On Neutrosophic Feebly Open Set In Neutrosophic Topological Spaces

P. Jeya Puvaneswari *1, Dr.K.Bageerathi 2

1 Department of Mathematics, Vivekananda College, Agasteeswaram – 629701, India.
2 Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur – 628216, India.

Abstract: *The focus of this paper is to introduce the concept of Neutrosophic point, Neutrosophic quasi coincident, Neutrosophic feebly open sets and Neutrosophic feebly closed sets in Neutrosophic Topological spaces. Also we analyse their characterizations and investigate their properties. This concept is the generalization of intuitionistic topological spaces and fuzzy topological spaces. Using this neutrosophic feebly open sets and neutrosophic feebly closed sets, we define a new class of functions namely neutrosophic feebly continuous functions. Further, relationships between this new class and the other classes of functions are established.*

Keywords: *Neutrosophic sets, Neutrosophic point, Neutrosophic quasi coincident, Neutrosophic Topological spaces, Neutrosophic feebly open set, Neutrosophic feebly closed set and Neutrosophic continuous functions .*

INTRODUCTION

Theory of fuzzy sets [18], theory of intuitionistic fuzzy sets [1-3], theory of neutrosophic sets [9] and the theory of interval neutrosophic sets [12] can be considered as tools for dealing with uncertainities. However, all of these theories have their own difficulties which are pointed out in [12]. In 1965, Zadeh [18] introduced fuzzy set theory as a mathematical tool for dealing with uncertainities where each element had a degree of membership. The intuitionistic fuzzy set was introduced by Atanassov [2] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. The neutrosophic set was introduced by Smarandache [9] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set. In 2012, Salama, Alblowi [16], introduced the concept of neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element. In 2014, Salama, Smarandache and Valeri [17] were introduced the concept of neutrosophic closed sets and neutrosophic continuous functions.

In this paper, we introduce and study the concept of neutrosophic feebly open sets and neutrosophic feebly continuous functions in neutrosophic topological spaces. This paper consists of four sections. The Section I consists of the basic definitions and the operations of neutrosophic sets which are used in the later sections. The Section II deals with the concept of Neutrosophic point, Neutrosophic quasi coincident, Neutrosophic quasi neighbourhood, Neutrosophic feebly open sets in Neutrosopic topological space and study their properties. The Section III deals with the complement of neutrosophic feebly open set namely neutrosophic feebly closed set. The Section IV consists of neutrosophic feebly continuous functions in neutrosophic topological spaces and its relations with other functions.

I. PRELIMINARIES

In this section, we give the basic definitions for neutrosophic sets and its operations.

Definition 1.1 [16] Let X be a non-empty fixed set. A neutrosophic set (NF for short) A is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 1.2 [16] A neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A \rangle$ in $]^-0,1^+[$ on X.

Remark 1.3 [16] For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ for the neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$.

Example 1.4 [16] Every intuitionistic fuzzy Set A is a non-empty set in X is obviously on neutrosophic set having the form $A = \{ \langle x, \mu_A(x), 1-(\mu_A(x) + \gamma_A(x)), \gamma_A(x) \rangle : x \in X \}$**.** Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the neutrosophic set 0_N and 1_N in X as follows :

0_N may be defined as:

(0_1) $0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$

(0_2) $0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$

(0_3) $0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$
(0_4) $0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$

1_N may be defined as:
(1_1) $1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$
(1_2) $1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$
(1_3) $1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$
(1_4) $1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$

Definition 1.5 [16] Let $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$ be a NF on X. Then the complement of the set A (C(A) for short) may be defined as three kinds of complements :
(C_1) $C(A) = \{ \langle x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \gamma_A(x) \rangle : x \in X \}$
(C_2) $C(A) = \{ \langle x, \gamma_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \}$
(C_3) $C(A) = \{ \langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X \}$
One can define several relations and operations between neutrosophic set follows :

Definition 1.6 [16] Let x be a non-empty set, and neutrosophic set A and B in the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X \}$. Then we may consider two possible definitions for subsets ($A \subseteq B$).
($A \subseteq B$) may be defined as :
(1) $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ $\forall x \in X$
(2) $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ $\forall x \in X$

Proposition 1.7 [16] For any neutrosophic set A the following are holds :
(1) $0_N \subseteq A$, $0_N \subseteq 0_N$
(2) $A \subseteq 1_N$, $1_N \subseteq 1_N$

Definition 1.8 [16] Let X be a non-empty set, and $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$, $B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$ are neutrosophic set. Then
(1) $A \cap B$ may be defined as:
(I_1) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\gamma_A(x) \vee \gamma_B(x) \rangle$
(I_2) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\gamma_A(x) \vee \gamma_B(x) \rangle$
(2) $A \cup B$ may be defined as:
(U_1) $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\gamma_A(x) \wedge \gamma_B(x) \rangle$
(U_2) $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\gamma_A(x) \wedge \gamma_B(x) \rangle$

We can easily generalize the operations of intersection and union in Definition 1.8 to arbitrary family of neutrosophic set as follows:

Definition 1.9 [16] Let $\{ A_j : j \in J \}$be a arbitrary family of neutrosophic set in X. Then
(1) $\cap A_j$ may be defined as:
(i) $\cap A_j = \langle x, \bigwedge_{j \in J} \mu_{A_j}(x), \bigwedge_{j \in J} \sigma_{A_j}(x), \bigvee_{j \in J} \gamma_{A_j}(x) \rangle$
(ii) $\cap A_j = \langle x, \bigwedge_{j \in J} \mu_{A_j}(x), \bigvee_{j \in J} \sigma_{A_j}(x), \bigvee_{j \in J} \gamma_{A_j}(x) \rangle$
(2) $\cup A_j$ may be defined as:
(i) $\cup A_j = \langle x, \vee, \vee, \wedge \rangle$
(ii) $\cup A_j = \langle x, \vee, \wedge, \wedge \rangle$

Proposition 1.10 [16] For all A and B are two neutrosophic sets then the following conditions are true :
(1) $C(A \cap B) = C(A) \cup C(B)$
(2) $C(A \cup B) = C(A) \cap C(B)$.
Here we extend the concepts of fuzzy topological space [5] and Intuitionistic fuzzy topological space [6,7] to the case of neutrosophic sets.

Definition 1.11 [16] A neutrosophic topology (NT for short) is a non-empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms :
(NT_1) $0_N, 1_N \in \tau$,
(NT_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(NT_3) $\cup G_i \in \tau$ for every $\{ G_i : i \in J \} \subseteq \tau$.
In this case the pair (X, τ) is called a neutrosophic topological space (NTS for short). The elements of τ are called neutrosophic open sets (NOS for short). A neutrosophic set F is closed if and only if it C (F) is neutrosophic open.

Example 1.12 [16] Any fuzzy topological space (X, τ_0) in the sense of Chang is obviously a NTS in the form $\tau = \{ A : \mu_A \in \tau_0 \}$ wherever we identify a fuzzy set in X whose membership function is μ_A with its counterpart.

Remark 1.13 [16] Neutrosophic topological spaces are very natural generalizations of fuzzy topological spaces allow more general functions to be members of fuzzy topology.

Example 1.14 [16] Let X ={ x } and
$A = \{ \langle x, 0.5, 0.5, 0.4 \rangle : x \in X \}$
$B = \{ \langle x, 0.4, 0.6, 0.8 \rangle : x \in X \}$
$D = \{ \langle x, 0.5, 0.6, 0.4 \rangle : x \in X \}$
$C = \{ \langle x, 0.4, 0.5, 0.8 \rangle : x \in X \}$
Then the family $\tau = \{ 0_N, A, B, C, D, 1_N \}$of neutrosophic sets in X is neutrosophic topology on X.

Definition 1.15 [16] The complement of a neutrosophic open set A(C (A) for short) is called a neutrosophic closed set (NCS for short) in X.
Now, we define neutrosophic closure and interior operations in neutrosophic topological spaces.

Definition 1.16 [17] Let (X, τ) be NTS and $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ be a NF in X. Then the neutrosophic closure and neutrosophic interior of A are defined by

NCl (A) = $\cap$ { K : K is a NCS in X and A $\subseteq$ K }
NInt (A) = $\cup$ {G : G is a NOS in X and G $\subseteq$ A }.
It can be also shown that NCl (A) is NCS and NInt (A) is a NOS in X. That is,
a) A is NCS in X if and only if A = NCl (A).
b) A is NOSin X if and only if A = NInt(A).

Proposition 1.17 [17] For any neutrosophic set A in (X, τ) we have
(a) NCl (C (A))= C (NInt (A),
(b) NInt (C (A)) = C (NCl (A)).

Proposition 1.18 [17] Let (X, τ) be a NTS and A,B be two neutrosophic sets in X. Then the following properties holds :
(a) NInt (A) $\subseteq$ A,
(b) A $\subseteq$ NCl (A),
(c) A $\subseteq$ B $\Rightarrow$ NInt (A) $\subseteq$ NInt (B),
(d) A $\subseteq$ B $\Rightarrow$ NCl (A) $\subseteq$ NCl (B),
(e) NInt (A $\cap$ B)) = NInt (A) $\wedge$ NInt (B),
(f) NCl (A $\cup$ B) = NCl (A) $\vee$ NCl (B),
(g) NInt (1_N) = 1_N,
(h) NCl (0_N) = 0_N,
(i) A $\subseteq$ B $\Rightarrow$ C (B) $\subseteq$ C (A),
(j) NCl (A $\cap$ B) $\subseteq$ NCl (A) $\cap$ NCl (B),
(k) NInt (A $\cup$ B) $\supseteq$ NInt (A) $\cup$ NInt (B),

Definition 1.19 [5] A Neutrosophic subset A is Neutrosophic semi open if A $\leq$ NClNInt A.
Definition 1.20 [5] A Neutrosophic topological space (X, τ) is product related to another Neutrosophic topological space (Y, σ) if for any Neutrosophic subset ν of X and ζ of Y, whenever $\lambda^C \not\geq \nu$ and $\mu^C \not\geq \zeta$ imply $\lambda^C \times 1 \vee 1 \times \mu^C \geq \nu \times \zeta$, where $\lambda \in \tau$and $\mu \in \sigma$, there exist $\lambda_1 \in \tau$and$\mu_1 \in \sigma$such that $\lambda_1{}^C \geq \nu$ or $\mu_1{}^C \geq \zeta$ and $\lambda_1{}^C \times 1 \vee 1 \times \mu_1{}^C = \lambda^C \times 1 \vee 1 \times \mu^C$.
Definition 1.21 [5] Let X and Y be two nonempty neutrosophic sets and f : X $\rightarrow$ Y be a function.
(i) If B = {(y, μ_B(y), σ_B(y), γ_B(y)): y $\in$ Y } is a Neutrosophic set in Y , then the pre image of B under f is denoted and defined by f^{-1}(B) = { (x, f^{-1}(μ_B)(x), f^{-1}(σ_B)(x), f^{-1}(γ_B)(x)) : x $\in$ X }.
(ii) If A = { < x, α_A(x), δ_A(x), λ_A(x)) : x $\in$ X } is a NS in X , then the image of A under f is denoted and defined by f (A) = {(y, f (α_A)(y), f (δ_A)(y), f _(λ_A)(y)): y $\in$ Y } where f_(λ_A) = C (f (C (A))).
In (i), (ii), since μ_B, σ_B, γ_B, α_A, δ_A, λ_A are neutrosophic sets, we explain that f^{-1}(μ_B)(x) = μ_B (f (x)),
and f (α_A)(y) =

$$= \begin{cases} \sup\left\{\alpha_A(x) : x \in f^{-1}(y)\right\} & \text{if } f^{-1}(y) \neq \varphi \\ 0 & \text{otherwise} \end{cases}$$

Definition 1.22 [5] Let $f_1 : X_1 \rightarrow Y_1$ and $f_2 : X_2 \rightarrow Y_2$. The neutrosophic product $f_1 \times f_2 : X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is defined by ($f_1 \times f_2$) (x_1, x_2) = ($f_1(x_1)$, $f_2(x_2)$) for all (x_1, x_2) $\in X_1 \times X_2$.

Definition 1.23 [5] Let A, A_i (i $\in$ J) be NSs in X and B, Bj (j $\in$ K) be NSs in Y and f : X $\rightarrow$ Y be a function. Then
(i) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$,
(ii) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$,
(iii) $f^{-1}(1_N) = 1_N$, $f^{-1}(0_N) = 0_N$,
(iv) $f^{-1}(C(B)) = C(f^{-1}(B))$,
(v) $f(\cup A_i) = \cup f(A_i)$.

Definition 1.24 [5]Let f : X $\rightarrow$ Y be a function. The neutrosophic graph g : X $\rightarrow$ X $\times$ Y of f is defined by g(x) = (x, f (x)) for all x $\in$ X.

Lemma 1.25 [5]Let $f_i : X_i \rightarrow Y_i$ (i = 1, 2) be functions and A, B be Neutrosophic subsets of Y_1, Y_2 respectively. Then $(f_1 \times f_2)^{-1} = f_1^{-1}(A) \times f_2^{-1}(B)$.

Lemma 1.26 [5] Let g : X$\rightarrow$ X x Y be the graph of a function f : X $\rightarrow$ Y . If A is the NS of X and B is the NS of Y , then $g^{-1}(A \times B)(x) = (A \cap f^{-1}(B))(x)$.

II. NEUTROSOPHIC FEEBLY OPEN SET

In this section, the concept of Neutrosophic feebly open set is introduced.

Definition 2.1 Let $\alpha, \beta, \gamma \in [0, 1]$ and $\alpha+\beta+\gamma \leq 1$. A Neutrosophic point with support $x_{(\alpha,\beta,\gamma)} \in X$ is a neutrosophic set of X is defined by $x_{(\alpha,\beta,\gamma)} = \begin{cases} (\alpha,\beta,\gamma), & y = x \\ (0,0,1), & y \neq x \end{cases}$

In this case, x is called the support of $x_{(\alpha,\beta,\gamma)}$ and α, β and γ are called the value, intermediate value and the non – value of $x_{(\alpha,\beta,\gamma)}$ respectively. A Neutrosophic point $x_{(\alpha,\beta,\gamma)}$ is said to belong to a neutrosophic set A = { $\langle$ x, μ_A(x), σ_A(x), γ_A(x) $\rangle$: x$\in$X } is denoted by two ways
(i) $x_{(\alpha,\beta,\gamma)} \in A$ if $\alpha \leq \mu_A(x)$, $\beta \leq \sigma_A(x)$ and $\gamma \geq \gamma_A(x)$.
(ii) $x_{(\alpha,\beta,\gamma)} \in A$ if $\alpha \leq \mu_A(x)$, $\beta \geq \sigma_A(x)$ and $\gamma \geq \gamma_A(x)$.

Clearly a Neutrosophic point can be represented by an ordered triple of Neutrosophic set as follows : $x_{(\alpha,\beta,\gamma)} = (x_\alpha, x_\beta, C(x_{c(\gamma)}))$. A class of all neutrosophic points in X is denoted as NP(X).

Definition 2.2 For any two Neutrosophic subsets A and B, we shall write AqB to mean that A is quasi-coincident (q- coincident, for short) with B if there exists x$\in$X such that A(x) + B(x) > 1. That is { $\langle$ x, $\mu_A(x)+\mu_B(x)$, $\sigma_A(x)+\sigma_B(x)$, $\gamma_A(x)+\gamma_B(x)$ $\rangle$: x$\in$X } > 1.

Definition 2.3 Let λ and μ be any two Neutrosophic subsets of a Neutrosophic topological space. Then A is q-neighbourhood with B (q-nbd, for short) if there exists a Neutrosophic open set O with AqO$\leq$B.

Proposition 2.4 Let (X, τ) be a Neutrosophic topological space. Then for a Neutrosophic set A of a Neutrosophic topological space X, NSClA is the union of all Neutrosophic points $x_{(\alpha, \beta, \gamma)}$ such that every Neutrosophic semi open set O with $x_{(\alpha, \beta, \gamma)}$ qO is Neutrosophic q–coincident with A.

Proof : Let $x_r \in$NSClA. Suppose there is a Neutrosophic semi open set O such that $x_{(\alpha, \beta, \gamma)}$ qO and $0 \not{q}$ A. That implies that $O^C \geq A$, where O^C is Neutrosophic semi closed. $O^C \geq$ NSCl A. By using Definition 2.6, $x_{(\alpha, \beta, \gamma)} \notin O^C$ implies that $x_{(\alpha, \beta, \gamma)} \notin$NSClA. This is a contradiction to our assumption. Therefore for every semi open O with $x_{(\alpha, \beta, \gamma)}$ qO is q-coincident with A.

Conversely, for every semi open O with $x_{(\alpha, \beta, \gamma)} q_C O$ is q-coincident with A. Suppose $x_r \notin$NSCl A. Then there is a neutrosophic semi closed set $G \geq A$ with $x_{(\alpha, \beta, \gamma)} \notin G$. G^C is neutrosophic semi open set with $x_{(\alpha, \beta, \gamma)} q(G^C)$ and $A \not{q} (G^C)$. That is $A(x) > (G^C)^C = G$. This is a contradiction to the assumption. Therefore $x_{(\alpha, \beta, \gamma)} \in$NSClA.

Proposition 2.5 Let (X, τ) be a Neutrosophic topological space. Let A and B be Neutrosophic subsets of a Neutrosophic topological space X. Then

If $A \wedge B = 0$ then $A \not{q} B$

$A \leq B \Leftrightarrow x_{(\alpha, \beta, \gamma)}$ qB for each $x_{(\alpha, \beta, \gamma)}$ qA

$A \not{q} B \Leftrightarrow A \leq B^C$

$x_{(\alpha, \beta, \gamma)}$ q $(V_{\propto 0 \in \Delta} A_{\propto}) \Leftrightarrow$ there is $\propto_0 \in \Delta$ such that $x_{(\alpha, \beta, \gamma)}$ $qA_{\propto 0}$

Proof: Let $(A \wedge B)(x) = 0$. Then min { A(x), B(x)} = 0. This implies that A(x) = 0 and $B(x) \leq 1$ (or) B(x) = 0 and $A(x) \leq 1$. $B^C \geq 1^C = A$ (or) $A^C \geq 1^C = B$. That implies $A \leq B^C$. That shows $A \not{q}$ B. This proves (i).

Let $A \leq \mu$. Then $x_{(\alpha, \beta, \gamma)}$ qA implies that $A^C(x) < (\alpha(x), \beta(x), \gamma(x))$ and $A \leq B$ implies that $A^C \geq B^C$ that gives $B^C < (\alpha, \beta, \gamma)$. Therefore $x_{(\alpha, \beta, \gamma)}$qB. Now x qA implies that $x_{(\alpha, \beta, \gamma)}$qB. So, $B^C < (\alpha, \beta, \gamma)$. Suppose A(x) >B (x). Then $A^C < (\alpha, \beta, \gamma)$ does not implies $B^C < (\alpha, \beta, \gamma)$. This is a contradiction. Therefore $A(x) \leq B(x)$. This proves (ii).

By using Definition 2.2,$A \not{q}$ B if and only if for each $x \in X$, $A(x) \leq B^C(x)$. That is $A \leq B^C$. This proves (iii).

Now $x_{(\alpha, \beta, \gamma)} q_C$ $(V_{\propto \in \Delta} A_{\propto})$if and only if $(V_{\propto \in \Delta} A_{\propto})^C$ (x) < (α, β, γ), for some $\alpha_0 \in \Delta$. $\wedge_{\propto \in \Delta} A_{\propto}^C < (\alpha, \beta, \gamma)$, for every $\propto_0 \in \Delta$. By using Definition 2.1, $x_{(\alpha, \beta, \gamma)}$ $qA_{\propto o}$.

Proposition 2.6 Let (X, τ) be a Neutrosophic topological space. Let A be a Neutrosophic subset of a Neutrosophic topological space X. Then

NIntNClNIntNClA = NIntNClA and NClNIntNClNIntA = NClNIntA

$(\text{NIntNClA})^C$ = NClNIntA^C and $(\text{NClNIntA})^C$ = NIntNClA^C

Proof : We know that NIntNClA≤NClA. By using Definition 2.6, NClNIntNClA≤ NCl(NClA)=NClA. This implies that NInt(NClNIntClA) ≤NInt(NClA). Since NIntNClAis Neutrosophic open and NIntNClA≤NClNIntNClA, NIntNClA = NInt(NIntClA) ≤NInt(NClNIntNClA). From the above NInt(NClNIntNClA) = NIntNClA. This proves (i).

(ii) follows from Proposition 1.17 [2].

Proposition 2.7 Let (X, τ) be a Neutrosophic topological space.

(a) Let x_r and A be a Neutrosophic point, a Neutrosophic subset, resp., of a Neutrosophic topological space X. Then $x_{(\alpha, \beta, \gamma)} \in A$, if and only if $x_{(\alpha, \beta, \gamma)}$ is not q -coincident with A^C.

(b) Let A and B be any two Neutrosophic open subsets of a Neutrosophic topological space X with $A \not{q}$ B. Then $A \not{q}$ NClB and NCl$A \not{q}$ B.

Proof : Let $x_{(\alpha, \beta, \gamma)} \in A$. Then $x_{(\alpha, \beta, \gamma)} \in A$ if and only if $A(x) \geq (\alpha(x), \beta(x), \gamma(x))$.$(A^C(x)^C) \geq (\alpha(x), \beta(x), \gamma(x))$. By using Definition 2.1, $x_{(\alpha, \beta, \gamma)} \not{q} A^C$. This proves (a).

Suppose $A \not{q}$ B. This implies that $A(x) \leq B^C(x)$ for all x. Let $x_{(\alpha, \beta, \gamma)} \in A(x)$ implies $A(x) \geq (\alpha, \beta, \gamma)$. Taking complement on both sides implies $A^C(x) < (\alpha, \beta, \gamma)^C$. Since $A^C$is Neutrosophic closed, $\text{NCl}A^C(x) < (\alpha, \beta, \gamma)^C$. That implies $(\text{NClB}(x)^C) \geq (\alpha, \beta, \gamma)$. This implies that $x_{(\alpha, \beta, \gamma)} \in (\text{NClB})^C$. That shows $A(x) \leq (\text{NClB}(x))^C$. From the above conclusion, $A \not{q}$ NClB. Let $x_{(\alpha, \beta, \gamma)} \in$NClA. Then by using Definition 2.1, $\text{NClA}(x) \geq (\alpha, \beta, \gamma)$. Since $A \not{q}_C$ B, we have $\text{NClA}(x) \leq B^C(x)$.This implies that $B^C(x) \geq (\alpha, \beta, \gamma)$. It follows that $\text{NClA} \leq B^C$, this shows NCl$A \not{q}$ B.

Proposition 2.8 Let (X, τ) be a Neutrosophic topological space. Let A be a Neutrosophic subset of a Neutrosophic topological space (X, τ). Then NIntNClA≤NSClA.

Proof : Let $x_{(\alpha, \beta, \gamma)} \in$NIntNClA. Then by using Definition 2.6, $(\alpha(x), \beta(x), \gamma(x)) \leq$NIntNClA(x). This can be written as $(\alpha(x), \beta(x), \gamma(x)) \leq$NClA(x). This implies that $x_{(\alpha, \beta, \gamma)} \in$NSClA. This shows that $x_{(\alpha, \beta, \gamma)} \in$NSClA.

Theorem 2.9 Let (X, τ) be a Neutrosophic topological space. If a Neutrosophic subset A is Neutrosophic open, then NIntNClA = NSClA.

Proof: By using Proposition 2.8, it suffices to show that NSClA≤NIntNClA. Let $x_{(\alpha, \beta, \gamma)} \notin$NIntNClA.

Then $x_{(\alpha,\beta,\gamma)}$ q $(NIntNClA)^C$. By using Proposition 2.4, $x_{(\alpha,\beta,\gamma)}$q $(NClNIntA^C)$. By using Proposition 2.5, NCl NInt A^C=NCl NInt NCl NInt A^C. This can be written as NCl NInt $A^C \leq$ NCl NInt (NCl NInt A^C). By using Definition 1. 19, NCl Nint A^C is Neutrosophic semi open. By using Proposition 2.6, A $\not{q}$ NClNIntA^C, that implies $x_{(\alpha,\beta,\gamma)} \notin$NSClA. That shows NSClA≤NIntNClA. Therefore NIntNClA = NSClA.

Definition 2.10 A Neutrosophic subset A of a Neutrosophic topological Space (X , τ) is Neutrosophic feebly open if there is a Neutrosophic open set U in X such that U≤ A≤NSClU .

Proposition 2.11 A Neutrosophic subset A is Neutrosophic feebly open iffA≤ NIntNClNInt A.

Proof: Necessity: If A is Neutrosophic feebly open, then by Definition 2.10, there is a Neutrosophic open set U such that U ≤A ≤NSCl U. Now U≤ A≤ NIntNCl U. Since U is Neutrosophic open, U = NIntU ≤ NInt A, it follows that NCl U ≤ NClNInt A. This implies that NIntNCl U≤ NIntNClNInt A. Thus A ≤ NIntNCl U≤ NIntNClNInt A.

Sufficiency: Assume that A≤ NInt NCl NInt A. Now NInt A ≤ A. That implies NInt A ≤ NInt NCl NInt A. Take U = NInt A. Then U is a Neutrosophic open set in X such that U ≤A ≤ NIntNClU. By Proposition 2.8, U ≤A ≤ NSClU. Therefore A is Neutrosophic feebly open.

Example 2.12 The following example is one of the Neutrosophic feebly-open set.
Let X = {x} and τ = { ⟨x, 0, 0, 1⟩ , ⟨x, 1, 1, 0⟩ , ⟨x, 0.5, 0.2, 0.6⟩, ⟨x, 0.7, 0.4, 0.8⟩, ⟨x, 0.7, 0.4, 0.6 ⟩, ⟨x, 0.5, 0.2, 0.8⟩}. Then (X, τ) is a Neutrosophic topological space.
Let A= ⟨x, 0.7, 0.6, 0.8⟩ .ThenNInt A = ⟨x, 0.7, 0.4, 0.8⟩ .The corresponding Neutrosophic closed sets τ^l ={⟨x,1,0,0⟩ , ⟨x,0,1,1⟩ , ⟨x,0.6,0.2,0.5⟩ , ⟨x ,0.8,0.4,0.7 ⟩ , ⟨x ,0.6,0.4,0.7 ⟩ , ⟨x ,0.8,0.2,0.5 ⟩ }.NowNClNInt A = ⟨x, 0.8,0.4,0.7 ⟩ ,NIntNClNInt A=⟨x, 0.7,0.4, 0.8⟩. ⟨x,0.7,0.6,0.8⟩≤ ⟨x,0.7,0.4,0.8⟩, A ≤NIntNClNInt A.Hence A = ⟨x ,0.7,0.6,0.8 ⟩ is Neutrosophic feebly-open set.

Proposition 2.13 Every Neutrosophic open set is Neutrosophic feebly- open set.

Proof: Let A be a Neutrosophic open set in X. Then A =NInt A.Since A≤NCl A,A≤NClNInt A. SinceNInt A ≤NIntNClNInt A, A≤NIntNClNInt A. Hence A is Neutrosophic feebly open set.

Example 2.14 The following example shows that the reverse implication is not true .That is ,A is Neutrosophic feebly open set but A is not a Neutrosophic open set.Let X = {x}and τ ={⟨x,0,0,1⟩ , ⟨x ,1,1,0 ⟩ , ⟨x ,0.5,0.2,0.6 ⟩ , ⟨ x ,0.7,0.4,0.8 ⟩ , ⟨x,0.7,0.4,0.6 ⟩, ⟨x,0.5,0.2,0.8 ⟩ }. Then (X, τ) is a Neutrosophic topological space. Let A = ⟨ x ,0.7,0.6,0.8 ⟩ is not a Neutrosophic open set .ThenNInt A = ⟨ x ,0.7,0.4,0.8 ⟩.The corresponding Neutrosophic closed sets τ^l ={⟨x,1,0,0 ⟩ ,⟨x,0,1,1⟩ , ⟨x ,0.6,0.2,0.5 ⟩ , ⟨x ,0.8,0.4,0.7 ⟩ ,⟨x ,0.6,0.4,0.7 ⟩ , ⟨x,0.8,0.2,0.5 ⟩ }. Now NClNIntA =⟨ x,0.8,0.2,0.7⟩ .NIntNClNInt A= ⟨x, 0.7,0.4, 0.8⟩. This implies thatA≤ NIntNClNIntA. Hence A is Neutrosophic feebly open set .

Proposition 2.15 If A and B be two Neutrosophic feebly open set then A⋃B is Neutrosophic feebly open set.

Proof: If A and B be two Neutrosophic feebly open set .Then by Proposition 2.11,
A ≤ NIntNClNInt A and B ≤ NIntNClNInt B. Now A⋃B ≤ (NIntNClNIntA)⋃(NIntNClNInt B). Since NInt A ⋃NInt B ⊆NInt (A⋃B), A⋃B ≤ NInt (NClNInt A⋃NClNIntB). Again by Proposition 1.18, A⋃B ≤ NInt **(**NCl(NIntA⋃NIntB)**)**. By using Proposition 1.18, A⋃B ≤ NInt NCl NInt (A⋃B). Hence A⋃B is Neutrosophic feebly open set.

Proposition 2.16 Arbitrary union of Neutrosophic feebly open sets is a Neutrosophic feebly open set.

Proof: Let $\{A_\alpha\}$ be a collection of Neutrosophic feebly open sets of a Neutrosophic topological space X. Then by Definition 2.10, There exists a Neutrosophic open set V_α such that $V_\alpha \leq A_\alpha \leq$ NSCl V_α for each α. Now, ⋃ $V_\alpha \leq$⋃$A_\alpha \leq$ ⋃NSClV_α . By Proposition 6.5 in [6] , ⋃ $V_\alpha \leq$⋃$A_\alpha \leq$ NSCl(⋃ V_α). Hence ⋃$A_\alpha$is a Neutrosophic feebly open set.

Example 2.17 Intersection of any two Neutrosophic feebly open sets need not be a Neutrosophic feebly open set as shown by the following example.
Let X = {x} and τ = { ⟨x, 0,0,1⟩, ⟨x, 1,1,0⟩ , ⟨x, 0.5,0.5,0.4⟩ , ⟨x, 0.4,0.6,0.8⟩ , ⟨x, 0.5,0.6,0.4⟩ , ⟨x, 0.4,0.5,0.8⟩ } .Then (X, τ) is a Neutrosophic topological space. Let A =⟨x, 0.5,0.5,0.4⟩. Then NInt A =⟨x, 0.5,0.5,0.4⟩. The corresponding Neutrosophic closed sets τ^l = { ⟨x, 1, 0, 0 ⟩ , ⟨x, 0, 1, 1 ⟩ , ⟨x, 0.4, 0.5, 0.5 ⟩ , ⟨x, 0.8, 0.6, 0.4 ⟩, ⟨x, 0.4, 0.6, 0.5 ⟩, ⟨x, 0.8, 0.5, 0.4 ⟩ }. Now NClNIntA = ⟨x, 0.8, 0.5, 0.4 ⟩. NIntNClNInt A= ⟨x, 0.5,0.6, 0.4⟩ This implies that A ≤ NIntNClNIntA. Hence A = ⟨x, 0.5,0.5,0.4⟩ is Neutrosophic feebly open set. Let B = ⟨x, 0.6,0.5,0.6⟩ .NIntB= ⟨x, 0.4,0.6,0.8⟩ . NClNIntB = ⟨x, 0.8, 0.5, 0.4 ⟩ .NIntNClNInt B= ⟨x, 0.5,0.6, 0.4⟩ .B≤ NIntNClNInt B. Henc B = ⟨x, 0.6,0.5,0.6⟩ is Neutrosophic feeblyopen set.A⋂B = ⟨x, 0.5,0.5,0.6⟩ .NInt(A⋂B)=

⟨x, 0.4,0.5,0.8⟩ .NClNInt(A∩B)= ⟨x, 0.4,0.5,0.5⟩.A∩B⊄NClNInt(A∩B). Hence A∩B =⟨x, 0.5,0.5,0.6⟩ is not a Neutrosophic feebly open set.

Example 2.18 The following example shows that Intersection of a Neutrosophic feebly open set with a Neutrosophic open set may fail to be a Neutrosophic feebly open set.
Let X = {x} and τ ={ ⟨x ,0,0,1 ⟩ , ⟨x ,1,1,0 ⟩ , ⟨x ,0.2,0.4,0.3 ⟩ , ⟨x ,0.7,0.5,0.6⟩ , ⟨x ,0.7,0.5,0.3 ⟩ ,⟨x,0.2,0.4,0.6 ⟩ } . Then (X, τ) is a Neutrosophic topological space. Let A = ⟨x ,0.8,0.6,0.5⟩. Then NIntA = ⟨x ,0.7,0.5,0.6 ⟩ . The corresponding Neutrosophic closed sets τ^I ={⟨ x,1,0,0 ⟩ ,⟨x,0,1,1 ⟩, ⟨ x,0.3,0.4,0.2⟩ , ⟨ x,0.6,0.5,0.7⟩ , ⟨ x,0.3,0.5,0.7 ⟩ , ⟨x,0.6,0.4,0.2⟩}. Now NClNIntA = ⟨x,1,0, 0 ⟩. This implies that A ⊆NClNIntA .Hence A = ⟨x,0.8,0.6,0.5⟩is aNeutrosophic feebly open set .Let B = ⟨x,0.7,0.5,0.6⟩ be aNeutrosophic open set.A∩B= ⟨ x,0.7,0.5,0.6 ⟩. ThenNInt(A∩B) = ⟨x,0.7,0.5,0.6⟩.NClNInt(A∩B)= ⟨x,0.1,0,0 ⟩. This implies that A∩B⊆NClNInt(A∩B). HenceA∩B= ⟨x,0.7,0.5,0.6⟩is not a Neutrosophic feebly open set.

Proposition 2.19 The Neutrosophic closure of a Neutrosophic open set is a Neutrosophic feebly open set.

Proof: Let A be a Neutrosophic open set in X. Then A =NIntA .NCl A =NClNInt A. Since A ≤ NCl A, NInt A ≤ NIntNCl A. Hence A ≤NIntNClNInt A . Hence A is Neutrosophic feebly open set.

Proposition 2.20 Let A be Neutrosophic feebly open in the Neutrosophic topological space (X, τ) and suppose A≤B≤NSCl A , then B is Neutrosophic feebly open.
Proof: Let A be Neutrosophic feebly open set in the Neutrosophic topological space (X,τ). Then there exist a Neutrosophic open set U such that U≤A≤NSClU. Since U≤B, NSClA≤NSCl B and thus B≤NSCl U. Hence U ≤B≤NSCl U. Hence B is Neutrosophic feebly open.

Theorem 2.21 Let (X, τ) and (Y, σ) be any two Neutrosophic topological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of a Neutrosophic feebly open set A_1 of X and a Neutrosophic feebly open set A_2 of Y is a Neutrosophic feebly open set of the Neutrosophic product space X × Y.

Proof: Let A_1 be a Neutrosophic feebly open subset of X and A_2 be a Neutrosophic feebly open subset of Y. Then by using Proposition 2.11, we have A_1 ≤NIntNClNIntA_1 and A_2 ≤NIntNClNIntA_2. By using Theorem 2.17 in [6], implies that $A_1 \times A_2$ ≤NIntNClNInt($A_1 \times A_2$). By using Proposition 2.11, $A_1 \times A_2$ is a Neutrosophic feebly open set of the Neutrosophic product space X × Y.

III. NEUTROSOPHIC FEEBLY CLOSED SET

In this section, the concept of Neutrosophic feebly closed set is introduced.

Definition 3.1 A Neutrosophic subset A of a Neutrosophic topological Space (X , τ) is Neutrosophic feebly closed if there is a Neutrosophic closed set U in X such that NSInt U≤A≤U.

Proposition 3.2 A Neutrosophic subset A is Neutrosophic feebly closed ifNClNIntNCl A≤ A .

Proof: Necessity: If A is Neutrosophic feebly closed, then by Definition 3.1, there is a Neutrosophic closed set U such that NSInt U ≤ A ≤ U. Now NClNInt U ≤ A ≤ U. NCl A ≤ U= NCl U. NClNIntNCl A≤NClNInt U≤ A. Hence NClNIntNCl A ≤A.

Sufficiency: Assume that NClNIntNCl A ≤ A. Take U = NCl A. Then U is a Neutrosophic closed set in X such that NSInt U ≤A ≤U. Therefore A is Neutrosophic feebly closed set.
Proposition 3.3 Let A be a Neutrosophic feebly closed set if A^C is Neutrosophic feebly open set.
Proof: A is Neutrosophic feebly closed set, NClNIntNCl A ≤A. Taking complement on both sides, $(\text{NClNIntNCl } A)^C \geq A^C$. A^C≤NIntNClNInt A^C. Hence A^C is Neutrosophic feebly open set. Conversely, A^C is Neutrosophic feebly open set, A^C≤NIntNClNInt A^C. Taking complement on both sides,$(A^C)^C$≥(NIntNClNInt $A^C)^C$. A ≥ NCl NIntNClA . Therefore sNClNIntNCl A≤A .Hence A is Neutrosophic feebly closed set.
Example3.4 The following example is one of the Neutrosophic feebly closed set.
Let X = {x} andτ ={ ⟨ x,0,0,1 ⟩ , ⟨ x,1,1, 0⟩ , ⟨x,0.5,0.5,0.4⟩,⟨x,0.4,0.6,0.8⟩,⟨x,0.5,0.6,0.4⟩,⟨x,0.4,0.5,0.8⟩}. Then (X, τ) is a Neutrosophic topological space. Let A =⟨x,0.6,0.3,0.5⟩ .The corresponding Neutrosophic closed sets τ^I ={ ⟨ X,1,0,0 ⟩ ,⟨x,0,1,1⟩,⟨x,0.4,0.5,0.5⟩,⟨x, 0.8, 0.6, 0.4⟩,⟨x, 0.4, 0.6, 0.5 ⟩ , ⟨ x,0.8,0.5,0.4 ⟩ }. Then NCl A = ⟨X,0.8,0.5,0.4⟩.NIntNCl A= ⟨X,0.4,0.5,0.8⟩. This implies that NIntNClA ⊆A. Hence A = ⟨x,0.6,0.3,0.5⟩ is Neutrosophic feebly closed set.

Proposition 3.5 Every Neutrosophic closed set is a Neutrosophic feebly closed set.

Proof: Let A be a Neutrosophic closed set in X. Then A =NClA. Since NInt A≤A, NIntNClA≤A. That implies NClNIntNClA≤NCl A. Thus

NClNIntNCl A≤A. Hence A is Neutrosophic feebly closed set.

Example 3.6 The following example shows that the reverse implication is not true. That is, A is Neutrosophic feebly closed set, but A is not a Neutrosophic closed set. Let X = {x} and τ ={ ⟨x,0,0,1⟩,⟨x,1,1,0 ⟩, ⟨x,0.6,0.6,0.5 ⟩, ⟨x,0.5,0.7,0.9 ⟩, ⟨x 0.2, 0.3, 0.5 ⟩, ⟨ x,0.6,0.7,0.5 ⟩, ⟨ x,0.5,0.6,0.9⟩ }. Then (X, τ) is a Neutrosophic topological space. The corresponding Neutrosophic closed setsτ^l ={⟨ x,1,0,0 ⟩ ,⟨x,0,1,1 ⟩ , ⟨x,0.5,0.6,0.6 ⟩ , ⟨x,0.9,0.7,0.5 ⟩ , ⟨ x,0.5,0.7,0.6 ⟩ , ⟨ x,0.9,0.6,0.5 ⟩ }. Let A = ⟨x,0.7,0.5,0.6⟩ is not a Neutrosophic closed set. Then NCl A = ⟨ x,0.9,0.6,0.5⟩⟩. Now NIntNClA =⟨ x,0.5,0.6,0.9 ⟩ .This implies that NIntNClA⊆A. Hence A is Neutrosophic feebly closed set .

Proposition 3.7 If A and B be two Neutrosophic feebly closed set then A∩B is Neutrosophic feebly closed set.

Proof: If A and B be two Neutrosophic feebly closed set. Then by Proposition3.2, NCl NInt NCl A ≤ A and NCl NInt NCl B ≤ B. (NClNIntNCl A)∩ (NClNIntNCl B)≤A∩B. By Proposition 1.18, NCl(NInt(NClA∩NClB))≤ A∩B. Again by Proposition 1.18, NClNInt(NCl(A∩B))≤A∩B. That implies NClNIntNCl(A∩B) ≤ A∩B. Hence A∩B is Neutrosophic feebly closed set.

Proposition 3.8 Finite intersection of a Neutrosophic feebly closed sets is a Neutrosophic feebly closed set.

Proof: Let $\{A_i\}$ be a collection of Neutrosophic feebly closed sets of a Neutrosophic topological space X. Then by Definition 3.1, there exists a Neutrosophic closed set V_i such that NSInt $V_i \leq A_i \leq V_i$ for each i. Now, ∩NSInt $V_i \leq \cap A_i \leq \cap V_i$. By Theorem 5.3 in [6], NSInt $(\cap V_i) \leq \cap A_i \leq \cap V_i$. Hence $\cap A_i$ is a Neutrosophic feebly closed set.

Example 3.9 Union of any two Neutrosophic feebly closed sets need not be a Neutrosophic feebly closed set as shown by the following example.
Let X = {x} and τ ={ ⟨x ,0,0,1 ⟩ , ⟨x ,1,1,0 ⟩ , ⟨x ,0.2,0.5,0.7⟩ , ⟨x ,0.8,0.4,0.5⟩ , ⟨x ,0.2,0.4,0.7 ⟩ , ⟨x,0.8,0.5,0.5⟩⟩ } . Then (X, τ) is a Neutrosophic topological space. The corresponding Neutrosophic closed setsτ^l ={⟨x,1,0,0 ⟩ ,⟨x,0,1,1 ⟩ , ⟨x,0.7,0.5,0.2 ⟩ ,⟨⟨x,0.5,0.4,0.8 ⟩ , ⟨x,0.7,0.4,0.2 ⟩ , ⟨x,0.5,0.5,0.8 ⟩}. Let A = ⟨ x ,0.4,0.3,0.9 ⟩ . Then NCl A = ⟨x,0.5,0.4,0.8⟩⟩ . Now NIntNClA =⟨x,0,0,1 ⟩.This implies thatNIntNClA⊆A. Hence A is Neutrosophic feebly closed set. Let B = ⟨x,0.4,0.5,0.6⟩.Then NCl B = ⟨x ,0.7,0.5,0.2 ⟩ .Now NIntNClB = ⟨x ,0.2,0.5,0.7 ⟩ .This implies that NIntNClB⊆B. Hence B is Neutrosophic feebly closed set. AUB =⟨x,0.5,0.5,0.6⟩⟩.Then NInt(AUB)= ⟨x,0.4,0.5,0.8⟩. NowNClNInt(AUB)= ⟨x ,0.4,0.5,0.5 ⟩ .This implies that NClNInt(AUB)⊄AUB. Hence AUB = ⟨x,0.5,0.5,0.6⟩ is not a Neutrosophic feebly closed set.

Example 3.10 Union of a Neutrosophic feebly closed set with a Neutrosophic open set may fail to be a Neutrosophic feebly closed set as shown by the following example.
Let X = {x} and τ ={ ⟨x ,0,0,1 ⟩ , ⟨x ,1,1,0 ⟩ , ⟨x,0.2,0.4,0.3 ⟩ , ⟨x,0.7,0.5,0.6 ⟩ , ⟨x,0.7,0.5,0.3 ⟩ , ⟨x,0.2,0.4,0.6 ⟩ }. Then (X, τ) is a Neutrosophic topological space. Let A = ⟨x,0.8,0.6,0.5⟩ .NInt A = ⟨x ,0.7,0.5,0.6 ⟩ .The corresponding Neutrosophic closed setsτ^l ={⟨x,1,0,0 ⟩ ,⟨x,0,1,1⟩⟩ , ⟨x,0.3,0.4,0.2 ⟩ , ⟨x,0.6,0.5,0.7 ⟩ , ⟨x,0.3,0.5,0.7 ⟩ , ⟨x,0.6,0.4,0.2 ⟩ }. NClNIntA =⟨x,.1,0,0 ⟩ . A ⊆NClNIntA .Henc A = ⟨x,0.8,0.6,0.5⟩is a Neutrosophic feebly closed set .Let B = ⟨x ,0.7,0.5,0.6⟩ be aNeutrosophic closed set.AUB = ⟨x ,0.7,0.5,0.6 ⟩ .NInt(AUB)= ⟨x,0.7,0.5,0.6⟩.NClNInt(AUB)= ⟨x,.1,0,0 ⟩NClNInt(AUB)⊄ AUB. Hence AUB = ⟨x,0.7,0.5,0.6 ⟩ is not a Neutrosophic feebly closed set .

Proposition 3.11 The neutrosophic interior of a neutrosophic closed set is a neutrosophic feebly closed set.

Proof: Let A be a Neutrosophic closed set in X. Then A =NClA .NInt A =NIntNClA . By Proposition 2.15, NIntA ≤ A, NInt NCl A≤A, NClNIntNCl A≤NCl A, NClNIntNCl A≤A. Hence A is Neutrosophic feebly closed set.

IV. NEUTROSOPHIC FEEBLY CONTINUOUS FUNCTIONS IN NEUTROSOPHIC TOPOLOGICAL SPACES

We shall now consider some possible definitions for neutrosophic feebly continuous functions.

Definition 4.1 [15] Let (X,τ) and (Y,σ) be two NTSs. Then a map f : (X,τ) → (Y,σ) is called neutrosophic continuous (in short N-continuous) function if the inverse image of every neutrosophic open set in (Y,σ) is neutrosophic open set in (X,τ).

Definition 4.2 Let (X,τ) and (Y,σ) be two neutrosophic topological space. Then a map f : (X,τ) → (Y,σ) is called neutrosophic feebly continuous (in short NF-continuous) function if the inverse image of every neutrosophic open set in (Y,σ) is neutrosophic feebly open set in (X,τ).

Theorem 4.3 Every N-continuous function is NF-continuous function.

Proof : Let f : $(X,\tau) \to (Y,\sigma)$ be N-continuous function. Let V be a neutrosophic open set in (Y,σ). Then $f^{-1}(V)$ is neutrosophic open set in (X,τ). Since every neutrosophic open set is neutrosophic feebly open set, $f^{-1}(V)$ is neutrosophic feebly open set in (X,τ). Hence f is neutrosophic feebly -continuous function.

Remark 4.4 The converse of the above theorem is need not be true as shown by following example.

Example 4.5 Let X = Y = { a , b, c }. Define the neutrosophic sets as follows :

$A = \langle (0.4, 0.5, 0.2), (0.3, 0.2, 0.1), (0.9, 0.6, 0.8) \rangle$

$B = \langle (0.2, 0.4, 0.5), (0.1, 0.1, 0.2), (0.6, 0.5, 0.8) \rangle$

$C = \langle (0.5, 0.4, 0.2), (0.2, 0.3, 0.1), (0.6, 0.9, 0.8) \rangle$ and

$D = \langle (0.4, 0.2, 0.5), (0.1, 0.1, 0.2), (0.5, 0.6, 0.8) \rangle$. Now $T = \{ 0_N, A, B, 1_N \}$ and $S = \{ 0_N, C, D, 1_N \}$ are neutrosophic topologies on X. Thus (X,τ) and (Y,σ) are NTSs. Also we define $f : (X,\tau) \to (Y,\sigma)$ as follows : f (a) = b, f (b) = a, f (c) = c. Clearly f is NF-continuous function. But f is not N-continuous function. Since $E = \langle (0.5, 0.6, 0.1), (0.4, 0.3, 0.1), (0.9, 0.8, 0.5) \rangle$ is a neutrosophic open in (Y,σ), $f^{-1}(E)$ is not neutrosophic open set in (X,τ).

Definition 4.6 Let (X, τ) be NTS and $A = \langle x, B_A(x), \sigma_A(x), \gamma_A(x) \rangle$ be a NF in X. Then the neutrosophic feebly-closure and neutrosophic feebly-interior of A are defined by

$NFCl (A) = \cap \{ K : K$ is a NFC set in X and $A \subseteq K \}$

$NFInt (A) = \cup \{G : G$ is a NFO set in X and $G \subseteq A \}$.

REFERENCES

[1] K. Atanassov, "Intuitionistic fuzzy sets", in V.Sgurev, ed.,Vii ITKRS Session, Sofia(June 1983 central Sci. and Techn. Library, Bulg.Academy of Sciences (1984)).

[2] K. Atanassov, "Intuitionistic fuzzy sets", Fuzzy Sets and Systems 20(1986)87-96.

[3] K. Atanassov, "Review and new result on intuitionistic fuzzy sets", preprint IM-MFAIS-1-88, Sofia, 1988.

[4] K.K.Azad, "On semi continuity, fuzzy Almost Continuity and fuzzy Weekly Continuity", Journal Of Mathematical Analysis And Applications 82, 14-32 (1981).

[5] Byung Sik In, "On fuzzy FC compactness" , comm. Korean Math. soc. 13 (1998) , No. 1, pp _137_150.

[6] C.L.Chang , "Neutrosophic Topological Spaces", Journal of Mathematical Analysis and Applications, 24, 182-190(1968).

[7] P.Iswarya, k. Bageerathi, "On Neutrosophic semi open sets in Neutrosophic Topological Spaces", International Journal of Mathematics Trends and Technology – Volume 37 Number 3 – September 2016.

[8] Dogan Coker, "An introduction to intuitionistic fuzzy topological spaces", Fuzzy Sets and Systems, 88(1997)81-89.

[9] Florentin Smarandache , "Neutrosophy and Neutrosophic Logic" , First International Conference on Neutrosophy , Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA(2002) , smarand@unm.edu

[10] FloretinSmarandache, "A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press, Rehoboth, NM, 1999.

[11] FloretinSmaradache, "Neutrosophic Set: A Generalization of Intuitionistic Fuzzy set", Journal of Defense Resourses Management. 1(2010),107-116.

[12] F.G.Lupianez, "Interval Neutrosophic Sets and Topology", Proceedings of 13th WSEAS , International conference on Applied Mathematics(MATH'08) Kybernetes, 38(2009), 621-624.

[13] N. Levine, "Semi-open sets and semi-continuity in topological spaces", Amer. Math. Monthly 70 (1963), 36-41.

[14] Reza Saadati, Jin HanPark, "On the intuitionistic fuzzy topological space", Chaos, Solitons and Fractals 27(2006)331-344 .

[15] A.A. Salama and S.A. Alblowi, "Generalized Neutrosophic Set and Generalized Neutrousophic Topological Spaces ", Journal computer Sci. Engineering, Vol. (2) No. (7) (2012).

[16] A.A.Salama and S.A.Alblowi, "Neutrosophic set and neutrosophic topological space", ISOR J. mathematics,Vol.(3), Issue(4),(2012). pp-31-35.

[17] A.A. Salama, F. Smarandache and K. Valeri, "Neutrosophic closed set and Neutrosophic continuous functions, Neutrosophic sets and systems", 4(2014), 4-8.

[18] L.A. Zadeh, "Fuzzy Sets", Inform and Control 8(1965)338-353.