On general Eulerian integral of certain products of two multivariable Aleph-functions and a class of multivariable polynomials

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ABSTRACT
The object of this paper is to establish an general Eulerian integral involving the product of two multivariable Aleph-functions, a class of multivariable polynomials and a generalized hypergeometric function which provide unification and extension of numerous results. We will study the particular cases concerning the multivariable I-function and the Srivastava-Daoust polynomial

Keywords: Eulerian integral, multivariable I-function, Lauricella function of several variables, multivariable Aleph-function, generalized hypergeometric function, Srivastava-Daoust polynomial

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1. Introduction

The Aleph-function of several variables generalize the multivariable I-function defined by Sharma and Ahmad [2] , itself is an a generalisation of G and H-functions of several variables definned by Srivastava et al [7]. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z'_1,\cdots,z'_s)=\aleph^{0,\mathfrak{n}':m'_1,n'_1,\cdots,m'_s,n'_s}_{p'_i,q'_i,\tau_i;R:p'_{i(1)},q'_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p'_{i(s)},q'_{i(s)};\tau_{i(s)};R^{(s)}}\left(\begin{array}{c} z'_1 \\ \cdot \\ \cdot \\ \cdot \\ z'_s \end{array}\right|$

$$[(a'_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}'}],[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}'+1,p_i}]:$$
$$\cdots\cdots\cdots,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m'+1,q_i}]:$$

$$[(c'^{(1)}_j,\gamma'^{(1)}_j)_{1,n'_1}],[\tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}},\gamma^{(1)}_{ji^{(1)}})_{n'_1+1,p'^{(1)}_i}];\cdots;[(c'^{(s)}_j,\gamma'^{(s)}_j)_{1,n'_s}],[\tau_{i^{(s)}}(c^{(s)}_{ji^{(s)}},\gamma^{(s)}_{ji^{(s)}})_{n'_s+1,p'^{(s)}_i}]$$
$$\left.[(d'^{(1)}_j,\delta'^{(1)}_j)_{1,m'_1}],[\tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}},\delta^{(1)}_{ji^{(1)}})_{m'_1+1,q'^{(1)}_i}];\cdots;[(d'^{(s)}_j,\delta'^{(s)}_j)_{1,m'_s}],[\tau_{i^{(s)}}(d^{(s)}_{ji^{(s)}},\delta^{(s)}_{ji^{(s)}})_{m'_s+1,q'^{(s)}_i}]\right)$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L'_1}\cdots\int_{L'_s}\psi(t_1,\cdots,t_s)\prod_{k=1}^{s}\zeta_k(t_k){z'_k}^{t_k}\,\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{\mathfrak{n}'}\Gamma(1-a'_j+\sum_{k=1}^{r}\alpha_j^{(k)}t_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}'+1}^{p'_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}t_k)\prod_{j=1}^{q'_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}t_k)]} \tag{1.2}$$

and $$\zeta_k(t_k)=\frac{\prod_{j=1}^{m'_k}\Gamma(d'^{(k)}_j-\delta'^{(k)}_j x_k)\prod_{j=1}^{n'_k}\Gamma(1-c^{(k)}_j+\gamma^{(k)}_j x_k)}{\sum_{i^{(k)}=1}^{r^{(k)}}[\tau_{i^{(k)}}\prod_{j=m'_k+1}^{q'_{i^{(k)}}}\Gamma(1-d'^{(k)}_{ji^{(k)}}+\beta'^{(k)}_{ji^{(k)}}x_k)\prod_{j=n'_k+1}^{p'_{i^{(k)}}}\Gamma(c^{(k)}_{ji^{(k)}}-\gamma^{(k)}_{ji^{(k)}}x_k)]} \quad (1.3)$$

Suppose , as usual , that the parameters

$a'_j, j=1,\cdots,p'; b'_j, j=1,\cdots,q';$

$c^{(k)}_j, j=1,\cdots,m'_k; c^{(k)}_{ji^{(k)}}, j=n_k+1,\cdots,p_{i^{(k)}}$

$d^{(k)}_j, j=1,\cdots,n'_k; d^{(k)}_{ji^{(k)}}, j=m'_k+1,\cdots,q_{i^{(k)}}$

with $k=1\cdots,s, i=1,\cdots,R$, $i^{(k)}=1,\cdots,R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)}=\sum_{j=1}^{\mathfrak{n}'}\alpha'^{(k)}_j+\tau_i\sum_{j=\mathfrak{n}'+1}^{p'_i}\alpha^{(k)}_{ji}+\sum_{j=1}^{n'_k}\gamma'^{(k)}_j+\tau_{i^{(k)}}\sum_{j=n'_k+1}^{p'_{i^{(k)}}}\gamma^{(k)}_{ji^{(k)}}-\tau_i\sum_{j=1}^{q'_i}\beta^{(k)}_{ji}-\sum_{j=1}^{m'_k}\delta'^{(k)}_j$$

$$-\tau_{i^{(k)}}\sum_{j=m'_k+1}^{q'_{i^{(k)}}}\delta^{(k)}_{ji^{(k)}}\leqslant 0 \quad (1.4)$$

The reals numbers τ_i are positives for $i=1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d'^{(k)}_j-\delta'^{(k)}_j s_k)$ with $j=1$ to m_k are separated from those o $\Gamma(1-a'_j+\sum_{i=1}^{r}\alpha^{(k)}_j s_k)$f with $j=1$ to n and $\Gamma(1-c'^{(k)}_j+\gamma'^{(k)}_j s_k)$ with $j=1$ to n'_k to the left of the contour L'_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz'_k|<\frac{1}{2}A^{(k)}_i\pi$, where

$$A^{(k)}_i=\sum_{j=1}^{\mathfrak{n}'}\alpha'^{(k)}_j-\tau_i\sum_{j=\mathfrak{n}'+1}^{p'_i}\alpha^{(k)}_{ji}-\tau_i\sum_{j=1}^{q'_i}\beta^{(k)}_{ji}+\sum_{j=1}^{n'_k}\gamma'^{(k)}_j-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma^{(k)}_{ji^{(k)}}$$

$$+\sum_{j=1}^{m'_k}\delta'^{(k)}_j-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta^{(k)}_{ji^{(k)}}>0, \text{ with } k=1\cdots,s, i=1,\cdots,R \text{ , } i^{(k)}=1,\cdots,R^{(k)} \quad (1.5)$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z'_1,\cdots,z'_s)=0(|z'_1|^{\alpha'_1},\cdots,|z'_s|^{\alpha'_s}), max(|z'_1|,\cdots,|z'_s|)\rightarrow 0$

$\aleph(z'_1,\cdots,z'_s)=0(|z'_1|^{\beta'_1},\cdots,|z_s|^{\beta'_s}), min(|z'_1|,\cdots,|z'_s|)\rightarrow\infty$

where $k = 1, \cdots, s : \alpha'_k = min[Re(d'^{(k)}_j / \delta'^{(k)}_j)], j = 1, \cdots, m'_k$ and

$$\beta'_k = max[Re((c'^{(k)}_j - 1)/\gamma'^{(k)}_j)], j = 1, \cdots, n'_k$$

We will use these following notations in this paper

$$U = p'_i, q'_i, \tau_i; R \ ; \ V = m'_1, n'_1; \cdots; m'_r, n'_r \tag{1.6}$$

$$W = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.7}$$

$$A = \{(a'_j; \alpha'^{(1)}_j, \cdots, \alpha'^{(r)}_j)_{1,n'}\}, \{\tau_i(a_{ji}; \alpha^{(1)}_{ji}, \cdots, \alpha^{(r)}_{ji})_{n+1,p_i}\} \tag{1.8}$$

$$B = \{\tau_i(b_{ji}; \beta^{(1)}_{ji}, \cdots, \beta^{(r)}_{ji})_{m+1,q_i}\} \tag{1.9}$$

$$C = \{(c^{(1)}_j; \gamma^{(1)}_j)_{1,n_1}\}, \tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}}; \gamma^{(1)}_{ji^{(1)}})_{n'_1+1,p_{i^{(1)}}}\}, \cdots,$$

$$\{(c^{(s)}_j; \gamma^{(s)}_j)_{1,n_s}\}, \tau_{i^{(s)}}(c^{(s)}_{ji^{(s)}}; \gamma^{(s)}_{ji^{(s)}})_{n'_s+1,p_{i^{(s)}}}\} \tag{1.10}$$

$$D = \{(d'^{(1)}_j; \delta'^{(1)}_j)_{1,m'_1}\}, \tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}}; \delta^{(1)}_{ji^{(1)}})_{m'_1+1,q_{i^{(1)}}}\}, \cdots$$

$$, \{(d'^{(s)}_j; \delta'^{(s)}_j)_{1,m'_s}\}, \tau_{i^{(s)}}(d^{(s)}_{ji^{(s)}}; \delta^{(s)}_{ji^{(s)}})_{m'_s+1,q_{i^{(s)}}}\} \tag{1.11}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots, z_r) = \aleph^{0,\mathfrak{n}':V}_{U:W}\left(\begin{array}{c|c} z'_1 & A : C \\ \cdot & \cdots \\ \cdot & \\ z'_s & B : D \end{array}\right) \tag{1.12}$$

Consider the Aleph-function of s variables

$$\aleph(z''_1, \cdots, z''_u) = \aleph^{0,N:M_1,N_1,\cdots,M_u,N_u}_{P_i,Q_i,\iota_i;r':P_{i^{(1)}},Q_{i^{(1)}},\iota_{i^{(1)}};r^{(1)};\cdots;P_{i^{(u)}},Q_{i^{(u)}};\iota_{i^{(u)}};r^{(u)}}\left(\begin{array}{c|} z''_1 \\ \cdot \\ \cdot \\ \cdot \\ z''_u \end{array}\right.$$

$$\begin{array}{ll} [(\mathrm{u}_j; \mu^{(1)}_j, \cdots, \mu^{(u)}_j)_{1,N}] & ,[\iota_i(u_{ji}; \mu^{(1)}_{ji}, \cdots, \mu^{(u)}_{ji})_{N+1,P_i}] : \\ \cdots\cdots\cdots\cdots & ,[\iota_i(v_{ji}; \upsilon^{(1)}_{ji}, \cdots, \upsilon^{(u)}_{ji})_{M+1,Q_i}] : \end{array}$$

$$\left.\begin{array}{l} [(\mathrm{a}''^{(1)}_j; \alpha''^{(1)}_j)_{1,N_1}], [\iota_{i^{(1)}}(a^{(1)}_{ji^{(1)}}; \alpha^{(1)}_{ji^{(1)}})_{N_1+1,P^{(1)}_i}]; \cdots; \ [(\mathrm{a}^{(u)}_j); \alpha^{(u)}_j)_{1,N_u}], [\iota_{i^{(u)}}(a^{(u)}_{ji^{(u)}}; \alpha^{(u)}_{ji^{(u)}})_{N_u+1,P^{(u)}_i}] \\ [(\mathrm{b}''^{(1)}_j; \beta''^{(1)}_j)_{1,M_1}], [\iota_{i^{(1)}}(b^{(1)}_{ji^{(1)}}; \beta^{(1)}_{ji^{(1)}})_{M_1+1,Q^{(1)}_i}]; \cdots; \ [(\mathrm{b}^{(u)}_j); \beta^{(u)}_j)_{1,M_u}], [\iota_{i^{(u)}}(b^{(u)}_{ji^{(u)}}; \beta^{(u)}_{ji^{(u)}})_{M_u+1,Q^{(u)}_i}] \end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^u} \int_{L''_1} \cdots \int_{L''_u} \zeta(x_1, \cdots, x_u) \prod_{k=1}^{u} \phi_k(x_k) z''^{x_k}_k \, \mathrm{d}x_1 \cdots \mathrm{d}x_u \tag{1.13}$$

with $\omega = \sqrt{-1}$

$$\zeta(x_1,\cdots,x_u)=\frac{\prod_{j=1}^{N}\Gamma(1-u''_j+\sum_{k=1}^{u}\mu_j^{(k)}x_k)}{\sum_{i=1}^{r'}[\iota_i\prod_{j=N+1}^{P_i}\Gamma(u_{ji}-\sum_{k=1}^{u}\mu_{ji}^{(k)}x_k)\prod_{j=1}^{Q_i}\Gamma(1-\upsilon''_{ji}+\sum_{k=1}^{u}\upsilon_{ji}^{(k)}x_k)]} \quad (1.14)$$

and

$$\phi'_k(x_k)=\frac{\prod_{j=1}^{M_k}\Gamma(b_j''^{(k)}-\beta_j''^{(k)}x_k)\prod_{j=1}^{N_k}\Gamma(1-a_j''^{(k)}+\alpha_j''^{(k)}x_k)}{\sum_{i^{(k)}=1}^{r^{(k)}}[\iota_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-b''_{ji^{(k)}}{}^{(k)}+\beta''_{ji^{(k)}}{}^{(k)}x_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(a_{ji^{(k)}}^{(k)}-\alpha_{ji^{(k)}}^{(k)}x_k)]} \quad (1.15)$$

Suppose , as usual , that the parameters

$u_j, j=1,\cdots,P; v_j, j=1,\cdots,Q;$

$a_j''^{(k)}, j=1,\cdots,N_k; a_{ji^{(k)}}^{(k)}, j=N_k+1,\cdots,P_{i^{(k)}};$

$b_{ji^{(k)}}^{(k)}, j=M_k+1,\cdots,Q_{i^{(k)}}; b_j^{(k)}, j=1,\cdots,M_k;$

with $k=1\cdots,u, i=1,\cdots,r', i^{(k)}=1,\cdots,r^{(k)}$

are complex numbers , and the $\alpha's,\beta's,\gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i'^{(k)}=\sum_{j=1}^{N}\mu_j^{(k)}+\iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)}+\sum_{j=1}^{N_k}\alpha_j''^{(k)}+\iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}-\iota_i\sum_{j=1}^{Q_i}\upsilon_{ji}^{(k)}-\sum_{j=1}^{M_k}\beta_j''^{(k)}$$

$$-\iota_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)}\leqslant 0 \quad (1.16)$$

The reals numbers τ_i are positives for $i=1,\cdots,u$, $\iota_{i^{(k)}}$ are positives for $i^{(k)}=1\cdots r^{(k)}$

The contour L_k is in the t_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(b_j''^{(k)}-\beta_j''^{(k)}t_k)$ with $j=1$ to M_k are separated from those o $\Gamma(1-u''_j+\sum_{i=1}^{s}\mu_j^{(k)}t_k)$f with $j=1$ to N and $\Gamma(1-a_j''^{(k)}+\alpha_j''^{(k)}t_k)$ with $j=1$ to N_k to the left of the contour L''_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz''_k|<\frac{1}{2}B_i^{(k)}\pi$, where

$$B_i^{(k)}=\sum_{j=1}^{N}\mu_j^{(k)}-\iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)}-\iota_i\sum_{j=1}^{Q_i}\upsilon_{ji}^{(k)}+\sum_{j=1}^{N_k}\alpha_j''^{(k)}-\iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{M_k}\beta_j''^{(k)}-\iota_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)}>0, \quad \text{with } k=1,\cdots,s, i=1,\cdots,u, i^{(k)}=1,\cdots,r^{(k)} \quad (1.17)$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1'', \cdots, z_u'') = 0(|z_1''|^{\alpha_1''}, \cdots, |z_u''|^{\alpha_s''}), max(|z_1''|, \cdots, |z_u''|) \rightarrow 0$$

$$\aleph(z_1'', \cdots, z_u'') = 0(|z_1''|^{\beta_1''}, \cdots, |z_u''|^{\beta_s''}), min(|z_1''|, \cdots, |z_u''|) \rightarrow \infty$$

where $k = 1, \cdots, z : \alpha_k'' = min[Re(b_j''^{(k)}/\beta_j''^{(k)})], j = 1, \cdots, M_k$ and

$$\beta_k'' = max[Re((a_j''^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, N_k$$

We will use these following notations in this paper

$$U' = P_i, Q_i, \iota_i; r' ; V' = M_1, N_1; \cdots ; M_u, N_u \tag{1.18}$$

$$W' = P_{i^{(1)}}, Q_{i^{(1)}}, \iota_{i^{(1)}}; r^{(1)}, \cdots, P_{i^{(u)}}, Q_{i^{(u)}}, \iota_{i^{(u)}}; r^{(u)} \tag{1.19}$$

$$A' = \{(u_j; \mu_j^{(1)}, \cdots, \mu_j^{(u)})_{1,N}\}, \{\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots, \mu_{ji}^{(u)})_{N+1,P_i}\} \tag{1.20}$$

$$B' = \{\iota_i(v_{ji}; v_{ji}^{(1)}, \cdots, v_{ji}^{(u)})_{M+1,Q_i}\} \tag{1.21}$$

$$C' = (a_j''^{(1)}; \alpha_j''^{(1)})_{1,N_1}, \iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_{i^{(1)}}}, \cdots,$$

$$(a_j^{(u)}; \alpha_j^{(u)})_{1,N_u}, \iota_{i^{(u)}}(a_{ji^{(u)}}^{(u)}; \alpha_{ji^{(u)}}^{(u)})_{N_u+1,P_{i^{(u)}}} \tag{1.22}$$

$$D' = (b_j''^{(1)}; \beta_j''^{(1)})_{1,M_1}, \iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)}; \beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_{i^{(1)}}}, \cdots,$$

$$(b_j^{(u)}; \beta_j^{(u)})_{1,M_u}, \iota_{i^{(u)}}(\beta_{ji^{(u)}}^{(u)}; \beta_{ji^{(u)}}^{(u)})_{M_u+1,Q_{i^{(u)}}} \tag{1.23}$$

The multivariable Aleph-function write :

$$\aleph(z_1'', \cdots, z_u'') = \aleph_{U':W'}^{0,N:V'}\left(\begin{array}{c|c} \text{z'}_1 & \text{A' : C'} \\ \cdot & \cdot\ \cdot\ \cdot \\ \cdot & \\ \text{z'u} & \text{B' : D'} \end{array}\right) \tag{1.24}$$

Srivastava and Garg [4] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_u}[z_1, \cdots, z_u] = \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u \leqslant L} (-L)_{h_1R_1+\cdots+h_uR_u} B(E; R_1, \cdots, R_u) \frac{z_1^{R_1} \cdots z_u^{R_u}}{R_1! \cdots R_u!} \tag{1.25}$$

The coefficients are $B[E; R_1, \ldots, R_v]$ arbitrary constants, real or complex.

2. Integral representation of Lauricella function of several variables

The Lauricella function $F_D^{(k)}$ is defined as , see Srivastava et al [5]

$$F_D^{(k)}[a, b_1, \cdots, b_k; c; x_1, \cdots, x_k] = \frac{\Gamma(c)}{\Gamma(a)\prod_{j=1}^k \Gamma(b_j)} \frac{1}{(2\pi\omega)^k} \int_{L_1} \cdots \int_{L_k} \frac{\Gamma\left(a + \sum_{j=1}^k \zeta_j\right) \Gamma(b_1 + \zeta_1), \cdots, \Gamma(b_k + \zeta_k)}{\Gamma\left(c + \sum_{j=1}^k \zeta_j\right)}$$

$$\prod_{j=1}^{k}\Gamma(-\zeta_j)(-x_j)^{\zeta_i}\,\mathrm{d}\zeta_1\cdots\mathrm{d}\zeta_k \tag{2.1}$$

where $max\,[|arg(-x_1)|,\cdots,|arg(-x_k)|]<\pi, c\neq 0,-1,-2,\cdots$.
In order to evaluate a number of integrals of multivariable I-functions, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}\mathrm{d}t=(b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}$$

$$\times F_D^{(k)}\left[\alpha,-\sigma_1,\cdots,-\sigma_k;\alpha+\beta;-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right] \tag{2.2}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i\in\mathbb{C},(i=1,\cdots,k)$; $min(Re(\alpha),Re(\beta))>0$ and

$$\max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1$$

$F_D^{(k)}$ is a Lauricella's function of k-variables, see Srivastava et al ([6], page60)

The formula (2.2) can be establish by expanding $\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.3}$$

integrating term by term with the help of the integral given by Saigo and Saxena [1, page 93, eq.(3.2)] and applying the definition of the Lauricella function $F_D^{(k)}$ [6, page 60].

3. Eulerian integral

In this section , we evaluate a general Eulerian integral with the product of two multivariable Aleph-functions and a class of multivariable polynomials. We given the expansion serie concerning the last function.

We note

$$K_1=(1-\alpha-\sum_{i=1}^{r}R_i(\mu_i+\mu_i');\mu_1,\cdots,\mu_s,\mu_1',\cdots,\mu_u',1,\cdots,1,\upsilon_1,\cdots,\upsilon_l) \tag{3.1}$$

$$K_2=(1-\beta-\sum_{i=1}^{r}R_i(\rho_i+\rho_i');\rho_1,\cdots,\rho_s,\rho_1',\cdots,\rho_u',0,\cdots,0,\tau_1,\cdots,\tau_l) \tag{3.2}$$

$$K_P=[1-A_j;0,\cdots,0,0,\cdots,0,0,\cdots,0,1,\cdots,1]_{1,P} \tag{3.3}$$

$$K_j=[1+\sigma_j-\sum_{i=1}^{r}R_i(\lambda_i^{(j)}+\lambda_i'^{(j)});\lambda_j^{(1)},\cdots,\lambda_j^{(s)},\lambda_j'^{(1)}\cdots,\lambda_j'^{(u)},0\cdots,1,\cdots,0,\zeta_j',\cdots,\zeta_j^{(l)}]_{1,k} \tag{3.4}$$

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$$L_1=(1-\alpha-\beta-\sum_{i=1}^{r}R_i(\mu_i+\mu_i'+\rho_i+\rho_i');\mu_1+\rho_1,\cdots,\mu_s+\rho_s,\mu_1'+\rho_1',\cdots,\mu_u'+\rho_u',$$

$$1,\cdots,1,\upsilon_1+\tau_1,\cdots,\upsilon_l+\tau_l) \tag{3.5}$$

$$L_Q = [1 - B_j; 0, \cdots, 0, 0, \cdots, 0, 0, \cdots, 0, 1 \cdots, 1]_{1,Q} \tag{3.6}$$

$$L_j = [1 + \sigma_j - \sum_{i=1}^{r} R_i(\lambda_i^{(j)} + \lambda_i'^{(j)}); \lambda_j^{(1)}, \cdots, \lambda_j^{(s)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(u)}, 0 \cdots, 0, \zeta_j', \cdots, \zeta_j^{(l)}]_{1,k} \tag{3.7}$$

$$V_1 = V; V'; 1, 0; \cdots; 1, 0; 1, 0; \cdots; 1, 0 \ ; W_1 = W; W'; 0, 1; \cdots; 0, 1; 0, 1; \cdots; 0, 1 \tag{3.8}$$

$$C_1 = C; C'; (1,0), \cdots, (1,0); (1,0), \cdots, (1,0) \ ; D_1 = D; D'; (0,1), \cdots, (0,1); (0,1), \cdots, (0,1) \tag{3.9}$$

A, B, A' and B' are defined respectively by (1.8), (1.9), (1.20) and (1.21), and C, D, C' and D' are defined respectively by (1.10), 1.11), (1.22) and (1.23)

$$B_r = \frac{(-L)_{h_1 R_1 + \cdots + h_r R_r} B(E; R_1, \cdots, R_r)}{R_1! \cdots R_r!} \tag{3.10}$$

We have the following result

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1} \prod_{j=1}^{k} (f_j t + g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_r}\left(\begin{array}{c} \mathrm{x}_1(t-a)^{\mu_1+\mu_1'}(b-t)^{\rho_1+\rho_1'} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)}-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{x}_r(t-a)^{\mu_u+\mu_u'}(b-t)^{\rho_r+\rho_r'} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(r)}-\lambda_j'^{(r)}} \end{array}\right)$$

$$\aleph\left(\begin{array}{c} \mathrm{z}_1(t-a)^{\mu_1}(b-t)^{\rho_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_s(t-a)^{\mu_s}(b-t)^{\rho_s} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(s)}} \end{array}\right)$$

$$\aleph\left(\begin{array}{c} \mathrm{z'}_1(t-a)^{\mu_1'}(b-t)^{\rho_1'} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z'}_u(t-a)^{\mu_u'}(b-t)^{\rho_u'} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'^{(u)}} \end{array}\right)$$

$${}_PF_Q\left[(A_P); (B_Q); -\sum_{i=1}^{l} z_i''(t-a)^{\upsilon_i}(b-t)^{\tau_i} \prod_{j=1}^{k} (f_j t + g_j)^{-\zeta_j^{(i)}}\right] \mathrm{d}t = (b-a)^{\alpha+\beta-1}$$

$$\sum_{R_1,\cdots,R_r=0}^{h_1R_1+\cdots h_rR_r\leqslant L}(b-a)^{\sum_{i=1}^r R_i(\mu_i+\mu_i'+\rho_i+\rho_i')}\frac{\prod_{j=1}^Q\Gamma(B_j)}{\prod_{j=1}^P\Gamma(A_j)}\prod_{j=1}^k(af_j+g_j)^{\sigma_j-\sum_{i=1}^r R_i(\lambda_j^{(i)}+\lambda_j'^{(i)})}B_r z_1^{R_1}\cdots z_r^{R_r}$$

$$\aleph_{U;U';l+k+2,l+k+1:W_1}^{0+n'+N+l+k+2:V_1}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(1)}}} & \mathrm{A}\ ;\ \mathrm{A'};\ \mathrm{K}_1,K_2,K_P,K_j:C_1\\ \cdots & \cdot\\ \cdots & \cdot\\ \frac{z_s(b-a)^{\mu_s+\rho_s}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(s)}}} & \cdot\\ \frac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j'^{(1)}}} & \cdot\\ \cdots & \cdot\\ \cdots & \cdot\\ \frac{z_u'(b-a)^{\mu_u'+\rho_u'}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j'^{(u)}}} & \cdot\\ \frac{(b-a)f_1}{af_1+g_1} & \cdot\\ \cdots & \cdot\\ \cdots & \cdot\\ \frac{(b-a)f_k}{af_k+g_k} & \cdot\\ \frac{z_1''(b-a)^{\tau_1+\upsilon_1}}{\prod_{j=1}^k(af_j+g_j)^{\zeta_j^{(1)}}} & \cdot\\ \cdots & \cdot\\ \cdots & \cdot\\ \frac{z_l''(b-a)^{\tau_l+\upsilon_l}}{\prod_{j=1}^k(af_j+g_j)^{\zeta_j^{(l)}}} & \mathrm{B}\ ;\mathrm{B'};\ \mathrm{L}_1,L_j,L_Q:D_1 \end{array}\right) \quad (3.11)$$

Provided that

(A) $a,b\in\mathbb{R}(a<b);\mu_i,\rho_i,\mu_j',\rho_j'\lambda_v^{(i)};\lambda_v'^{(i)}\in\mathbb{R}^+$, $f_i,g_j,\tau_v,\sigma_j\in\mathbb{C}$ $(i=1,\cdots,s;j=1,\cdots;u;v=1,\cdots,k)$ $\zeta_j^{(i)}>0(i=1,\cdots,l;j=1,\cdots,k)$

(B) **See I**

(C) $\max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1$

(D) $Re\big[\alpha+\sum_{i=1}^s\mu_i\min_{1\leqslant j\leqslant m_i'}\frac{d_j'^{(i)}}{\delta_j'^{(i)}}+\sum_{i=1}^u\mu_i'\min_{1\leqslant j\leqslant m_i''}\frac{b_j''^{(i)}}{\beta_j''^{(i)}}\big]>0$

$$Re[\beta+\sum_{i=1}^{s}\rho_i\min_{1\leqslant j\leqslant m'_i}\frac{b_j'^{(i)}}{\beta_j'^{(i)}}+\sum_{i=1}^{u}\rho'_i\min_{1\leqslant j\leqslant m''_i}\frac{d_j''^{(i)}}{\delta_j''^{(i)}}]>0$$

(E) $U_i^{(k)}=\sum_{j=1}^{\mathfrak{n}'}\alpha_j'^{(k)}+\tau_i\sum_{j=\mathfrak{n}'+1}^{p'_i}\alpha_{ji}^{(k)}+\sum_{j=1}^{n'_k}\gamma_j'^{(k)}+\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}-\tau_i\sum_{j=1}^{q'_i}\beta_{ji}^{(k)}-\sum_{j=1}^{m'_k}\delta_j'^{(k)}$

$$-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}\leqslant 0$$

$$U_i'^{(k)}=\sum_{j=1}^{N}\mu_j^{(k)}+\iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)}+\sum_{j=1}^{N_k}\alpha_j''^{(k)}+\iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}-\iota_i\sum_{j=1}^{Q_i}\upsilon_{ji}^{(k)}-\sum_{j=1}^{M_k}\beta_j''^{(k)}$$

$$-\iota_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)}\leqslant 0$$

(F) $\left|arg\left(z_i\prod_{j=1}^{h}(p_jt+q_j)^{-\lambda_j^{(i)}}\right)\right|<\frac{1}{2}A_i^{(k)}\pi$, $A_i^{(k)}$ is defined by (1.12) and

$\left|arg\left(z'_i\prod_{j=1}^{h}(p_jt+q_j)^{-\lambda_j'^{(i)}}\right)\right|<\frac{1}{2}B_i^{(k)}\pi$, $B_i^{(k)}$ is defined by (1.24)

(H) $P\leqslant Q+1$. The equality holds, when , in addition,

either $P>Q$ and $\left|z''_i\left(\prod_{j=1}^{k}(f_jt+g_j)^{-\zeta_j^{(i)}}\right)\right|^{\frac{1}{Q-P}}<1 \qquad (a\leqslant t\leqslant b)$

or $P\leqslant Q$ and $\max_{1\leqslant i\leqslant k}\left[\left|\left(z''_i\prod_{j=1}^{k}(f_jt+g_j)^{-\zeta_j^{(i)}}\right)\right|\right]<1 \qquad (a\leqslant t\leqslant b)$

Proof

First expressing the class of multivariable polynomials defined by Srivastava et al [4] in serie with the help of (1.25), expressing the Aleph-function of s-variables and u-variables by the Mellin-Barnes contour integral with the help of the equation (1.1) and (1.13) respectively, the generalized hypergeometric function ${}_PF_Q(.)$ in Mellin-Barnes contour integral with the help of (2.1). Now collect the power of (f_jt+g_j) with $j=1,\cdots,k$ and use the equations (2.1) and (2.2) and we obtain $k-$Mellin-Barnes contour integral. Changing the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process) and interpreting $(r+s+k+l)-$Mellin-barnes contour integral to multivariable Aleph-function, we obtain the desired result.

4. Particular case

a) If $\tau,\tau_{(1)},\cdots,\tau_{(s)},\iota,\iota_{(1)},\cdots,\iota_{(u)}\rightarrow 1$, the Aleph-function of s-variables and the Aleph-function of u-variables reduces respectively to I-function of s-variables and I-function of u-variables defined by Sharma et al [2]. We have the following integral under the same notations and conditions that (3.11) with :

$\tau,\tau_{(1)},\cdots,\tau_{(s)},\iota,\iota_{(1)},\cdots,\iota_{(u)}\rightarrow 1$

We have the following integral

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^k (f_jt+g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_r}\left(\begin{array}{c} \mathrm{x}_1(t-a)^{\mu_1+\mu'_1}(b-t)^{\rho_1+\rho'_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(1)}-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{x}_r(t-a)^{\mu_u+\mu'_u}(b-t)^{\rho_r+\rho'_r}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(r)}-\lambda_j'^{(r)}} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_s(t-a)^{\mu_s}(b-t)^{\rho_s}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(s)}} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z'}_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z'}_u(t-a)^{\mu'_u}(b-t)^{\rho'_u}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(u)}} \end{array}\right)$$

$${}_PF_Q\left[(A_P);(B_Q);-\sum_{i=1}^l z''_i(t-a)^{\upsilon_i}(b-t)^{\tau_i}\prod_{j=1}^k(f_jt+g_j)^{-\zeta_j^{(i)}}\right]\mathrm{d}t=(b-a)^{\alpha+\beta-1}$$

$$\sum_{R_1,\cdots,R_r=0}^{h_1R_1+\cdots h_rR_r\leqslant L}(b-a)^{\sum_{i=1}^r R_i(\mu_i+\mu'_i+\rho_i+\rho'_i)}\frac{\prod_{j=1}^Q\Gamma(B_j)}{\prod_{j=1}^P\Gamma(A_j)}\prod_{j=1}^k(af_j+g_j)^{\sigma_j-\sum_{i=1}^r R_i(\lambda_j^{(i)}+\lambda_j'^{(i)})}B_r z_1^{R_1}\cdots z_r^{R_r}$$

$$I_{U;U';l+k+2,l+k+1:W_1}^{0+n'+N+l+k+2:V_1}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \text{A ; A'; } K_1, K_2, K_P, K_j : C_1 \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z_s(b-a)^{\mu_s+\rho_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(s)}}} & \cdot \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(1)}_j}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z'_u(b-a)^{\mu'_u+\rho'_u}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(u)}_j}} & \cdot \\ \frac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{(b-a)f_k}{af_k+g_k} & \cdot \\ \frac{z''_1(b-a)^{\tau_1+\upsilon_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\varsigma_j^{(1)}}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z''_l(b-a)^{\tau_l+\upsilon_l}}{\prod_{j=1}^{k}(af_j+g_j)^{\varsigma_j^{(l)}}} & \text{B ;B'; } L_1, L_j, L_Q : D_1 \end{array}\right) \tag{4.1}$$

b) If $B(L;R_1,\cdots,R_r)$ =

$$\frac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_r\theta_j^{(r)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(r)}}(b_j^{(r)})_{R_r\phi_j^{(r)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{R_1\psi'_j+\cdots+R_r\psi_j^{(r)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(r)}}(d_j^{(r)})_{R_r\delta_j^{(r)}}} \tag{4.2}$$

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_r}[z_1,\cdots,z_r]$ reduces to generalized Lauricella function defined by Srivastava et al [3]. We have

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$F_{\bar{C}:D';\cdots;D^{(r)}}^{1+\bar{A}:B';\cdots;B^{(r)}}\left(\begin{array}{c|} \mathrm{x}_1(t-a)^{\mu_1+\mu'_1}(b-t)^{\rho_1+\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}-\lambda'^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{x}_r(t-a)^{\mu_u+\mu'_u}(b-t)^{\rho_r+\rho'_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}-\lambda'^{(r)}_j} \end{array}\right.$$

$$\left.\begin{matrix} [(\text{-L});\text{R}_1,\cdots,R_r][(a);\theta',\cdots,\theta^{(r)}]:[(b');\phi'];\cdots;[(b^{(r)});\phi^{(r)}] \\ [(\text{c});\psi',\cdots,\psi^{(r)}]:[(d');\delta'];\cdots;[(d^{(r)});\delta^{(r)}] \end{matrix}\right)$$

$$\aleph\begin{pmatrix} \text{z}_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \text{z}_s(t-a)^{\mu_s}(b-t)^{\rho_s}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(s)}} \end{pmatrix}\aleph\begin{pmatrix} \text{z'}_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \text{z'}_u(t-a)^{\mu'_u}(b-t)^{\rho'_u}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(u)}} \end{pmatrix}$$

$${}_PF_Q\left[(A_P);(B_Q);-\sum_{i=1}^{l}z''_i(t-a)^{\upsilon_i}(b-t)^{\tau_i}\prod_{j=1}^{k}(f_jt+g_j)^{-\zeta_j^{(i)}}\right]\text{d}t=(b-a)^{\alpha+\beta-1}$$

$$\sum_{R_1,\cdots,R_r=0}^{h_1R_1+\cdots h_rR_r\leqslant L}(b-a)^{\sum_{i=1}^r R_i(\mu_i+\mu'_i+\rho_i+\rho'_i)}\frac{\prod_{j=1}^Q\Gamma(B_j)}{\prod_{j=1}^P\Gamma(A_j)}\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j-\sum_{i=1}^r R_i(\lambda_j^{(i)}+\lambda_j'^{(i)})}B'_r z_1^{R_1}\cdots z_r^{R_r}$$

$$\aleph_{U;U';l+k+2,l+k+1:W_1}^{0+n'+N+l+k+2:V_1}\left(\begin{matrix} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(1)}}} \\ \cdots \\ \cdots \\ \frac{z_s(b-a)^{\mu_s+\rho_s}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(s)}}} \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j'^{(1)}}} \\ \cdots \\ \cdots \\ \frac{z'_u(b-a)^{\mu'_u+\rho'_u}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j'^{(u)}}} \\ \frac{(b-a)f_1}{af_1+g_1} \\ \cdots \\ \cdots \\ \frac{(b-a)f_k}{af_k+g_k} \\ \frac{z''_1(b-a)^{\tau_1+\upsilon_1}}{\prod_{j=1}^k(af_j+g_j)^{\zeta_j^{(1)}}} \\ \cdots \\ \cdots \\ \frac{z''_l(b-a)^{\tau_l+\upsilon_l}}{\prod_{j=1}^k(af_j+g_j)^{\zeta_j^{(l)}}} \end{matrix}\left|\begin{matrix} \text{A ; A'; K}_1,K_2,K_P,K_j:C_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \text{B ;B'; L}_1,L_j,L_Q:D_1 \end{matrix}\right.\right) \tag{4.3}$$

under the same notations and conditions that (3.11)

where $B'_r = \frac{(-L)_{h_1 R_1+\cdots+h_r R_r} B(E; R_1, \cdots, R_r)}{R_1! \cdots R_r!}$, $B[E; R_1, \ldots, R_v]$ is defined by (4.2)

Remark:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable I-functions and a class of multivariable polynomials defined by Srivastava et al [4].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable Aleph-function, a class of multivariable polynomials defined by Srivastava et al [4] and a generalized hypergeometric function with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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