$\Lambda - \theta - I - Closed - Sets$

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Abstract

We define $locally\ \theta - I - closed$ sets and $\Lambda - \theta - I - closed$ sets and discuss their properties. Using these sets we characterize $T_{1/2} - spaces$ and $T_I - spaces$.

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1 Introduction and preliminaries

An ideal I on a topological space (X, τ) is a non empty collection of subsets of X which satisfies (i) $A \in I$ and $B \subset A$ implies $B \in I$ and (ii) $A, B \in I$ implies $A \cup B \in I$. Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(\cdot)^* : P(X) \to P(X)$ called a local function [8] of A with respect to τ and I is defined as follows: for $A \subset X, A^*(X, \tau) = \{x \in X \mid U \cap A \notin I,$ for every $U \in \tau(x)\}$, where

$\tau(x) = \{U \in \tau \mid x \in U\}$. A Kuratowski closure operator $cl^*(\cdot)$ for a topology $\tau^*(I,\tau)$ called the $\star - topology$, finer than τ, is defined by $cl^*(A) = A \cup A^*(I,\tau)$ [16]. When there is no confusion we will simply write A^* for $A^*(I,\tau)$ and τ^* for $\tau^*(I,\tau)$. If I is an ideal on X, then (X,τ,I) is called an *ideal space*. A subset A of an ideal space $(X\tau, I)$ is said to be $\star - closed$ [7] $(resp. \star -dense$[6]) if $A^* \subset A$ $(resp. cl^*(A) = X)$. A subset A of an ideal space (X,τ,I) is said to be $I_g - closed$ [2] if $A^* \subset U$ whenever $A \subset U$ and U is open. A subset A of an ideal space (X,τ,I) is said to be $I_g - open$ if $(X - A)$ is $I_g - closed$. An ideal space (X,τ,I) is said to be a $T_I - space$ [2] if every $I_g - closed$ set is $\star - closed$.

By a space, we always mean a topological space (X,τ) with no separation properties assumed. If $A \subset X, cl(A)$ and $int(A)$ will respectively, denote the closure and interior of A in (X,τ) and $int^*(A)$ will denote the interior of A in (X,τ^*). A subset A of a topological space (X,τ) is said to be a $g - closed$ set [9] if $cl(A) \subset U$ whenever $A \subset U$ and U is open. A subset A of a topological space (X,τ) is said to be a $g - open$ set if $X - A$ is a $g - closed$ set. A space (X,τ) is said to be a $T_{1/2} - space$ [9] if every $g - closed$ set is a closed set.

For a subset A of a space $(X,\tau), the$ $\theta - interior$ [17] of A is the union of all open sets of X whose closures contained in A and is denoted by $int_\theta(A)$. The subset A is called $\theta - open$ if $A = int_\theta(A)$. The complement of a $\theta - open$ set is called a $\theta - closed$ set. Equivalently, $A \subset X$ is called $\theta - closed$ [17] if $A = cl_\theta(A) = \{x \in X \mid cl(U) \cap A \neq \phi$ for all $U \in \tau(x)\}$. The family of all $\theta - open$ sets of X forms a topology [17] on X, which is coarser than τ and is denoted by τ_θ. A subset A of a topological space (X,τ) is said to be a $\theta - g - closed$ set [3] if $cl_\theta(A) \subset U$ whenever $A \subset U$ and U is open. A subset A of a space (X,τ) is said to be a $\theta - g - open$ set [3] if $X - A$ is a $\theta - g - closed$ set.

A subset A of an ideal space (X,τ,I) is said to be $\theta - I - closed$ [1] if $cl^*_\theta(A) = A$, where $cl^*_\theta(A) = \{x \in X \mid A \cap cl^*(U) \neq \phi$ for all $U \in \tau(x)\}$. A is said to be $\theta - I - open$ if $X - A$ is $\theta - I - closed$. A subset A of an ideal space (X, τ, I) is said to be $\theta - I_g - closed$ [13] if $cl^*_\theta(A) \subset U$, whenever $A \subset U$ and U is open. The complement of $\theta - I_g - closed$ is said to be $\theta - I_g - open$. If $I = \{\phi\}, cl^*_\theta(A) = cl_\theta(A)$. If $I = P(X), cl^*_\theta(A) = cl(A)$. For a subset A of X, $int_\theta I(A) = \cup\{U \in \tau \mid cl^*((U) \subset A\}$ [1]. We denote this $int_\theta I(A)$ by $int^*_\theta(A)$. The family of all $\theta - I - open$ sets of (X, τ, I) is a topology and it is denoted by $\tau_{\theta - I}$ (see [1, Theorem 1]).

A subset A of a space (X, τ) is said to be $\Lambda - set(resp.V - set)$ [10,11] if $A = A^\Lambda(resp.A = A^V)$, where $A^\Lambda = \cap\{U \in \tau \mid A \subset U\}$ and $A^V = \cup\{F \mid F \subset A$ and $X - F \in \tau\}$

A subset A of an ideal space (X, τ, I) is said to be an $I.\Lambda - set$ [12] if $A^\Lambda \subset F$ whenever $A \subset F$ and F is $\star - closed$. A subset A of a topological space (X, τ) is said to be a $g.\Lambda - set$ [10] if $A^\Lambda \subset A$ whenever $A \subset F$ and F is closed. A subset A of an ideal space (X, τ, I) is said to be $\Lambda - \star - closed$ [14], if there exist an open set B and a $\star - closed$ set C such that $A = B \cap C$. If $I = \{\phi\}$, then $\Lambda - \star - closed$ sets coincide with $\Lambda - closed$ sets.

Lemma 1.1. *[15, Theorem 2.13]. Let (X, τ, I) be an ideal space. Then every subset of X is $I_g - closed$ if and only if every open set is $\star - closed$.*

Lemma 1.2. *[3, Theorem 3.3]. An ideal space (X, τ, I) is $T_I - space$ if and only if every singleton subset of X is open or $\star - closed$.*

Lemma 1.3. *Let (X, τ) be a topological space. Then the following properties are valid.*

(a) If B_i is a $\Lambda - set(i \in I)$, then $\cup_{i \in I} B$ is a X - set

(b) If B_i is a $\Lambda - set(i \in I)$, then $\cap_{i \in I} B_i$ is a $\Lambda - set$

(c) B is a $\Lambda - set$ if and only if $X - B$ is a $V - set$

(d) For any subset A of $X, A^{\Lambda\Lambda} = A^{\Lambda}$.

Lemma 1.4. *[1, Lemma 2.1] For a subset A of a topological space* (X, τ) *the following are equivalent*

(a) A *is a* $\Lambda - closed$

(b) $A = L \cap cl(A)$, *where* L *is a* $\Lambda - set$

(c) $A = A^{\Lambda} \cap cl(A)$

Lemma 1.5. *[13, Theorem 2.20]. A subset A of an ideal space* (X, τ, I) *is* $\theta - I_g - closed$ *if and only if* $cl_\theta^*(A) \subset A^{\Lambda}$.

Lemma 1.6. *[13, Theorem 2.6]. If* (X, τ, I) *is a* $T_1 - space$ *and* A *is* $\theta - I_g - closed$ *then* A *is a* $\theta - I - closed$ *set.*

2 *Locally* $\theta - I - closed$ sets

In this section, we define and study a new class of generalized Locally closed sets in an ideal topological space (X, τ, I). A subset A of an ideal space (X, τ, I) is said to be *locally* $\theta - I - closed$ if there exist an open set U and a $\theta - I - closed$ set F such the $A = U \cap F$ A subset A of a space (X, τ) is said to be *locally* $\theta - closed$ there exist open set U and a $\theta - closed$ set F such that $A = U \cap F$.

If $I = \{\phi\}$, then *locally* $\theta - I - closed$ sets coincide with *locally* $\theta - closed$. If $I = P(X)$, then *locally* $\theta - I - closed$ sets coincide with locally closed sets.

Theorem 2.1. *Let* (X, τ, I) *be an ideal space and* A *be a subset of* X. *Then the following are equivalent.*

(a) A *is locally* $\theta - I - closed$

(b) $A = U \cap cl_\theta^*(A)$, *for some open set* U

(c) $cl_\theta^*(A) - A$ *is closed*

(d) $A \cup (X - cl^*_\theta(A))$ *is open*

(e) $A \subset int(A \cup (X - cl^*_\theta(A)))$

Proof. $(a) \Rightarrow (b)$. If A is *locally* $\theta - I - closed$, then there exist an open set U and a $\theta - I - closed$ set F such that $A = U \cap F$. Clearly, $A \subset U \cap cl^*_\theta(A)$. Since F is $\theta - I - closed, cl^*_\theta(A) \subset cl^*_\theta(F) = F$ and so $U \cap cl^*_\theta(A) \subset U \cap F = A$. Therefore, $A = U \cap cl^*_\theta(A)$.

$(b) \Rightarrow (c)$. Since, $cl^*_\theta(A) - A = cl^*_\theta(A) \cap (X - A) = cl^*_\theta(A) \cap (X - (U \cap cl^*_\theta(A)))$ $= cl^*_\theta(A) \cap (X - U)$, it is closed.

$(c) \Rightarrow (d)$. Since $X - (cl^*_\theta(A) - A) = A \cup (X - cl^*_\theta(A)), A \cup (X - cl^*_\theta(A))$ is open.

$(d) \Rightarrow (e)$. $A \subset A \cup (X - cl^*_\theta(A)) = int(A \cup (X - cl^*_\theta(A)))$

$(e) \Rightarrow (a)$. $X - cl^*_\theta(A) = int(X - cl^*_\theta(A)) \subset int(A \cup *(X - cl^*_\theta(A)))$ Therefore, $A \cup (X - cl^*_\theta(A)) \subset int(A \cup (X - cl^*_\theta(A)))$. So $A \cup (X - cl^*_\theta(A))$ is open. Since, $A = (A \cup (X - cl^*_\theta(A))) \cap cl^*_\theta(A)$, A is *locally* $\theta - I - closed$.

If we put $I = \{\phi\}$ in Theorem 2.1, we get Corollary 2.2. If we put $I = P(X)$ in Theorem 2.1, we get Corollary 2.3.

Corollary 2.2. *Let* (X, τ) *be a topological space and* A *be a Subset of* X*. Then the following are equivalent.*

(a) A *is locally* $\theta - closed$

(b) $A = U \cap cl_\theta(A)$, *for some open set* U

(c) $cl_\theta(A) - A$ *is closed*

(d) $A \cup (X - cl_\theta(A))$ *is open*

(e) $A \subset int(A \cup (X - cl_\theta(A)))$.

Corollary 2.3. *Let* (X, τ) *be a topological space and* A *be a subset of* X*. Then the following are equivalent.*

(a) A is locally closed

(b) $A = U \cap cl(A)$, *for some open set* U

(c) $cl(A) - A$ *is closed*

(d) $A \cup (X - cl(A))$ *is open*

(e) $A \subset int(A \cup (X - cl(A)))$.

Theorem 2.4. *Let* (X, τ, I) *be an ideal space and* A *be a subset of* X. *If* A *is* $locally\ \theta - I - closed$ *and* $I - dense$, *then* A *is open.*

Proof. If A is $locally\ \theta - I - closed$, then by Theorem 2.1, $A \subset int(A \cup (X - cl^*_\theta(A)))$. Since A is $I - dense, A^* = X$ and hence $cl^*(A) = X$. Since $cl^*(A) \subset cl^*_\theta(A)$, we have $cl^*_\theta(A) = X$. So $A \subset int(A)$, which implies that A is open.

Corollary 2.5. *Let* (X, τ, I) *be an ideal space and* A *be an* $I - dense$ *subset of* X. *Then,* A *is* $locally\ \theta - I - closed$ *if and only if* A *is open.*

Theorem 2.6. *Let* (X, τ, I) *be an ideal space and* A *be a* $\theta - I_g - closed$ *subset of* X. *Then,* A *is* $locally\ \theta - I - closed$ *if and only if* A *is* $\theta - I - closed$.

Proof. If A is $\theta - I - closed$, then A is $locally\ \theta - I - closed$. Conversely, suppose A is $locally\ \theta - I - closed$ and $\theta - I_g - closed$. By Theorem 2.3 $cl^*_\theta(A) - A$ has no nonempty closed set. By, Theorem 2.1 (c), $cl^*_\theta(A) - A$ is closed. Therefore, $cl^*_\theta(A) - A = \phi$, which implies that $cl^*_\theta(A) \subset A$ and so A is $\theta - I - closed$.

If we put $I = \{\phi\}$ in Theorem 2.6, we set the following Corollary 2.7. If we put $I = P(X)$ in Theorem 2.6, we get the Corollary 2.8.

Corollary 2.7. *Let* (X, τ) *be a topological space and* A *be a* $\theta - g - closed$ *subset of* X. *Then,* A *is* $locally\ \theta - closed$ *if and only if* A *is* $\theta - closed$.

Corollary 2.8. *Let (X, τ) be a topological space and A be a $g-closed$ subset of X. Then, A is locally closed if and only if A is closed.*

Theorem 2.9. *An ideal space (X, τ, I) is a $T_1 - space$ if and only if every $\theta - I_g - closed$ set is locally $\theta - I - closed$.*

Proof. Suppose, A is any $\theta - I_g - closed$ subset of X. Then by Theorem 2.3 [13], $cl^*_\theta(A) - A$ contains no nonempty closed set. By hypothesis, A is $locally\ \theta - I - closed$. So, by Theorem 2.1, $cl^*_\theta(A) - A$ is closed. Therefore, $cl^*_\theta(A) - A = \phi$ and hence $cl^*_\theta(A) = A$. Then A is $\theta - I - closed$ and hence A is $\star - closed$. Therefore, every $\theta - I_g - closed$ set of X is $\star - closed$. By Theorem 3.2 [13], (X, τ, I) is a $T_I - space$. Conversely, suppose A is a $\theta - I_g - closed$ set. Then by Lemma 1.6, A is $\theta - I - closed$ and hence $locally\ \theta - I - closed$.

If we put $I = \{\phi\}$ in Theorem 2.9, we get the Corollary 2.10. If we put $I = P(X)$ in Theorem 2.9, we get the Corollary 2.11.

Corollary 2.10. *In a topological space (X, τ), if every $\theta - g - closed$ set is locally $\theta - closed$, then (X, τ) is a $T_1 - space$.*

Corollary 2.11. *In a topological space (X, τ), if every $g - closed$ set is locally closed, then (X, τ) is a $T_{1/2} - space$.*

Since X is $\theta - I - closed$, every open set is locally $\theta - I - closed$. Since X is open, every $\theta - I - closed$ set is locally $\theta - I - closed$.

Example 2.12, shows that $locally\ \theta - I - closed$ need not be $\theta - I - closed$. Example 2.13 , show that a $locally\ \theta - I - closed$ set need not be an open set.

Example 2.12. *Let $X = \{a, b, c, d\}, \tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\}$ and $I = \{\phi, \{a\}, \{c\}, \{a, c\}\}$. Let $A = \{b, c\}$. Then, A is open and hence locally $\theta - I - closed$. Since, $cl^*_\theta(A) = X$, A is not $\theta - I - closed$.*

Example 2.13. *Let* $X = \{a, b, c, d\}, \tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\}$ *and* $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. *Let* $A = \{c\}$. *Then* A *is* $\theta - I - closed$. *But it is not open.*

3 $\Lambda - \theta - I - closed$ sets

A subset A of an ideal space (X, τ, I) is said to be a $\Lambda - \theta - I - closed$ set if $A = B \cap C$, where B is a $\Lambda - set$ and C is $\theta - I - closed$. The complement of a $\Lambda - \theta - I - closed$ set is said to be a $\Lambda - \theta - I - open$ set. A subset A of a topological space (X, τ) is said to be a $\Lambda - \theta - closed$ if $A = B \cap C$, where B is a $\Lambda - set$ and C is a $\theta - closed$ set. The complement of a $\Lambda - \theta - closed$ set is said to be a $\Lambda - \theta - open$ set.

If $I = \{\phi\}$, then the $\Lambda - \theta - I - closed$ sets coincide with $\Lambda - \theta - closed$ sets. If $I = P(X)$, then the $\Lambda - \theta - I - closed$ sets coincide with $\Lambda - closed$ sets.

Theorem 3.1. *If* A *is a subset of an ideal space* (X, τ, I), *then the following are equivalent.*

(a) A *is* $\Lambda - \theta - I - closed$

(b) $A = L \cap cl^*_\theta(A)$, *where* L *is a* $\Lambda - set$

(c) $A = A^\Lambda \cap cl^*_\theta(A)$.

Proof. $(a) \Rightarrow (b)$. Suppose A is a $\Lambda - \theta - I - closed$ set. Then $A = L \cap B$, where L is a $\Lambda - set$ and B is a $\theta - I - closed$ set. Since, $cl^*_\theta \subset cl^*_\theta(B) = B$, we have $L \cap cl^*_\theta(A) \subset L \cap B = A$. On the other hand, $A \subset L \cap cl^*_\theta(A)$. Therefore, $A = L \cap cl^*_\theta(A)$.

$(b) \Rightarrow (c)$. Suppose $A = L \cap cl^*_\theta(A)$, where L is a $\Lambda - set$. Clearly, $A \subset A^\Lambda \cap cl^*_\theta(A)$. Since $A \subset L$, $A^\Lambda \subset L^\Lambda = L$. Hence, $A^\Lambda \cap cl^*_\theta(A) \subset L \cap cl^*_\theta(A) = A$. Therefore, $A = A^\Lambda \cap cl^*_\theta(A)$.

$(c) \Rightarrow (a)$ follows from the fact that A^{Λ} is a $\Lambda - set$ for every subset A of X.

If we put $I = \{\phi\}$ in Theorem 3.1, we get Corollary 3.2. If we put $I = P(X)$ in Theorem 3.1, we get Corollary 3.3.

Corollary 3.2. *If A is a subset of a topological space (X, τ), then the following are equivalent.*

(a) A is $\Lambda - \theta - closed$

(b) $A = L \cap cl_{\theta}(A)$, where L is a $\Lambda - set$

(c) $A = A^{\Lambda} \cap cl_{\theta}(A)$.

Corollary 3.3. *If A is a subset of a topological space (X, τ), then the following are equivalent.*

(a) A is $\Lambda - closed$

(b) $A = L \cap cl(A)$, where L is a Λ set

(c) $A = A^{\Lambda} \cap cl(A)$

Theorem 3.4. *A $\star-dense$ subset A of an ideal space (X, τ, I) is $\Lambda-\theta-I-closed$ if and only if it is a $\Lambda - set$*

Proof. If A is a $\Lambda - set$ then it is $\Lambda - \theta - I - closed$. Conversely, suppose A is $\Lambda - \theta - I - closed$ and $\star - dense$. Then $A = A^{\Lambda} \cap cl_{\theta}^{*}(A)$. Since A is $\star - dense$, we have $cl^{*}(A) = X$. Since $cl^{*}(A) \subset cl_{\theta}^{*}(A)$, we have $cl_{\theta}^{*}(A) = X$. Therefore, $A^{\Lambda} \cap cl_{\theta}^{*}(A) = A^{\Lambda} \cap X = A^{\Lambda}$. Therefore, $A = A^{\Lambda}$. Therefore A is a $\Lambda - set$.

If we put $I = \{\phi\}$ in Theorem 3.4, we get Corollary 3.5. If we put $I = P(X)$ in Theorem 3.4, we get Corollary 3.6.

Corollary 3.5. *A dense subset A of a space (X, τ) is $\Lambda - \theta - closed$ if and only if it is a $\Lambda - set$.*

Corollary 3.6. *A dense subset A of a space (X, τ) is $\Lambda - closed$ if and only if it is a $\Lambda - set$*

Since X is a $\theta - I - closed$ set, every $\Lambda - set$ is a $\Lambda - \theta - I - closed$ set. Since X is a $\Lambda - set$, every $\theta - I - closed$ set is a $\Lambda - \theta - I - closed$ set. Example 3.7 show that a $\Lambda - \theta - I - closed$ set need not be $\Lambda - set$. Example 3.8 shows that a $\Lambda - \theta - I - closed$ set need not be a $\theta - I - closed$ set.

Example 3.7. *Let $X = \{a, b, c, d\}, \tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\}$ and $I = \{\phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$.*

Let $A = \{a, c, d\}$. Then $cl^*_\theta(A) = A$. Therefore, A is $\theta - I - closed$ and hence $\Lambda - \theta - I - closed$. But $A^\Lambda = X \neq A$. Therefore, A is not a $\Lambda - set$.

Example 3.8. *Let (X, τ, I) be same as in Example 3.7. Let $A = \{a, b, c\}$. Since A is an open set, A is $\Lambda - \theta - I - closed$. But $cl^*_\theta(A) = X \neq A$. Therefore, A is not $\theta - I - closed$.*

Since every $\theta - I - closed$ set is a $\star - closed$ set, every $\Lambda - \theta - I - closed$ set is $\Lambda - \star - closed$ set. Example 3.9 shows that a $\Lambda - \star - closed$ set need not be a $\Lambda - \theta - I - closed$ set.

Example 3.9. *Let (X, τ, I) be same as in Example 3.7. Let $A = \{b, c, d\}$. Then $A^* = \phi \subset A$. So A is $\star - closed$. and hence $\Lambda - \star - closed$. Now $A^\Lambda = X$ and $cl^*_\theta(A) = X$. So $A^\Lambda \cap cl^*_\theta(A) = X \neq A$. Therefore, by Theorem 3.1, A is not $\Lambda - \theta - I - closed$.*

Theorem 3.10. *Let (X, τ, I) be an ideal space and A be a $\star - dense$ subset of X. Then A is $\Lambda - \star - closed$ if and only if it is $\Lambda - \theta - I - closed$.*

Proof. Every $\Lambda - \theta - I - closed$ is $\Lambda - \star - closed$. Conversely, suppose A is $\Lambda - \star - closed$ and a $\star - dense$ subset of X. Then, $A = A^\Lambda \cap cl^*(A)$. Since

X is $\star - dense$, $cl^*(A) = X$. But $cl^*(A) \subset cl^*_\theta(A)$. Therefore $Cl^*_\theta(A) = X$. So, $A = A^\Lambda \cap X$ and hence A is $\Lambda - \theta - I - closed$.

A subset A of an ideal space (X, τ, I) is said to be $\theta - I.\Lambda - set$ if $A^\Lambda \subset F$ whenever $A \subset F$ and F is $\theta - I - closed$. A subset A of (X, τ, I) is said to be $\theta - g.\Lambda - set$ if $A^\Lambda \subset F$ whenever $A \subset F$ and A is $\theta - closed$.

Theorem 3.11. *A $\Lambda - \theta - I - closed$ set of an ideal space (X, τ, I) is a $\Lambda - set$. if and only if it is a $\theta - I.\Lambda - set$*

Proof. Let A be a $\Lambda - \theta - I - closed$ set. If A is a $\Lambda - set$, then A is $\theta - I.\Lambda - set$. Conversely, suppose A is a $\theta - I.\Lambda - set$. Then, by Lemma 1.5, $cl^*_\theta(A) \subset A^\Lambda$. So $A^\Lambda \cap cl^*_\theta(A) = A^\Lambda$. Since A is a $\Lambda - \theta - I - closed$ set, $A^\Lambda \cap cl^*_\theta(A) = A$. Therefore, $A = A^\Lambda$ and hence A is a $\Lambda - set$.

If we put $I = \{\phi\}$ in Theorem 3.11, we get Corollary 3.12. If we put $I = P(X)$ in Theorem 3.11, we get Corollary 3.13.

Corollary 3.12. *A $\Lambda - \theta - closed$ set of a topological space (X, τ) is a $\Lambda - set$ if and only if it is a $\theta - g.\Lambda - set$.*

Corollary 3.13. *A $\Lambda - closed$ set of a topological space (X, τ) is a $\Lambda - set$ if and only if it is a $g.\Lambda - set$.*

Theorem 3.14. *For a subset A of an ideal space (X, τ, I), the following are equivalent.*

(a) A is $\Lambda - \theta - I - closed$.

(b) $cl^*_\theta(A) - A$ is a $V - set$

Proof. $(a) \Rightarrow (b)$. Suppose, A is a $\Lambda - \theta - I - closed$ set. Then $A = L \cap cl^*_\theta(A)$, for some $\Lambda - set$ L. Now $cl^*_\theta(A) - A = cl^*_\theta(A) \cap (X - A) = cl^*_\theta(A) \cap (X - (L \cap cl^*_\theta(A))) = cl^*_\theta(A) \cap ((X - L) \cup (X - cl^*_\theta(A))) = cl^*_\theta(A) \cap (X - L)$. Since $cl^*_\theta(A)$ is closed, it is a $V - set$. Therefore $cl^*_\theta(A) \cap (X - L)$ is a $V - set$ and hence $cl^*_\theta(A) - A$ is a $V - set$.

$(b) \Rightarrow (c)$ Suppose $cl^*_\theta(A) - A$ is a $V - set$. Then $X - (cl^*_\theta(A) - A) = A \cup (X - cl^*_\theta(A))$ is a $\Lambda - set$. Now, $(A \cup (X - cl^*_\theta(A)) \cap cl^*_\theta(A) = (A \cap cl^*_\theta(A)) \cup (X - cl^*_\theta(A)) \cap cl^*_\theta(A)) = A \cap cl^*_\theta(A) = A$. Therefore, A is a $\Lambda - \theta - I - closed$ set.

If we put $I = \{\phi\}$ and $I = P(X)$ in Theorem 3.14, respectively, we get Corollary 3.15 and Corollary 3.16.

Corollary 3.15. *For a subset A of a topological space (X, τ), the following are equivalent.*

(a) A is $\Lambda - \theta - closed$

(b) $cl_\theta(A) - A$ is a $V - set$

Corollary 3.16. *For a subset A of a topological space (X, τ), the following equivalent.*

(a) A is $\Lambda - closed$

(b) $cl(A) - A$ is a $V - set$.

Theorem 3.17. *For a subset A of an ideal space (X, τ, I), the following are equivalent.*

(a) A is $\Lambda - \theta - I - closed$

(b) $A \cup (X - cl^*_\theta(A))$ is $\Lambda - \theta - I - closed$.

Proof. $(a) \Rightarrow (b)$. If A is $\Lambda - \theta - I - closed$, then by Theorem 3.14, $A \cup (X - cl^*_\theta(A)) = X - (cl^*_\theta(A) - A)$ is a $\Lambda - set$ and hence a $\Lambda - \theta - I - closed$ set.

$(b) \Rightarrow (a)$. If $A \cup (X - cl^*_\theta(A))$ is a $\Lambda - \theta - I - closed$ set, then by Theorem 3.14, $cl^*_\theta(A \cup (X - cl^*_\theta(A))) - (A \cup (X - cl^*_\theta(A))) = X - (A \cup (X - cl^*_\theta(A))) = cl^*_\theta(A) - A$ is a $V - set$. Again by Theorem 3.14, A is $\Lambda - \theta - I - closed$.

If we $I = \{\phi\}$ and $I = P(X)$ in Theorem 3.17, respectively we get Corollary 3.18 and Corollary 3.19.

Corollary 3.18. *For a subset A of a topological space (X, τ), the following are equivalent.*

(a) A is $\Lambda - \theta - closed$

(b) $A \cup (X - cl_\theta(A))$ is $\Lambda - \theta - closed$.

Corollary 3.19. *For a subset A of a topological space (X, τ), the following are equivalent.*

(a) A is $\Lambda - closed$

(b) $A \cup (X - cl(A))$ is $\Lambda - closed$.

Theorem 3.20. *For a subset A of an ideal space (X, τ, I), the following are equivalent.*

(a) A is $\theta - I - closed$

(b) A is $\theta - I_g - closed$ and locally $\theta - I - closed$.

(c) A is $\theta - I_g - closed$ and $\Lambda - \theta - I - closed$.

Proof. *(a) $\Rightarrow$ (b) follows from the fact that every $\theta - I - closed$ set is both $\theta - I_g - closed$ and locally $\theta - I - closed$.*

(b) $\Rightarrow$ (c) follows from the fact that every locally $\theta - I - closed$ is $\Lambda - \theta - I - closed$.

(c) $\Rightarrow$ (a). Let A be a $\Lambda - \theta - I - closed$ set. Then $A = A^\Lambda \cap cl^*_\theta(A)$. Since A is $\theta - I_g - closed$, by Lemma 1.5, $cl^*_\theta(A) \subset A^\Lambda$. So $A = A^\Lambda \cap cl^*_\theta(A) = cl^*_\theta(A)$. Hence A is $\theta - I - closed$.

If we put $I = \{\phi\}$ and $I = P(X)$ in Theorem 3.20, respectively we get Corollary 3.21 and Corollary 3.22.

Corollary 3.21. *For a subset A of a space (X,τ), the following are equivalent.*

(a) A is $\theta-closed$

(b) A is $\theta-g-closed$ and locally $\theta-closed$

(c) A is $\theta-g-closed$ and $\Lambda-\theta-closed$.

Corollary 3.22. *For a subset A of a space (X,τ), the following are equivalent*

(a) A is closed

(b) A is $g-closed$ and locally closed

(c) A is $g-closed$ and $\Lambda-closed$.

Theorem 3.23. *Let (X,τ,I) be an ideal space and (X,τ) is a $T_1-space$. Then the following are equivalent.*

(a) (X,τ,I) is a $T_I-space$

(b) Every $\theta-I_g-closed$ set is a locally $\theta-I-closed$ set.

(c) Every $\theta-I_g-closed$ set is $\Lambda-\theta-I-closed$

(d) For each $x\in X$, $\{x\}$ is either closed or $\Lambda-\theta-I-open$.

(e) For each $x\in X$, $\{x\}$ is either closed or $\theta-I-open$.

Proof. $(a)\Rightarrow(b)$ *follows from the Theorem 2.8*

$(b)\Rightarrow(c)$ *follows from the fact that every locally $\theta-I-closed$ is $\Lambda-\theta-I-closed$.*

$(c)\Rightarrow(d)$. *Suppose $\{x\}$ is not closed. Then $X-\{x\}$ is not open and hence $\theta-I_g-closed$. By (c), $X-\{x\}$ is $\Lambda-\theta-I-closed$. Hence $\{x\}$ is $\Lambda-\theta-I-open$.*

$(d)\Rightarrow(e)$. *Suppose $\{x\}$ is not closed. Then by (d), $\{x\}$ is $\Lambda-\theta-I-open$. Therefore, $X-\{x\}$ is $\Lambda-\theta-I-closed$. Since $\{x\}$ is not closed, $X-\{x\}$ is $\theta-I_g-closed$. By Theorem 3.20, $X-\{x\}$ is $\theta-I-closed$. Therefore, $\{x\}$ is $\theta-I-open$.*

(e) $\Rightarrow$ (a). Suppose $\{x\}$ is not closed. Then by hypothesis, $\{x\}$ is $\theta - I -$open. Therefore, $\{x\}$ is open and hence $\star -$open. By Lemma 1.2, (X, τ, I) is a $T_I -$ space. This proves (a)

If we put $I = \{\phi\}$ in Theorem 3.23, we get Corollary 3.24.

Corollary 3.24. *In a T_1- topological space (X, τ), the following are equivalent.*

(a) (X, τ) is a $T_{1/2} -$ space

(b) Every $\theta - g -$closed set is locally $\theta -$closed set.

(c) Every $\theta - g -$closed set is $\Lambda - \theta -$closed.

(d) For each $x \in X$, $\{x\}$ is either closed or $\Lambda - \theta -$open

(e) For each $x \in X$, $\{x\}$ is either closed or $\theta -$open

Theorem 3.25. *In an ideal space (X, τ, I) the following are equivalent*

(a) Every $\Lambda - \theta - I -$closed set is $\theta - I -$closed

(b) Every open set is $\theta - I -$closed

(c) Every subset of X is $\theta - I_g -$closed.

Proof. *(a) $\Rightarrow$ (b). Suppose U is an open set. Then U is a $\Lambda -$set and hence is $\Lambda - \theta - I -$closed. By (a), U is a $\theta - I -$closed.*

(b) $\Rightarrow$ (c). Suppose A is a subset of X and U is an open set containing A. Then $cl^*_\theta(A) \subset cl^*_\theta(U)$. By (b), $cl^*_\theta(A) \subset U$. Therefore, A is $\theta - I_g -$closed.

(c) $\Rightarrow$ (a). Suppose A is $\Lambda - \theta - I -$closed. By (c), A is $\theta - I_g -$closed. Therefore, by Theorem 3.19, A is $\theta - I -$closed.

If we put $I = \{\phi\}$ and $I = P(X)$ in Theorem 3.25, respectively we get Corollary 3.26 and Corollary 3.27.

Corollary 3.26. *In a topological space (X, τ), the following are equivalent*

(a) Every $\Lambda - \theta -$closed set is $\theta -$closed

(b) Every open set is $\theta-closed$

(c) Every subset of X *is* $\theta-g-closed$

Corollary 3.27. *In a topological space* (X,τ), *the following are equivalent*

(a) Every $\Lambda-closed$ *set is closed*

(b) Every open set is closed

(c) Every subset of X *is* $g-closed$.

The proof of the following Theorem 3.28, follows from the fact that arbitrary intersection of $\Lambda-sets$ *is a* $\Lambda-set$.

Theorem 3.28. *Arbitrary intersection of* $\Lambda-\theta-I-closed$ *sets is* $\Lambda-\theta-I-closed$.

The following Theorem 3.30, gives the characterization of $\Lambda-\theta-I-open$ *sets.*

Theorem 3.29. *For a subset* A *of an ideal space* (X,τ,I) *the following are equivalent.*

(a) A *is* $\Lambda-\theta-I-open$

(b) $A=M\cup W$, *where* M *is a* $V-set$ *and* W *is a* $\theta-I-open$ *set.*

Proof. $(a)\Rightarrow(b)$. *Suppose* A *is* $\Lambda-\theta-I-Open$. *Then* $X-A$ *is* $\Lambda-\theta-I-closed$. *So* $X=B\cap C$, *where* B *is a* $\Lambda-set$ *and* C *is* $\theta-I-closed$. *So,* $A=M\cup W$, *where* $M=X-B$ *is a* $V-set$ *and* $W=X-C$ *is* $\theta-I-open$.

$(b)\Rightarrow(a)$. *Suppose,* $A=M\cup W$, *where* M *is a* $V-set$ *and* W *is* $\theta-I-open$. *Then* $X-A=(X-M)\cap(X-W)$, *where* $X-A$ *is a* $\Lambda-set$ *and* $X-W$ *is* $\theta-I-closed$. *Therefore,* $X-A$ *is* $\Lambda-\theta-I-closed$ *and hence* A *is* $\Lambda-\theta-I-open$

If we $I=\{\phi\}$ *and* $I=P(X)$ *in Theorem 3.29, respectively we get Corollary 3.30 and Corollary 3.31.*

Corollary 3.30. *For a subset A of a topological space (X, τ), the following are equivalent.*

(a) A is $\Lambda - \theta - open$

(b) $A = M \cup W$, where M is a $V - set$ and W is $\theta - open$

Corollary 3.31. *For a subset A of a topological space(X, τ), the following are equivalent*

(a) A is $\Lambda - open$

(b) $A = M \cup W$, where M is a $V - set$ and W is open.

Theorem 3.32. *For a subset A of an ideal space (X, τ, I), the following are equivalent.*

(a) A is $\Lambda - \theta - I - open$

(b) $A = M \cup int^*_\theta(A)$, where M is a $V - set$

(c) $A = A^\Lambda \cup int^*_\theta(A)$

Proof. $(a) \Rightarrow (b)$. *Suppose, A is $\Lambda - \theta - I - open$. Then $X - A$ is $\Lambda - \theta - I - closed$. Therefore, $X - A = L \cup cl^*_\theta(X - A)$, where L is a $\Lambda - set$. So, $A = (X - L) \cup (X - cl^*_\theta(X - A)) = M \cup int^*_\theta(A)$, where $M = X - L$ is a $V - set$.*

$(b) \Rightarrow (c)$. *Suppose, $A = M \cup int^*_\theta(A)$, where M is a $V - set$. Since $M \subset A$, $M = M^V \subset A^V$. Therefore, $A = M^V \cup int^*_\theta(A) \subset A^* \cup int^*_\theta(A)$. Hence, $A = A^V \cup int^*_\theta(A)$.*

$(c) \Rightarrow (a)$ *follows from the fact that A^V is a $V - set$ and $int^*_\theta(A)$ is $\theta - I - open$.*

If we put $I = \{\phi\}$ and $I = P(X)$ in Theorem 3.32, respectively we get corollary 3.33 and Corollary 3.34.

Corollary 3.33. *For a subset A of a topological space (X, τ), the following are equivalent.*

(a) A *is* $\Lambda - \theta - open$

(b) $M \cup int_\theta(A)$, *where* M *is a* $V - set$

(c) $A = A^V \cup int_\theta(A)$.

Corollary 3.34. *For a subset* A *of a topological space* (X, τ), *the following are equivalent.*

(a) A *is* $\Lambda - open$

(b) $A = M \cup int(A)$, *where* M *is a* $V - set$

(c) $A = A^V \cup int(A)$

The following Theorem 3.35, gives characterization of $\Lambda - \theta - I - open$ *sets in terms* $\theta - I_g - closed$ *sets.*

Theorem 3.35. *Let* (X, τ, I) *be an ideal space. Then for each* $x \in X$, *either* $\{x\}$ *is* $\Lambda - \theta - I - open$ *or* $X - \{x\}$ *is* $\theta - I_g - closed$.

Proof. *Suppose,* $\{x\}$ *is not* $\Lambda - \theta - I - open$. *Then* $A = X - \{x\}$ *is not* $\Lambda - \theta - I - closed$. *So,* $A^\Lambda \cup cl_\theta^*(A) \neq A$. *But* $A = X - \{x\} \subset A^\Lambda \cup cl_\theta^*(A)$. *Therefore,* $A^\Lambda \cup cl_\theta^*(A) = X$ *and hence* $A^\Lambda = cl_\theta^*(A) = X$. *Therefore,* X *is the only open set containing* A. *Hence* A *is* $\theta - I_g - closed$.

If we put $I = \{\phi\}$ *and* $I = P(X)$ *in Theorem 3.35, respectively we get the Corollary 3.36 and Corollary 3.37.*

Corollary 3.36. *Let* (X, τ) *be a topological space. Then for each* $x \in X$, *either* $\{x\}$ *is* $\Lambda - \theta - open$ *or* $X - \{x\}$ *is* $\theta - g - closed$.

Corollary 3.37. *Let* (X, τ) *be a topological space. Then for each* $x \in X$, *either* $\{x\}$ *is* $\Lambda - open$ *or* $X - \{x\}$ *is* $g - closed$.

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