a Steiner W-tree. Therefore, W is a Steiner dominating set of$G^*$and so $2 \le \gamma_s(G^*) \le m$.

Claim: There is no Steiner dominating set of G^* with $t<m$ elements.Let W^*be a Steiner dominating set of $G^*$with $t<m$ elements. If the vertices ofW^*form a clique in G^*, then $W^*$is not a Steiner dominating set of G^*. So, W^*contains at least two non-adjacent vertices. Since the subgraph induced by W in$G^*$is totally disconnected and $|W^*| <m$, there is a vertex in $\{v_1,v_2,...,v_m\} - W^*$which is not Steiner dominated by W^*. This is a contradiction to our assumption.Therefore, there is no Steiner dominating set of $G^*$with less than m elements.

Hence, $\gamma_s(G^*) = m$.

Theorem 3.7. If $G^*$is the graph obtained from$K_p(p\ge 3)$ by removing the edgesof a star with k end vertices $(2 \le k \le p-2)$ in K_p,then $\gamma_s(G^*) = k + 1$.

Proof. Let Hbe a star in G. Let $W=V(H)=\{v,v_1,v_2,...,v_k\}$, where v is thevertex of degree k in H. Every vertex v' of $V(G^*) - W$ lie in the Steiner W-tree.Hence, W is a Steiner dominating set of $G^*$and so $2 \le \gamma_s(G^*) \le |W| = k + 1$.

Claim: There is no Steiner dominating set of $G^*$with $t<k + 1$ elements.Suppose W' is a Steiner dominating set of $G^*$with $t<k + 1$ elements. If W' is a clique, then W' is not a Steiner dominating set of G^*. Suppose, W' contains atleast two non-adjacent vertices which is one of $\{v, v_i\}$, $1 \le i \le k$. Then, there is avertex of $\{v, v_1, v_2, ...,v_k\} - W'$ which is not Steiner dominated by W'. Therefore,W' is not a Steiner dominating set of G^*.Therefore, there is no Steiner dominating set of $G^*$with less than $k + 1$ elements.Hence, $\gamma_s(G^*) = k + 1$.

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