Solving a Multi Objective Linear Programming Problem Using Pentagonal Fuzzy Numbers

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Abstract *— The level sum method is based on the multi objective linear programming problem(MOLPP) and here pentagon fuzzy numbers is used for computing an optimal solution to the fuzzy linear programming problem(FLPP) without ranking functions. This is illustrated with a numerical example.*

Keywords — *Multi objective linear programming problem(MOLPP), pentagonal fuzzy numbers, Level sum method.*

I. INTRODUCTION

Linear programming (LP) is one of the most widely used optimization techniques. It deals with the optimization of a linear function while satisfying a set of linear equality and/or inequality constraints or restriction. In the literature, a variety of algorithms for solving FLP have been studied based on fuzzy ranking function and classical linear programming. Tanaka [6], Zimmerman[8],Buckley and Feuring[1], Thakrel[8] and Zhang[11] solved FLP problems using multi objective linear programming problem (MOLPP) technique. Pandian[5] has proposed a new approach, namely sum of objectives (SO) method for finding a properly efficient solution to multi objective linear programming problems.

In this paper, we use level sum method for finding optimal solution to the pentagon fuzzy numbers. we discuss this method with the pentagonal. the advantage of this method is the fuzzy ranking functions are not used to obtain and the results are satisfied by the constraints.

II. PRELIMINARIES

A. *Definition: [3]*

A fuzzy set is characterized by a membership function mapping the elements of domain, space or universe of discourse x to the unit interval [0, 1]. A fuzzy set Ã is set of ordered pairs $\{x, \mu_{\tilde{A}}(x) / x \in R\}$ where $\mu_{\tilde{A}}(x): R \to [0,1]$ is upper semi- continuous function $\mu_{\tilde{A}}(x)$ is called membership function of the fuzzy set.

B. *Definition: [3]*

A fuzzy number f in the real line R is a fuzzy set f : R→[0,1]that satisfies the following properties.

(i) f is piecewise continuous.
(ii) There exists an x∈R such that f (x) =1.
(iii) f is convex,(i.e).,if $x_1,x_2 \in R$, and a∈[0,1] then $f(\lambda x_1+(1- \lambda)x_2) \geq f(x_1) \wedge f(x_2)$

C. *Definition:[3]*

A pentagon fuzzy number $\tilde{A}_p = (a_1, a_2, a_3, a_4, a_5)$ where a_1, a_2, a_3, a_4, a_5 are real numbers and its membership is given below.

$$\mu_{\tilde{A}_p}(x) = \begin{cases} 0 & x \prec a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{y-a_2}{a_3-a_2}\right), & a_2 \leq x \leq a_3 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{a_4-y}{a_4-a_3}\right), & a_3 \leq x \leq a_4 \\ \frac{1}{2}\left(\frac{a_5-x}{a_5-a_4}\right), & a_4 \leq x \leq a_5 \end{cases}$$

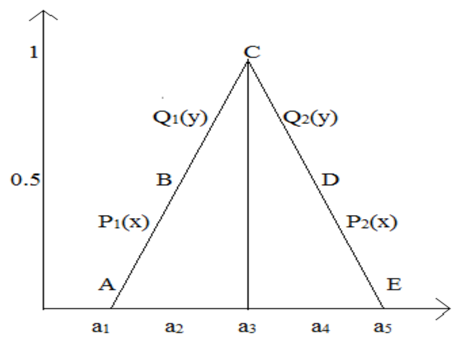

Fig 1:Graphical representation of a normal pentagonal fuzzy number for x ∈[0,1]

D. ***Definition:[3]***

Following are the three operations that can be performed on pentagon fuzzy numbers.

Let $\tilde{A}_p = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}_p = (b_1, b_2, b_3, b_4, b_5)$ be the two pentagon fuzzy numbers then

- Addition:
  $\tilde{A}_p \oplus \tilde{B}_p = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5).$
- Subtraction:
  $\tilde{A}_p (-) \tilde{B}_p = (a_1 - b_5, a_2 - b_4, a_3 - b_3, a_4 - b_2, a_5 - b_1).$
- Multiplication:
  $\tilde{A}_p (*) \tilde{B}_p = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5)$
- Division:
  $\tilde{A}_p \div \tilde{B}_p = (a_1 \div b_5, a_2 \div b_4, a_3 \div b_3, a_4 \div b_2, a_5 \div b_1)$

E. Definition:[3]

Let $\tilde{A}_p = (a_1, a_2, a_3, a_4, a_5)$ *and* $\tilde{B}_p = (b_1, b_2, b_3, b_4, b_5)$ be two pentagon fuzzy numbers if $\tilde{A}_p$ is identically equal to $\tilde{B}_p$ only if $a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4, a_5 = b_5.$

F. ***Definition***: **[5]**

Let $\tilde{A}_p = (a_1, a_2, a_3, a_4, a_5)$ *and* $\tilde{B}_p = (b_1, b_2, b_3, b_4, b_5)$ be in F(R), then $\tilde{A} \succ \tilde{B}$ iff $a_i \geq b_i, i = 1,2,3,4,5$ and $a_r > b_r$, for some $r \in \{1,2,3,4,5\}$.

G. ***Definition: [6]***

A feasible point x^0 is said to be efficient solution if there exists no other feasible point x in P such that $f_i(x) \leq f_i(x^0), i = 1,2,..k$ *and* $f_r(x) \prec f_r(x^0)$ for some $r \in \{1, 2, \ldots, k\}$.

III. THEOREM

Let $X^0 = \{u_j^0, v_j^0, w_j^0, x_j^0, y_j^0; j = 1,2,..m\}$ be an efficient solution. then, prove that $\tilde{X}^0 = \{(u_j^0, v_j^0, w_j^0, x_j^0, y_j^0); j = 1,2,..m\}$ is an optimal solution.

Proof:

Let $X^0 = \{u_j^0, v_j^0, w_j^0, x_j^0, y_j^0; j = 1,2,..m\}$ is an efficient solution , $\tilde{X}^0 = \{(u_j^0, v_j^0, w_j^0, x_j^0, y_j^0); j = 1,2,..m\}$ is a feasible solution

Assume that $\tilde{X}^0 = \{(u_j^0, v_j^0, w_j^0, x_j^0, y_j^0); j = 1,2,..m\}$ is not an optimal solution Then, there exists a feasible solution $\tilde{X} = \{(u_j^0, v_j^0, w_j^0, x_j^0, y_j^0); j = 1,2,..m\}$ such that $z(\tilde{X}) \succ z(\tilde{X}^0)$, that is

$$z_i(u, v, w, x, y) \geq z_i(u^0, v^0, w^0, x^0, y^0),$$
$$i = 1,2,3,4,5 \quad and$$
$$z_r(u, v, w, x, y) \succ z_r(u^0, v^0, w^0, x^0, y^0),$$
$$for\ some\ r \varepsilon \{1,2,3,4,5\},$$

$u^0 = \{u_j^0; j = 1,2..m\}, \quad v^0 = \{v_j^0; j = 1,2..m\},$
$w^0 = \{w_j^0; j = 1,2..m\}, \quad x^0 = \{x_j^0; j = 1,2..m\},$
$y^0 = \{y_j^0; j = 1,2..m\}, \quad u = \{u_j; j = 1,2,..m\},$
$v = \{v_j; j = 1,2,..m\}, \quad w = \{w_j; j = 1,2,..m\},$
$x = \{x_j; j = 1,2,..m\}, \quad y = \{y_j; j = 1,2,..m\},$

this means that $X^0 = \{u_j^0, v_j^0, w_j^0, x_j^0, y_j^0; j = 1,2,..m\}$ is not an efficient solution. which is the contradiction. Hence the theorem is proved.

3.1 Algorithm:

We propose level sum method for finding an optimal fuzzy solution to the FLP problem.

The method proceeds as follows:

Step 1: construct a crisp MOLP problem from the given FLP problem.

Step 2: Find an efficient solution to the MOLP problem obtained in step1 using the SO method.

Step 3: The efficient solution obtained from step2 to the MOLP problem yields an optimal fuzzy solution to the FLP problem by the above theorem.

3.2 Mathematical Formulation:

Consider the following fully FLP with m fuzzy inequality/equality constraints and n fuzzy variables may be formulated as follows

Maximize $\tilde{Z} = \tilde{c}\tilde{x}$

Subject to,

$$\tilde{A} \otimes \tilde{x} \{\leq, \equiv, \geq\} \tilde{b}; \quad \tilde{x} \geq \tilde{0}$$

Where

$\tilde{a}_{ij}, \tilde{c}_j, \tilde{x}_j, \tilde{b}_i\ \varepsilon\ F(R)$ *for all* $1 \leq j \leq n$ *and* $1 \leq i \leq m$, $\tilde{c}^T = (\tilde{c}_j)_{1xn}$, $\tilde{A} = (\tilde{a}_{ij})_{mxn}$,

$\tilde{x} = (\tilde{X}_j)_{nx1}$ $\tilde{b} = (\tilde{b}_j)_{mx1}$

Let the parameters $\tilde{z}, \tilde{a}_{ij}, \tilde{c}_j, \tilde{x}_j$ *and* $\tilde{b}_i$ be the pentagonal fuzzy numbers

$\tilde{z} = (z_1, z_2, z_3, z_4, z_5),$
$\tilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}, e_{ij}), \tilde{c}_j = (p_j, q_j, r_j, s_j, t_j),$
$\tilde{x}_j = (u_j, v_j, w_j, x_j, y_j), \tilde{b}_i = (b_i, e_i, f_i, g_i, h_i)$

respectively

Step1:

construct a crisp MOLP problem from the given FLP problem can be written as follows:
Maximize

$$(z_1,z_2,z_3,z_4,z_5)=\sum_{j=1}^{n}\begin{matrix}(p_j,q_j,r_j,s_j,t_j)\otimes\\(u_j,v_j,w_j,x_j,y_j)\end{matrix}$$

Subject to ,

$$\sum_{j=1}^{n}(a_{ij},b_{ij},c_{ij},d_{ij},e_{ij})\otimes(u_j,v_j,w_j,x_j,y_j)\{\le,\approx,\ge\}$$

$$(b_i,e_i,f_i,g_i,h_i)$$

for all i =1,2,… m

$$(u_j,v_j,w_j,x_j,y_j)\ge 0 \quad j=1,2\ \ldots.m$$

$$x_j\ge 0$$

Step:2

Now using the arithmetic operation and partial ordering we write the given FLPP as a MOLP problem which is given below

$$Maximize \quad z_1=\sum_{j=1}^{n} most\ \ lowest\ \ value\ \ of\ ((p_j,q_j,r_j,s_j,t_j)\otimes(u_j,v_j,w_j,x_j,y_j))$$

$$Maximize \quad z_2=\sum_{j=1}^{n} lowest\ \ value\ \ of\ ((p_j,q_j,r_j,s_j,t_j)\otimes(u_j,v_j,w_j,x_j,y_j))$$

$$Maximize \quad z_3=\sum_{j=1}^{n} middle\ \ value\ \ of\ ((p_j,q_j,r_j,s_j,t_j)\otimes(u_j,v_j,w_j,x_j,y_j))$$

$$Maximize \quad z_4=\sum_{j=1}^{n} upper\ \ value\ \ of\ ((p_j,q_j,r_j,s_j,t_j)\otimes(u_j,v_j,w_j,x_j,y_j))$$

$$Maximize \quad z_5=\sum_{j=1}^{n} most\ \ upper\ \ value\ \ of\ ((p_j,q_j,r_j,s_j,t_j)\otimes(u_j,v_j,w_j,x_j,y_j))$$

Subject to

$$\sum_{j=1}^{n} most\ \ lowest\ \ value\ \ of\ \begin{pmatrix}(a_{ij},b_{ij},c_{ij},d_{ij},e_{ij})\otimes\\(u_j,v_j,w_j,x_j,y_j)\end{pmatrix}\{\le,\approx,\ge\}\ b_i$$

$$for\ all\ i=1,2,..m;$$

$$\sum_{j=1}^{n} lowest\ \ value\ \ of\ \begin{pmatrix}(a_{ij},b_{ij},c_{ij},d_{ij},e_{ij})\otimes\\(u_j,v_j,w_j,x_j,y_j)\end{pmatrix}\{\le,\approx,\ge\}\ e_i$$

$$for\ all\ i=1,2,..m;$$

$$\sum_{j=1}^{n} middle\ \ value\ \ of\ \begin{pmatrix}(a_{ij},b_{ij},c_{ij},d_{ij},e_{ij})\otimes\\(u_j,v_j,w_j,x_j,y_j)\end{pmatrix}\{\le,\approx,\ge\}\ f_i$$

$$for\ all\ i=1,2,..m;$$

$$\sum_{j=1}^{n} upper\ \ value\ \ of\ \begin{pmatrix}(a_{ij},b_{ij},c_{ij},d_{ij},e_{ij})\otimes\\(u_j,v_j,w_j,x_j,y_j)\end{pmatrix}\{\le,\approx,\ge\}\ g_i$$

$$for\ all\ i=1,2,..m;$$

$$\sum_{j=1}^{n} most\ \ upper\ \ value\ \ of\ \begin{pmatrix}(a_{ij},b_{ij},c_{ij},d_{ij},e_{ij})\otimes\\(u_j,v_j,w_j,x_j,y_j)\end{pmatrix}\{\le,\approx,\ge\}h_i$$

$$for\ all\ i=1,2,..m;$$

$$z_2\ge z_1;z_3\ge z_2;z_4\ge z_3;z_5\ge z_4;u_j\le v_j,\ j=1,2,..m;v_j\le w_j,\ j=1,2,..m;w_j\le x_j,\ j=1,2,..m;$$
$$x_j\le y_j,\ j=1,2,..m;u_j\ge 0,\ j=1,2,..m$$

step:3

The above theorem establishes a relation between an optimal fuzzy solution to a fully FLP problem, and an efficient solution to its related MOLP problems. Here an efficient solution is obtained to the MOLP problem using the SO method.

The proposed method is illustrated by the following examples.

IV. NUMERICAL EXAMPLE

(A) Consider the following FFLP problem

Maximize

$$\tilde{z}\approx(0.8,0.7,0.3,0.3,0.2)\otimes\tilde{x}_1\oplus(0.2,0.3,0.4,0.1,0.2)\otimes\tilde{x}_2$$

Subject to,

$$(0.2,0.4,0.5,0.6,0.7)\otimes\tilde{x}_1\oplus(0.3,0.2,0.6,0.5,0.1)\otimes\tilde{x}_2\le(0.1,0.2,0.5,0.4,0.3);$$
$$(0.7,0.8,0.6,0.9,0.1)\otimes\tilde{x}_1\oplus(0.2,0.3,0.5,0.7,0.1)\otimes\tilde{x}_2\le(0.2,0.3,0.5,0.7,0.9);$$

$$\tilde{x}_1,\tilde{x}_2\ge 0$$

Solution;

Let

$\tilde{x}_1 \approx (u_1, v_1, w_1, x_1, y_1), \tilde{x}_2 \approx (u_2, v_2, w_2, x_2, y_2)$ *and* $\bar{z} \approx (z_1, z_2, z_3, z_4, z_5)$

Now using step 1 the MOLP problem related to the given fully FLP problem is given below:

$$\max z_1 \approx 0.2u_1 \oplus 0.1u_2$$
$$\max z_2 \approx 0.3v_1 \oplus 0.2v_2$$
$$\max z_3 \approx 0.3w_1 \oplus 0.2w_2$$
$$\max z_4 \approx 0.7x_1 \oplus 0.3x_2$$
$$\max z_5 \approx 0.8y_1 \oplus 0.4y_2$$

Subject to

$0.2u_1 \oplus 0.1u_2 \le 0.1$; $0.4v_1 \oplus 0.2v_2 \le 0.2$; $0.5w_1 \oplus 0.3w_2 \le 0.3$;

$0.6x_1 \oplus 0.5x_2 \le 0.4$; $0.7y_1 \oplus 0.6y_2 \le 0.5$; $0.1u_1 \oplus 0.1u_2 \le 0.2$;

$0.6v_1 \oplus 0.2v_2 \le 0.3$; $0.7w_1 \oplus 0.3w_2 \le 0.5$; $0.8x_1 \oplus 0.5x_2 \le 0.7$;

$0.9y_1 \oplus 0.7y_2 \le 0.9$; $z_2 \ge z_1$; $z_3 \ge z_2$; $z_4 \ge z_3$;

$z_5 \ge z_4$; $v_1 \ge u_1$; $w_1 \ge v_1$; $x_1 \ge w_1$; $y_1 \ge x_1$; $v_2 \ge u_2$;

$w_2 \ge v_2$; $x_2 \ge w_2$; $y_2 \ge x_2$

Now, by step 2, we consider the following LP problem related to the above MOLP problem as.

(B)

$Max\ \bar{z} \approx 0.2u_1 \oplus 0.1u_2 \oplus 0.3v_1 \oplus 0.2v_2 \oplus 0.3w_1 \oplus 0.2w_2 \oplus$
$0.7x_1 \oplus 0.3x_2 \oplus 0.8y_1 \oplus 0.4y_2$

Subject to,

$0.2u_1 \oplus 0.1u_2 \le 0.1$; $0.4v_1 \oplus 0.2v_2 \le 0.2$; $0.5w_1 \oplus 0.3w_2 \le 0.3$;

$0.6x_1 \oplus 0.5x_2 \le 0.4$; $0.7y_1 \oplus 0.6y_2 \le 0.5$; $0.1u_1 \oplus 0.1u_2 \le 0.2$;

$0.6v_1 \oplus 0.2v_2 \le 0.3$; $0.7w_1 \oplus 0.3w_2 \le 0.5$; $0.8x_1 \oplus 0.5x_2 \le 0.7$;

$0.9y_1 \oplus 0.7y_2 \le 0.9$;

$0.3v_1 \oplus 0.2v_2 - 0.2u_1 - 0.1u_2 \ge 0$; $0.3w_1 \oplus 0.2w_2 - 0.3v_1 - 0.2v_2 \ge 0$;

$0.7x_1 \oplus 0.3x_2 - 0.3w_1 - 0.2w_2 \ge 0$; $0.8y_1 \oplus 0.4y_2 - 0.7x_1 - 0.3x_2 \ge 0$

$v_1 \ge u_1$; $w_1 \ge v_1$; $x_1 \ge w_1$; $y_1 \ge x_1$; $v_2 \ge u_2$; $w_2 \ge v_2$; $x_2 \ge w_2$; $y_2 \ge x_2$

The optimal solution to the problem((B) is $u_1 = 0.5, u_2 = 0, v_1 = 0, v_2 = 1, w_1 = 0, w_2 = 1, x_1 = 0.67, x_2 = 0,$
$y_1 = 0.71, y_2 = 0$ *with* $Z = 1.54$

is an efficient solution to the problem(A).

4.1Comparison study

TABLE I

Maximum Z	*Proposed Method (Without Ranking)*	*Existing Method (With Ranking)*
z_1	**0.1**	**0.1**
z_2	**0.2**	**0.18**
z_3	**0.2**	**0.18**
z_4	**0.469**	**0.37**
z_5	**0.568**	**0.42**
$\tilde{z}$	**1.54**	**0.14**

V. CONCLUSION

In this paper, the level sum method is used in pentagon fuzzy numbers for finding optimal fuzzy solution to a FLP problem satisfying all constraints. Here the proposed method is the one without ranking on comparison with the existing ranking method. The main advantage of the proposed method is that the FLP problems can be solved by any LP solver using the level sum method. In future this method is used where a condition of more than five parameters to find an optimal solution

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