Fuzzy Supra α-open sets

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Abstract

In this paper fuzzy supra α-open sets and fuzzy supra α-closed sets are introduced and certain properties and relations of fuzzy supra α-open and α-closed sets are investigated.

Keywords - *Fuzzy supra topological space, fuzzy supra α-open set, fuzzy supra α-closed set.*

1. INTRODUCTION

In 1965, Njastad [7] introduced the notion of *α*-sets in topological spaces. In 1992, Singal and Rajvanshi [11] introduced the concept of fuzzy *α*-open sets.

In 1983, A.S. Mashhour et al. [6] introduced the concept of supra topological spaces and studied s-continuous functions and s∗-continuous functions. In 2008, Devi et al. [4] introduced supra *α*-open (closed) sets. In 1987, M. E. Abd El-Monsef et al. [1] introduced fuzzy supra topological spaces and studied fuzzy supra-continuous functions. In section 2 of this paper preliminary definitions and properties regarding fuzzy sets and fuzzy supra sets are given. In section 3 of this paper the concept of fuzzy supra α-open sets and fuzzy supra α-closed sets are introduced and studied.

2. Preliminary Definitions

Throughout the paper X denotes a non empty set.

Definition : 2.1 [12]

A fuzzy set in X is a map f: X → [0,1] = I. The family of fuzzy sets of X is denoted by I^X.

Following are some basic operations in fuzzy sets in X. For the fuzzy sets f and g in X,

1) f = g if and only if f(x) = g(x) for all x ϵ X
2) f ≤ g if and only if f(x) ≤ g(x) for all x ϵ X
3) (f v g)(x) = max { f(x),g(x) } for all x ϵ X
4) (f ∧ g)(x) = min { f(x),g(x) } for all x ϵ X
5) $f^c(x)$ = 1- f(x) for all x ϵ X here f^c denotes the complement of f.
6) For a family { f_λ/ λϵ∧ } of fuzzy sets defined on a set X
$$(\vee_{\lambda\in\Lambda} f_\lambda)(x) = \vee_{\lambda\in\Lambda}(f_\lambda(x))$$
$$(\wedge_{\lambda\in\Lambda} f_\lambda)(x) = \wedge_{\lambda\in\Lambda}(f_\lambda(x))$$
7) For any α ϵ I, the constant fuzzy set **α** in X is a fuzzy set in X defined by **α**(x) = α for all x ϵ X.

0 denotes null fuzzy set in X and **1** denotes universal fuzzy set in X.

Definition :2.2 **[3]**

A fuzzy topological space is a pair (X,δ) where X is a nonempty set and δ is a family of fuzzy set on X satisfying the following properties:

1) The constant functions **0** and **1** belong to δ.
2) f, g ϵδ implies f ∧ g ϵ δ.
3) f_λ ϵ δ for each λ ϵ ∧ implies $(\vee_{\lambda\in\Lambda} f_\lambda)\in\delta$.

Then δ is called a fuzzy topology on X. Every member of δ is called fuzzy open set. The complement of a fuzzy open set is called fuzzy closed set.

Definition :2.3 **[3]**

The closure and interior of a fuzzy set f ϵ I^X are defined respectively as

cl (f) = ∧ { g / g is a fuzzy closed set in X and f ≤ g }

int (f)= v { g / g is a fuzzy open set in X and g ≤ f }

Clearly cl(f) is the smallest fuzzy closed set containing f and int(f) is the largest fuzzy open set contained in f.

Definition :2.4 **[8]**

A collection δ^* of fuzzy sets in a set X is called fuzzy supra topology on X if the following conditions are satisfied:

1) **0** and **1** belongs to δ^*.
2) f_λϵ $\delta^*$for each λ ϵ ∧ implies $(\vee_{\lambda\in\Lambda} f_\lambda)\in\delta^*$.

The pair (X,δ^*) is called a fuzzy supra topological space. The elements of $\delta^*$are called fuzzy supra open sets and the complement of a fuzzy supra open set is called fuzzy supra closed set.

Definition :2.5 **[8]**

Let (X,δ^*) be a fuzzy supra topological space and f be a fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior of f defined respectively as

$cl^*(f) = \wedge$ { g / g is a fuzzy supra closed set in X and $f \le g$}
$int^*(f) = \vee$ { g / g is a fuzzy supra open set in X and $g \le f$}

Definition :2.6 **[8]**
Let (X,δ) be a fuzzy topological space and δ^* be a fuzzy supra topology on X .
We call δ^* a fuzzy supra topology associated with δ if $\delta \le \delta^*$

Remark :2.7 **[8]**
1) The fuzzy supra closure of a fuzzy set f in a fuzzy supra topological space is the smallest fuzzy supra closed set containing f
2) The fuzzy supra interior of a fuzzy set f in a fuzzy supra topological space is the largest fuzzy supra open set contained in f.
3) If $(X,\delta^*)$is an associated fuzzy supra topological space with the fuzzy topological space (X,δ) and f is any fuzzy set in X, then
$$int (f) \le int^*(f) \le f \le cl^*(f) \le cl (f)$$

Theorem :2.8 **[8]**
For any fuzzy sets f and g in a fuzzy supra topological space(X,δ^*),
1) f is a fuzzy supra closed set if and only if $cl^*(f) = f$.
2) f is a fuzzy supra open set if and only if $int^*(f) = (f)$.
3) $f \le g$ implies $int^*(f) \le int^*(g)$ and $cl^*(f) \le cl^*(g)$
4) $cl^*(cl^*(f)) = cl^*(f)$ and $int^*(int^*(f)) = int^*(f)$.
5) $cl^*(f \vee g) \ge cl^*(f) \vee cl^*(g)$
6) $cl^*(f \wedge g) \le cl^*(f) \wedge cl^*(g)$
7) $int^*(f \vee g) \ge int^*(f) \vee int^*(g)$
8) $int^*(f \wedge g) \le int^*(f) \wedge int^*(g)$
9) $cl^*(f^c) = [int^*(f)]^c$
10) $int^*(f^c) = [cl^*(f)]^c$

Definition :2.9 **[8]**
Let (X,δ^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra semi open set if
$$f \le cl^*(int^*(f))$$
The complement of a fuzzy supra semi open set is called a fuzzy supra semi closed set.

3. Fuzzy supra α-open sets

Definition :3.1
Let (X,δ^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra α-open set if
$$f \le int^*[cl^*(int^*(f))]$$
and a fuzzy set f is called a fuzzy supra α-closed set if
$$cl^*[int^*(cl^*(f))] \le f$$

Theorem :3.2
Every fuzzy supra open set is fuzzy supra α-open set.

Proof :
Let f be a fuzzy supra open set in (X,δ^*)
$\therefore int^*(f) = f$
Since $cl^*(f) \ge f$
From the above two relations
$\therefore cl^*[int^*(f)] \ge int^*(f)$
$\therefore cl^*[int^*(f)] \ge f$
By the property of fuzzy supra interior,
$f \le g$ implies $int^*(f) \le int^*(g)$
$\therefore int^*[cl^*(int^*(f))] \ge int^*(f)$
$\therefore int^*[cl^*(int^*(f))] \ge f$

Theorem :3.3
Every fuzzy supra closed set is fuzzy supra α-closed set.

Proof :
Let f be a fuzzy supra closed set in (X,δ^*)
$\therefore cl^*(f) = f$
Since $int^*(f) \le f$
From the above two relations
$\therefore int^*[cl^*(f)] \le cl^*(f)$
$\therefore int^*[cl^*(f)] \le f$
By the property of fuzzy supra closure,
$f \le g$ implies $cl^*(f) \le cl^*(g)$
$\therefore cl^*[int^*(cl^*(f))] \le cl^*(f)$
$\therefore cl^*[int^*(cl^*(f))] \le f$

Theorem :3.4
Every fuzzy supra α-open set is fuzzy supra semi open set.

Proof:
Let f be a fuzzy supra α-open set in (X,δ^*), then
$f \le int^*[cl^*(int^*(f))]$
It is obvious that
$int^*[cl^*(int^*(f))] \le cl^*(int^*(f))$
From the above relations
$f \le int^*[cl^*(int^*(f))] \le cl^*(int^*(f))$
$f \le cl^*(int^*(f))$

Theorem :3.5
1) Finite union of fuzzy supra α-open sets is a fuzzy supra α-open set.
2) Finite intersection of fuzzy supra α-closed sets is a fuzzy supra α- closed set.

Proof :

1) Let f and g be two fuzzy supra α-open sets. Then

$f \leq int^* [cl^*(int^*(f))]$

and $g \leq int^* [cl^*(int^*(g))]$

implies $f \vee g \leq int^* [cl^*(int^*(f \vee g))]$

∴ Finite union of fuzzy supra α-open sets is a fuzzy supra α-open set

2) From (1)

$f \vee g \leq int^* [cl^*(int^*(f \vee g))]$

By taking complement

$(f \vee g)^c \geq [int^* [cl^*(int^*(f \vee g))]]^c$

$f^c \wedge g^c \geq cl^*[(cl^*(int^*(f \vee g))^c]$

$\geq cl^*(int^*[(int^*(f \vee g))^c])$

$\geq cl^*(int^*(cl^*[(f \vee g)^c]))$

$\geq cl^*(int^*(cl^*(f^c \wedge g^c)))$

∴ Finite intersection of fuzzy supra α-closed sets is always a fuzzy supra α- closed set.

Definition :3.6

Let (X,δ^*) be a fuzzy supra topological space and f be a fuzzy set in X, then the fuzzy supra α-closure and fuzzy supra α- interior of f defined respectively as

$\alpha cl^*(f) = \wedge \{ g / g$ is a fuzzy supra α-closed set in X and $f \leq g\}$

$\alpha int^*(f) = \vee \{ g / g$ is a fuzzy supra α-open set in X and $g \leq f\}$

Remark :3.7

It is obvious that $\alpha int^*(f)$ is a fuzzy supra α-open set and $\alpha cl^*(f)$ is a fuzzy supra α-closed set.

Theorem :3.8

For any fuzzy set f in a fuzzy supra topological space(X,δ^*),

1) $[\alpha int^*(f)]^c = \alpha cl^*(f^c)$
2) $[\alpha cl^*(f)]^c = \alpha int^*(f^c)$

Proof :

1) consider

$\alpha int^*(f) = \vee \{ g / g$ is a fuzzy supra α-open set in X and $g \leq f\}$

$[\alpha int^*(f)]^c = 1 - \vee \{ g / g$ is a fuzzy supra α-open set in X and $g \leq f\}$

$= \wedge \{ g^c / g^c$ is a fuzzy supra α-closed set in X and $g^c \geq f^c\}$

$= \alpha cl^*(f^c)$

2)

$\alpha cl^*(f) = \wedge \{ g / g$ is a fuzzy supra α-closed set in X and $f \leq g\}$

$[\alpha cl^*(f)]^c = 1 - \wedge \{ g / g$ is a fuzzy supra α-closed set in X and $f \leq g\}$

$= \vee \{ g^c / g^c$ is a fuzzy supra α-open set in X and $f^c \geq g^c \}$

$= \alpha int^*(f^c)$

Theorem :3.9

For any fuzzy sets f and g in fuzzy supra topological space(X,δ^*)

1) $\alpha int^*(f) \vee \alpha int^*(g) = \alpha int^*(f \vee g)$
2) $\alpha cl^*(f) \wedge \alpha cl^*(g) = \alpha cl^*(f \wedge g)$

Proof :

Let $\alpha int^*(f) = \vee \{ h/ h$ is a fuzzy supra α-open set in X and $h \leq f\}$

$\alpha int^*(g) = \vee \{ k /k$ is a fuzzy supra α-open set in X and $k \leq g\}$

then the union of these sets

$\alpha int^*(f) \vee \alpha int^*(g) = \vee \{ h \vee k /h \vee k$ is a fuzzy supra α-open set in X and $h \vee k \leq f \vee g\}$

(if $j = h \vee k$)

$= \vee \{ j / j$ is a fuzzy supra α-open set in X and $j \leq f \vee g\}$

$= \alpha int^*(f \vee g)$

2) Let $\alpha cl^*(f) = \wedge \{ h/ h$ is a fuzzy supra α-closed set in X and $f \leq h\}$

$\alpha cl^*(g) = \wedge \{ k / k$ is a fuzzy supra α-closed set in X and $g \leq k\}$

then the intersection of these sets

$\alpha cl^*(f) \wedge \alpha cl^*(g) = \wedge \{ h \wedge k /h \wedge k$ is a fuzzy supra α-closed set in X and $f \wedge g \leq h \wedge k \}$

(if $j = h \wedge k$)

$= \wedge \{ j / j$ is a fuzzy supra α- closed set in X and $f \wedge g \leq j \}$

$= \alpha cl^*(f \wedge g)$

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