Modelling of Two Phase Flow over a Stretching Sheet with Analysis of Boundary Layer Flow and Heat Transfer Characteristics

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Abstract:
The steady boundary layer incompressible fluid (gas) with suspended particulate matter (SPM) over a stretching sheet has been investigated. The random motion of particles, electrification of particles, heat due to conduction and viscous dissipation for both fluid as well as particle phases have taken into consideration. The governing partial differential equations have reduced to a set of non-linear ordinary differential equations with the help of similarity transformations and solved numerically by using Runge-Kutta 4th order method adopting shooting technique. The effect of various parameters such as electrification parameter, Prandtl number, Eckert number, volume fraction, diffusion parameter and fluid-particle interactions parameter on the normalized velocity and the temperature of both phases have been analyzed and interpreted graphically. The effects of different parameters on the skin friction coefficient and Nusselt number also have tabulated. The comparisons of the present results are found to be an excellent agreement with the existing literature. It is worthy to note that, the electrification of particles enhance the temperature of the particle phase. Further it is also interpreted that, the negative value of skin friction coefficient indicates the solid surface exerts a drag force on the fluid.

AMS Clasifications: 76T10, 76T15

Keywords:
Electrification of particles, Diffusion parameter, Fluid–Particle interaction parameter, Shooting techniques, Stretching sheet, Volume fraction.

Nomenclature:

x,y-Cartesian coordinates.	P_r-Prandtl number.
η -Similarity variable.	G_r-Grashof number.
u-Velocity component of fluid along x- axis.	F_r-Froud number.
v -Velocity component of fluid along y- axis.	E_c -Eckert number.
u_p-Velocity component of the particle along x-axis.	ω -Density ratio.
v_p-Velocity component of the particle along y-axis.	φ -Volume fraction.
$U_w(x)$-Stretching sheet velocity.	c -Stretching rate.
M -Electrification parameter.	l -Characteristic length.
E -Electric field of force.	c_p-Specific heat of fluid.
m -Mass of particle.	c_s -Specific heat of particles.
e -Charge of particle.	k_s -Thermal conductivity of particle.
T -Temperature of fluid phase.	k -Thermal conductivity of fluid.
T_p-Temperature of particle phase.	β -Fluid particle interaction parameter.
T_w-Wall temperature.	β^*-volumetric coefficient of thermal expansion.
T_∞-Temperature at large distance from the wall.	ϵ-Diffusion parameter.
θ- Non-dimensional fluid phase temperature.	μ-Dynamic viscosity of fluid.
θ_p-Non-dimensionalparticle phase temperature.	ν -Kinematic viscosity of fluid.
ρ -Density of the fluid phase.	γ-Ratio of specific heat.
ρ_s-Material density.	τ -Relaxation time of particle phase.
ρ_p-Density of the particle phase.	τ_T -Thermal relaxation time i.e. the time required by the dust particle to adjust

	its temperature relative to the fluid.
A-Positive constant.	τ_p -Velocity relaxation time i.e. the time required by the dust particle to adjust its velocity relative to the fluid.

1. Introduction:

The investigation of momentum boundary layer flow and heat transfer of two phase flow with electrification of particles has numerous industrial applications viz; manufacture of cables insulated with pulp, artificial fibres, spinning of filaments , plasma studies, centrifugal separation of particles, flow through packed beds, sedimentation ,cooling or drying of papers, materials travelling on conveyer belt treated with heat etc. In fact the rate of heat transfer over a surface plays a pivotal role in making the final products qualitative. The mechanical properties of the final products are influenced by the stretching rate as well as the rate of cooling. The rate of stretching is important because the rapid stretching results in sudden solidification destroy the properties of the expected outcomes. Further, the heat transfer cannot be ignored in the application of industrial processes involved with high temperature regimes and its good knowledge will help to design pertinent equipments. These areas have potential contributions in the industrial sector which plays an important role towards the progress of the society.

In 1961, Sakiadis [5] has first initiated the study of boundary layer flow over a stretched surface moving with a constant velocity. Then after, many researchers extended his study with the effect of heat transfer, of which, some of the important studies have cited bellow. Tsou.et.al [10] have studied the effects of heat transfer and experimentally confirmed the numerical results of Sakiadis. Chen [8] has investigated the mixed convection of a power law fluid past a stretching surface in presence of the thermal radiation and magnetic field. Crane [13] has obtained the exponential solutions for planar viscous flow of linear stretching sheet. The problem of two phase suspension flow is solved in the frame work of a two-way coupling model or a two-fluid approach. Grubka et.al[14] have interpreted the temperature field in the flow over a stretching surface when subject to uniform heat flux. Sharidan et.al. [26] have presented similarity solutions for unsteady boundary layer flow and heat transfer due to stretching sheet. Gireesha.et.al[6] have studied the effect of boundary layer flow and heat transfer of a dusty fluid over a vertical stretching surface. They have examined the heat transfer characteristics for two types of boundary conditions namely variable wall temperature and variable heat flux. Gireesh et.al [7] have also studied the mixed convective flow of a dusty fluid over a stretching sheet in presence of thermal radiation and space dependent heat source/sink. Barik et.al [22] have studied the heat and mass transfer on MHD flow through a porous medium over a stretching surface with heat sources. Mohammad et.al.[17] have studied the heat transfer over an inclined stretching sheet in the presence of magnetic field . Sharma et.al [19] have investigated the momentum and heat transfer characteristics in MHD convective flow of dusty fluid over stretching sheet with heat source/sink .Soo [25] has studied the effect of electrification on the dynamics of a particulate system. Though many investigations have been made, but to the author's knowledge no study has been analyzed the effect of electrification of particles, particle-particle interactions, volume fraction and effect of diffusion parameter for fluid phase as well as particle phase. So our investigation will be a significant contribution to the literature which is not covered by the previous works. Here, it is mainly focused on the role of the inter particle electrostatic forces which have been given less attention by the previous investigators. At low temperature, electrification of solid particles occurs due to the impact of wall. Even a very small charge on the solid particles causes a pronounced effect on concentration distribution in the flow of a gas-solid system. Although electric charge on the solid particles can be excluded by definition in theoretical analysis or when dealt truly with a boundless system, electrification of the solid particles always occurs when it comes in contact and separation are made between the solid particles and a wall of different materials or similar materials but in different surface conditions. The electric charges on the solid particles cause deposition of the solid particles on a wall in a more significant manner than the gravity effect and are expected to affect the motion of a metalized propellant and its product of reaction through a rocket nozzle and the jet at the exit of the nozzle. The charged solid particles in the jet of a hot gas also affect radio communications. The above analysis has motivated to present this paper. Here the particles are allowed to diffuse through the carrier fluid. This can be done by applying the kinetic theory of gases as the motion of the particles across the streamline due to the concentration and pressure diffusion. The momentum equation for particulate phase in normal direction, heat due to conduction and viscous dissipation in the energy equation of the particle phase have been considered for better understanding of the boundary layer characteristics. The effects of electrification, volume fraction of particles on skin friction, heat transfer and other boundary layer characteristics also have been studied. The governing partial differential equations have reduced to a system of ordinary differential equations and solved with the help of Runge-Kutta Method by using shooting techniques.

2. Mathematical Formulation And Solution:

Here, a steady two dimensional laminar boundary layer flow of a viscous incompressible dusty fluid with electrification of particles over a stretching sheet has been taken for investigation. Further it is considered that, the sheet is being stretched with velocity $U_w(x)$ along the x-axis, keeping the origin fixed in the fluid of ambient temperature ' T_∞ '. The flow is generated by the action of two equal and opposite forces along the x-axis and y-axis being the normal to the flow. In this study both the fluid and the dust particle clouds are supposed to be static at the beginning. Moreover the dust particles are assumed to be spherical in shape and uniform in size throughout the flow.

The governing equations for the steady two dimensional boundary layer incompressible flows of dusty fluids are
stated to be;

$$\frac{\partial}{\partial x}\vec{F}(u_f) + \frac{\partial}{\partial y}\vec{G}(u_f) + H(u_f) = S(u_f, u_p, T, T_p) \quad (1)$$

$$\frac{\partial}{\partial x}\vec{F}(u_p) + \frac{\partial}{\partial y}\vec{G}(u_p) + H(u_p) = S_p(u_f, u_p, T, T_p) \quad (2)$$

Where, $H(u_f) = 0$, $H(u_p) = 0$, $\vec{F}(u_f) = \begin{bmatrix} u \\ (1-\varphi)\rho u^2 \\ \rho c_p uT \end{bmatrix}$, $\vec{F}(u_p) = \begin{bmatrix} \rho_p u_p \\ \rho_p u_p{}^2 \\ \rho_p u_p v_p \\ \rho_p c_s u_p T_p \end{bmatrix}$

$$\vec{G}(u_f) = \begin{bmatrix} v \\ (1-\varphi)\rho uv \\ \rho c_p vT \end{bmatrix}, \quad \vec{G}(u_p) = \begin{bmatrix} \rho_p v_p \\ \rho_p u_p v_p \\ \rho_p v_p{}^2 \\ \rho_p c_s v_p T_p \end{bmatrix}$$

$$S(u_f, u_p, T, T_p) = \begin{bmatrix} 0 \\ \mu\frac{\partial^2 u}{\partial y^2} - \frac{\rho_p}{\tau_p}(u - u_p) + g\beta^*(T - T_\infty) + \left(\frac{e}{m}\right)E \\ k\,(1-\varphi)\frac{\partial^2 T}{\partial y^2} + \frac{\rho_p c_s}{\tau_T}(T_p - T) + \frac{\rho_p}{\tau_p}(u_p - u)^2 + \mu(1-\varphi)\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{e}{m}\right)E u_p \end{bmatrix}$$

$$S_p(u_f, u_p, T, T_p) = \begin{bmatrix} 0 \\ \frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right) + \frac{\rho_p}{\tau_p}(u - u_p) + \varphi(\rho_s - \rho)g + \left(\frac{e}{m}\right)E \\ \frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial v_p}{\partial y}\right) + \frac{\rho_p}{\tau_p}(v - v_p) \\ \frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right) - \frac{\rho_p}{\tau_p}(u - u_p)^2 + \varphi\mu_s\left(u_p\frac{\partial^2 u_p}{\partial y^2} + \left(\frac{\partial u_p}{\partial y}\right)^2\right) - \frac{\rho_p c_s}{\tau_T}(T_p - T) + \left(\frac{e}{m}\right)E u_p \end{bmatrix}$$

With boundary conditions

$$\left.\begin{aligned} &u = U_w(x) = cx,\ v = 0 \ \ at\ y = 0 \\ &\rho_p = \omega\rho,\ u = 0,\ u_p = 0,\ v_p \to v\ as\ y \to \infty \end{aligned}\right\} \quad (3)$$

Where, 'ω' is the density ratio in the main stream.

Similarly, the corresponding boundary condition for T and T_p are given by

$$\left.\begin{aligned} &T = T_w = T_\infty + A\left(\frac{x}{l}\right)^2 \ \ at\ y = 0 \\ &T \to T_\infty,\ T_p \to T_\infty\ \ as\ y \to \infty \end{aligned}\right\} \quad (4)$$

Where, A is a positive constant, $l = \sqrt{\frac{v}{c}}$ is a characteristic length.

For most of the gases $\tau_p \approx \tau_T$, $k_s = k\frac{c_s}{c_p}\frac{\mu_s}{\mu}$ if $\frac{c_s}{c_p} = \frac{2}{3P_r}$

Introducing the following non dimensional variables in equation (1) and (2)

$$u = cxf'(\eta), v = -\sqrt{c\nu} f(\eta), \eta = \sqrt{\frac{c}{\nu}}\, y, u_p = cxF(\eta),$$

$$v_p = \sqrt{c\nu}\, G(\eta), \rho_r = H(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}, T - T_\infty = A\left(\frac{x}{l}\right)^2 \theta,$$

$T_p - T_\infty = A\left(\frac{x}{l}\right)^2 \theta_p, \beta = \frac{1}{c\tau_p}, \quad \epsilon = \frac{\nu_s}{\nu}, P_r = \frac{\mu c_p}{k}, E_c = \frac{\nu c}{A c_p}$

$, \quad F_r = \frac{c^2 x}{g}, \gamma = \frac{\rho_s}{\rho}, G_r = \frac{g\beta^*(T - T_\infty)}{c^2 x}, \nu = \frac{\mu}{\rho}, M = \left(\frac{e}{m}\right)\frac{E}{c^2 x}$

From the above, we get the following non dimensional equations.

$$HF + HG' + GH' = 0 \qquad (5)$$

$$f''' + ff'' - [f']^2 + \frac{1}{(1-\varphi)}\beta H[F - f'] + Gr\theta + \frac{H}{(1-\varphi)} M = 0 \qquad (6)$$

$$G(\eta)F'(\eta) + [F(\eta)]^2 - \epsilon F''(\eta) - \beta[f'(\eta) - F(\eta)] - \frac{1}{Fr}\left(1 - \frac{1}{\gamma}\right) - M = 0 \qquad (7)$$

$$GG' - \epsilon G'' + \beta[f + G] = 0 \qquad (8)$$

$$\theta'' - Pr(2f'\theta - f\theta') + \frac{2}{3}\frac{\beta H}{1-\varphi}(\theta_p - \theta) + \frac{Pr E_c \beta H}{1-\varphi}(F - f')^2 + Pr E_c f''^2 + \frac{HMPr E_c F}{(1-\varphi)} = 0 \qquad (9)$$

$2F\theta_p + G\theta_p' + \beta(\theta_p - \theta) - \frac{\epsilon}{Pr}\theta_p'' + \frac{3}{2}\beta EcPr[f' - F2-32\epsilon EcPrFF''+F2-32MPrEcF=0$

(10)

with boundary conditions

$$G' = 0, f = 0, f' = 1, F' = 0, \theta = 1, \theta_p' = 0 \ as\ \eta \to 0$$

$$f' = 0, F = 0, G = -f, H = \omega, \theta \to 0, \theta_p \to 0 \ as\ \eta \to \infty$$

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as

$C_f = \frac{\tau_w}{\rho U_w^2}$, $Nu_x = \frac{x q_w}{k(T_w - T_\infty)}$ where, the surface shear stress τ_w and surface heat flux q_w are given by $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$, $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$

3. Numerical Procedure And Validation Of The Result:

The coupled non-linear ordinary differential equations from (5) to (10) subject to the non dimensional boundary conditions are solved numerically using Runge-Kutta 4th order method with shooting integration scheme implemented on computer programme Fortran-77. The advantage of shooting technique is to solve a boundary value problem by converting it into an initial value problem. In this problem the values of $F(0), G(0), H(0), \theta_p(0), \theta'(0)$ and $f''(0)$ are not known, but $f'(\infty) = 0, F(\infty) = 0, G(\infty) = -f(\infty), H(\infty) = \omega, \theta(\infty) = 0, \theta_p(\infty) = 0$ are known. Here, the missing values of $F(0), G(0), H(0), \theta_p(0), \theta'(0)$ and $f''(0)$ for different set of values of parameters are chosen on Hit and Trial basis in such a way that the boundary conditions at other end i.e. the boundary conditions at infinity (η_∞) are satisfied. The most important step in this method is to choose an appropriate finite value of $\eta \to \infty$ in order to determine $\eta \to \infty$ for the boundary value problem which is described by equations (5) to (10). Further, we have started with the initial guess values for a particular set of physical parameters to obtain $F(0), G(0), H(0), \theta_p(0), \theta'(0)$ and $f''(0)$ .The solution procedure is repeated with another large value of $\eta \to \infty$ until two successive values of $F(0), G(0), H(0), \theta_p(0), \theta'(0)$ and $f''(0)$ differs, only by a specific significant digits. For instance, we have supplied $f''(0) = \propto_0$ and $f''(0) = \propto_1$ and the improved value of $f''(0) = \propto_2$ is determined by utilizing linear interpolation formula. Then the value of $f'(\propto_2, \infty)$ is determined by using Runge-Kutta method. It is noticed that, when $f'(\propto_2, \infty)$ is equal to $f'(\infty)$ up to a certain decimal accuracy, then $\propto_2$ i.e $f''(0)$ is determined, otherwise the above procedure is continued with $\propto_0 = \propto_1$ and $\propto_1 = \propto_2$ until a correct $\propto_2$ is obtained. The same procedure as explained above is adopted to determine the correct values of $F(0), G(0), H(0), \theta_p(0), \theta'(0)$ .If they agreed about six significant digits, the last value of η_∞ is used; otherwise the procedure is repeated until further change in η_∞ does not lead to any more change in the values of $F(0), G(0), H(0), \theta_p(0), \theta'(0)$ and $f''(0)$ .Depending upon the initial guess and number of steps N=81.0, the solution of the present problem is obtained by numerical computation after finding the infinite value for η. So, we performed our bulk of computations for $\eta_\infty = 3.0$ with step size $\Delta\eta$ =0.125. The values of η may change for a different set of physical parameters used. It has been observed from the numerical results that, the approximation to $F(0), G(0), H(0), \theta_p(0), \theta'(0)$ and $f''(0)$ is improved by increasing the infinite value of η, which is finally determined as η =3.0 with a step length of Δη =0.125 beginning from η = 0 to ensure the satisfactory convergence criterion of 1×10^{-6} and to achieve the far field boundary conditions asymptotically for all values of the parameters under consideration. For the sake of brevity, further details on the solution process are not presented here. It is also important to note that, the computational time for each set of input parameter values should be short because the physical domain in this problem is unbounded, and the computational domain has to be finite. Here, we applied the far field boundary conditions for the similarity variable η at finite value denoted by η_∞ and the solutions are obtained with an error tolerance of 10^{-6} in all cases. For zero values of parameter like β, G_r, E_c, M, ϕ and various non-zero values of P_r, the integration has been started with initial approximation for $f''(0)$ and $-\theta'(0)$ and determining η_∞ in each cases .The corrected approximation for various values of $-\theta'(0)$ is determined by shooting technique which are tabulated in table-1 to facilitate a comparison with the result obtained by Chen [8], Grubka et.al.[14], Subhas et.al.[4], Mukhopadhaya et.al.[24], Ishak et.al.[3] and Gireesha et.al.[6] for the local Nusselt number in the limiting conditions i.e. $\beta, G_r, E_c, M, \phi = 0.0$. From the table-1, it is observed that, the present result is well agreed with their results up to six decimal place accuracy as the criterion for convergence.

"TABLE-1": Comparison results for the wall temperature gradient $-\theta'(0)$ for $\beta, G_r, E_c, M, \phi = 0.0$.

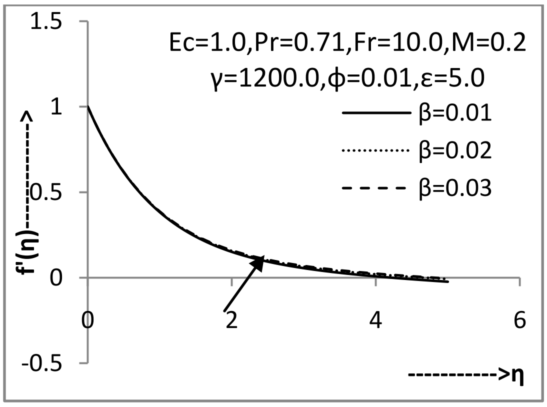
P_r	Ishak et.al. [3]	Subhas et.al. [4]	Gireesha et.al. [6]	Chen [8]	Grubka et.al. [14]	Mukhopadhaya et.al. [24]	Present Study $-\theta'(0)$
0.72	-	1.0885	1.0885	1.0885	1.0885	1.0885	1.08845
1.0	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333	1.333927
3.0	2.5097	-	2.5097	2.5097	-	2.5097	2.509684
10.0	4.7969	4.7968	4.7969	4.7968	4.7969	-	4.796353

"TABLE-2": Showing initial values of wall velocity gradient $-f''(0)$ and temperature gradient $-\theta'(0)$

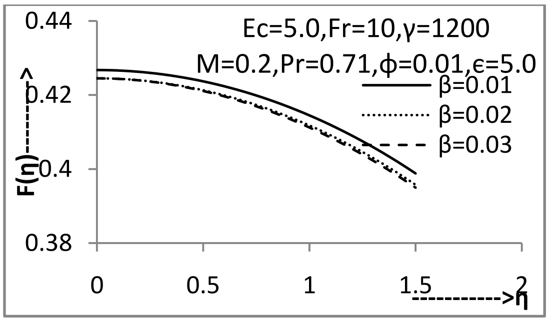
M	β	ε	P_r	E_c	φ	$-f''(0)$	$u_p(0)$	$-v_p(0)$	$-\theta'(0)$	$\theta_p(0)$
-	-	-	0.71	0.0	0.0	1.001397	-	-	1.082315	-
0.2	0.01	5.0	0.71	5.0	0.01	0.962271	0.426757	1.129012	0.102395	0.069452
				7.0		0.961090	0.427011	1.128881	0.435744	0.094296
				10.0		0.960392	0.427131	1.127352	1.028948	0.140825
			0.5			0.959273	0.426639	1.127921	0.008463	0.043226
0.2	0.01	5.0	0.71	5.0	0.01	0.962271	0.426757	1.129012	0.102395	0.069452
			1.0			0.968431	0.427068	1.132730	0.134761	0.114119
0.2	0.01	5.0	0.71	5.0	0.01	0.962271	0.426757	1.129012	0.102395	0.069452
					0.02	0.952341	0.426887	1.128631	0.029926	0.075235
					0.03	0.951430	0.427367	1.130123	0.014961	0.109427
0.2	0.01	5.0	0.71	5.0	0.01	0.962271	0.426757	1.129012	0.102395	0.069452
	0.02					0.975622	0.424477	1.112390	0.159525	0.050771
	0.03					0.982821	0.424432	1.097221	0.095656	0.044293
0.0						0.991962	0.194431	1.054230	0.147736	0.060731
0.1						0.969751	0.32304	1.096322	0.107213	0.016682
0.15						0.963383	0.377380	1.112736	0.107124	0.029381
0.2	0.01	5.0	0.71	5.0	0.01	0.962271	0.426757	1.129012	0.102395	0.069452
0.2	0.01	5.0	0.71	5.0	0.01	0.962271	0.426757	1.129012	0.102395	0.069452
				5.5		0.963181	0.410520	1.112626	0.102373	0.052954
				6.0		0.968741	0.394547	1.099636	0.101721	0.032915

4. GRAPHICAL REPRESENTATION AND DISCUSSION OF THE RESULT:

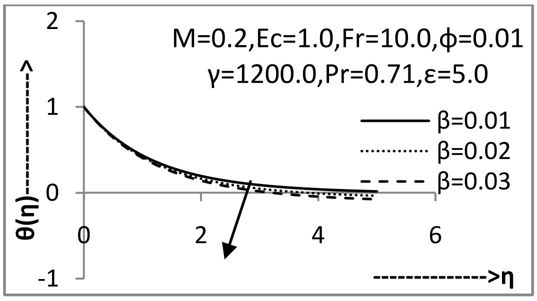
Here the boundary layer flow and heat transfer characteristics over a stretching sheet have been discussed extensively by considering various physical parameters like Prandtl number ‘Pr’,Eckert number ‘Ec’, Froud number ‘Fr’ , Electrification parameter ‘M’, volume fraction ‘φ’, particle interaction parameter ‘β’and diffusion parameter ‘ϵ’.Our model based on the motion and collisions of particles, as well as the charge transfer during these collisions.



“Fig-1”:Variation of non-dimensional velocity profile of fluid phase u w.r.t. β

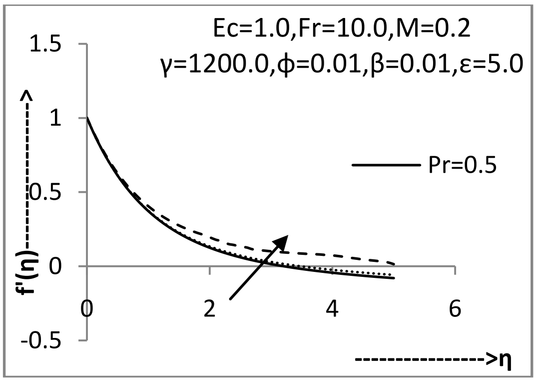


“Fig-2”:Variation of non-dimensional velocity profile of particle phase Up w.r.t. β

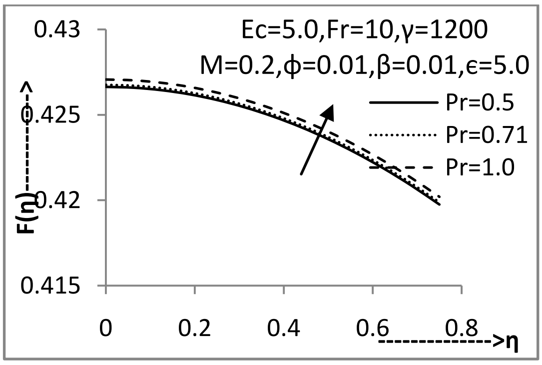


“Fig-3”:Variation of non-dimensional temperature profile of fluid phase θ w.r.t.β

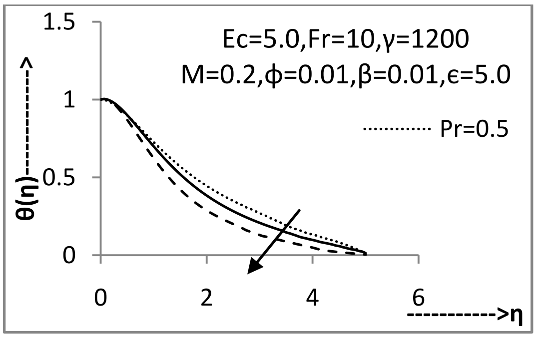
“Fig. 1” and “Fig. 2” demonstrate the effect of ‘β’ on fluid phase velocity and particle phase velocity respectively. Here it is interpreted that, the increasing values of ‘β’ increases the fluid phase velocity but decreases the particle phase velocity. “Fig. 3” illustrates that the increasing values of ‘β’ decreases the temperature of fluid phase because the delay in the particle relaxation time allows for greater access to the particles in fluid thereby decreasing the convective heat and mass transfer.



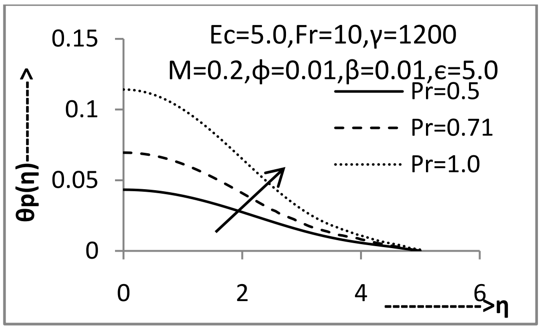
“Fig-4”:Variation of non-dimensional velocity profile of fluid phase u w.r.t.Pr



“Fig-5”:Variation of non-dimensional velocity profile of particle phase Up w.r.t.Pr

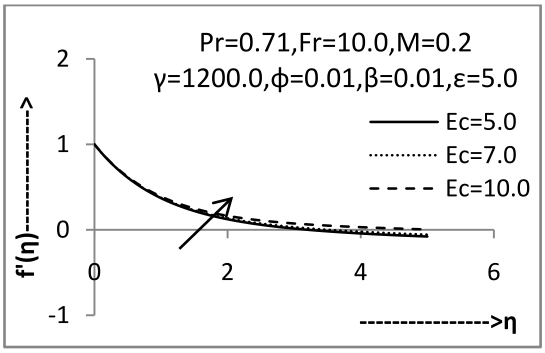


“Fig-6”:Variation of non-dimensional temperature profile of fluid phase θ w.r.t.Pr

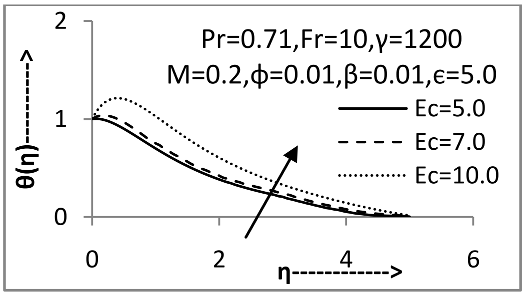


“Fig-7”:Variation of non-dimensional temperature profile of particle phase θp w.r.t. Pr

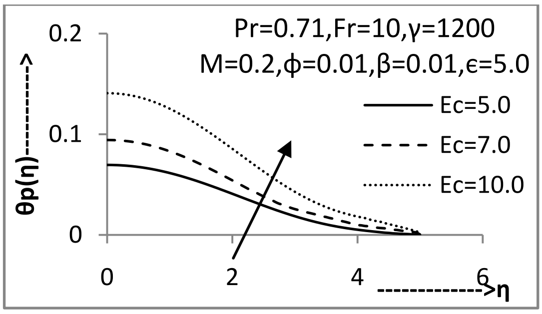
“Fig. 4” and “Fig. 5” indicates that, the increase of velocity profiles of both fluid and particle phases with the increasing values of ‘Pr’. “Fig. 6” and “Fig. 7” depicts the effect of ‘Pr’ on temperature profile of fluid phase and particle phase respectively. From the aforesaid figures, we observe that the increasing values of ‘Pr’ decreases the temperature of the fluid phase but the reverse trend occurs in case of particle phase. This occurs due to the smaller values of ‘Pr’ which are equivalent to the larger values of thermal conductivities. Hence it is noticed that, this phenomenon leads to decrease of energy ability which reduces the thermal boundary layer and the heat is able to diffuse away from the stretching sheet.



“Fig-8”:Variation of non-dimensional velocity profile of fluid phase u w.r.t. Ec

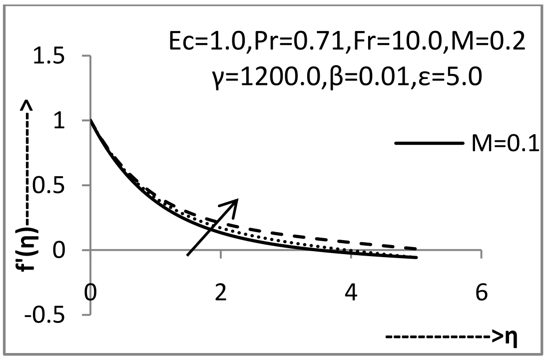


“Fig-9”:Variation of non-dimensional temperature profile of fluid phase θ w.r.t. Ec

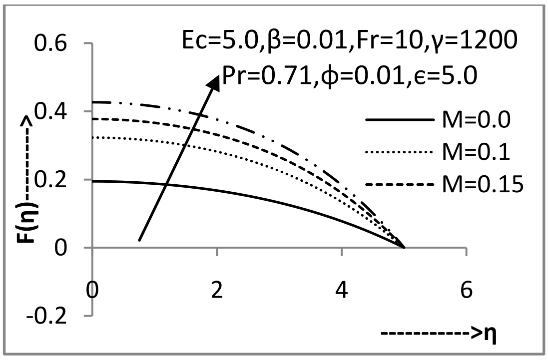


“Fig-10”:Variation of non-dimensional temperature profile of particle phase θp w.r.t.Ec

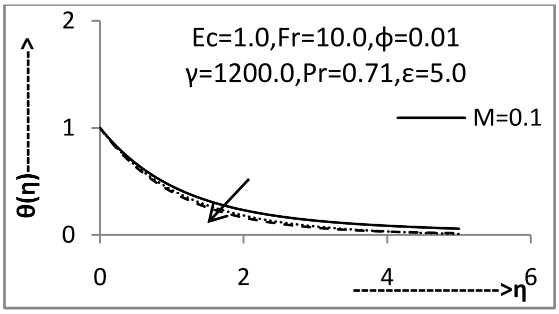
“Fig. 8” depicts the velocity profile of fluid phase which shows the increasing values of ‘Ec’ increases the velocity profile of fluid phase. “Fig. 9” and “Fig. 10” explain about the increasing values of ‘Ec’ allows to increase the temperature of fluid phase as well as the particle phase. Here, the larger values of ‘Ec’ gives rise to a strong viscous dissipation effect, as a result of which the heat energy is stored in the fluid thereby enhancing the thickness of the temperature and thermal boundary layers.



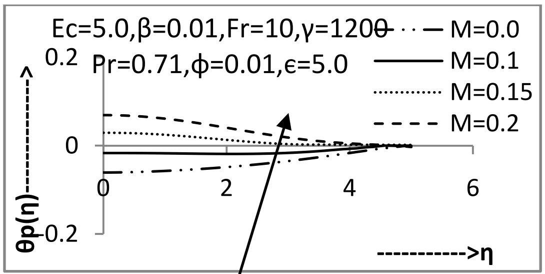
“Fig-11”:Variation of non-dimensional velocity profile of fluid phase u w.r.t. M



“Fig-12”: Variation of non-dimensional velocity profile of particle phase Up w.r.t. M

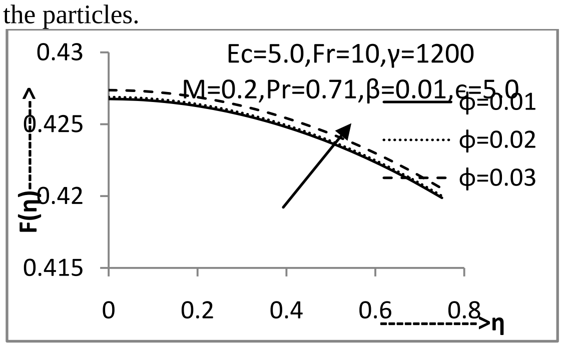


"Fig-13": Variation of non-dimensional temperature profile of fluid phase θ w.r.t. M.

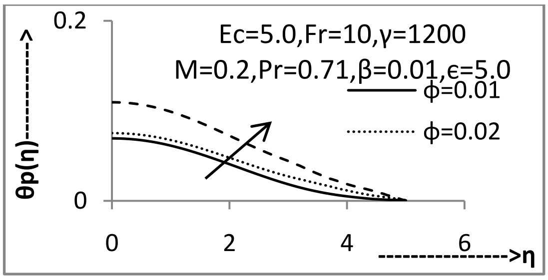


"Fig-14": Variation of non-dimensional temperature profile of particle phase θp w.r.t. M.

"Fig. 11" and "Fig. 12" represents the velocity profile of both fluid and particles phases respectively. The increasing values of electrification parameter 'M' causes to increase the velocity of both fluid phase and particle phase. "Fig. 13" demonstrates the temperature of fluid phase. It is noticed that, the temperature of fluid phase decreases with increasing in the values of electrification parameter 'M'. But "Fig. 14" reveals that, the temperature of particle phase increases on increasing the values of electrification parameter 'M'. Here it is also observed that, the enhancement in the values of 'M' enhances the temperature of particle phase due to the applied transverse electric field that opposes the motion of the particles.

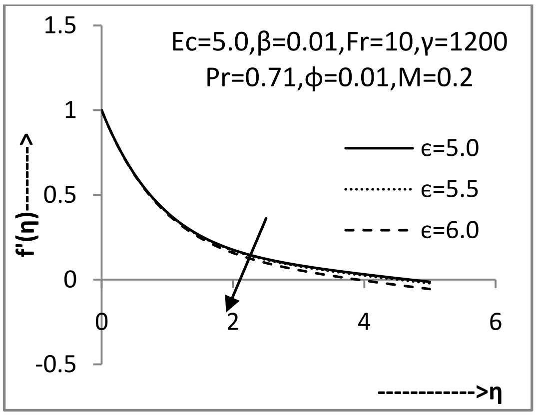


"Fig-15": Variation of non-dimensionalvelocity profile of particle phase Up w.r.t.φ.

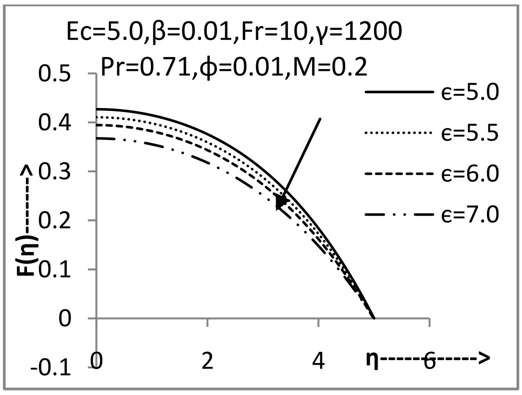


"Fig-16": Variation of non-dimensional temperature profile of particle phase θp w.r.t. φ.

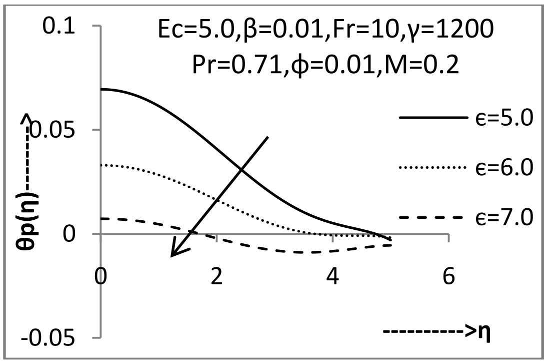
"Fig. 15" shows the effect of volume fraction 'φ' on velocity of particle phase. From this it can be perceived that the velocity of both phases increases with the increasing values of 'φ', due to increase in the number of particles at constant Stocks number. "Fig. 16" witnesses the increase in temperature of the particles phase with the increasing values of 'φ', because the high values of' 'φ' causes the fluid to becomes more viscous, and as a result of which the natural convection increased.



"Fig-17": Variation of non-dimensional velocity profile of fluid phase u w.r.t. ϵ



"Fig-18": Variation of non-dimensional velocity profile of particle phase Up w.r.t. ϵ



"Fig-19": Variation of non-dimensional temperature profile of particle phase θp w.r.t. **ϵ**.

"Fig. 17" and "Fig. 18" explain about the effect of 'ϵ' on the velocity of fluid phase and particle phase respectively. It can be revealed from the above figures that, the increasing values of '**ϵ**' are responsible for decreasing in the velocity of fluid phase and particle phase. This also seen that, when the length is increased, the driving force decreases while we approach to equilibrium and this leads to lower the mass transfer coefficient. "Fig. 19" shows the temperature of particle phase decreases with the increasing values of 'ϵ' due to heat transfer from particle phase to fluid phase.

CONCLUSION:

The novelty of our study is the consideration of electrification of particles which is absolutely different from the electrically conducting fluid that has been investigated by many authors. Again, the parameters like electrification of particles, Froud number, Eckert number, Prandtle number, diffusion parameter, volume fraction, fluid particle interactions parameter and Grashof number etc have been taken into account while conducting this investigation. Though the computations are cumbersome but consideration of all these parameters may give a new dimension for further study of stretching sheet complex problems. Last but not the least, the results thus obtained from this study may inspire future researchers in this area including investigations in three dimensional problems and other fields of stretching sheet problems. In our study, it is claimed that, the velocity profiles of both phases and temperature profile of particle phase increases on increasing the values of electrification parameter. However, the velocity profiles decrease on increasing the value of electrification parameter due to a retarding Lorentz force in the study of electrically conducting fluid that investigated by the previous authors so far. Another significant finding of this study is the consideration of diffusion parameter i.e. 'ϵ' that has been ignored by other researchers. In addition to the above findings, the following conclusions are also drawn from the table and graphs.

i. The negative values of $f''(0)$ indicate the solid surface exerts a drag force on the fluid.
ii. Increasing values of Ec increase the temperature of both fluid phases as well as particle phase due to the frictional heating; as a result the heat energy is stored.
iii. The thermal boundary layer thickness decreases on the increase of Pr i.e. slow rate of thermal diffusion.
iv. Increasing values of β decrease the particle phase velocity and temperature profile of fluid phases.
v. The increasing values of φ increases the velocity and temperature profile of particle phase.
vi. The increasing values of electrification parameter M increase the temperature of dust phase.

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